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$\alpha g\omega$ - closed sets in topological spaces

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ABSTRACT

New classes of sets called αgω-closed sets and αgω -open sets are introduced and study some of their properties. Moreover the notions of αgω-Continuity and αgω-irresolute are introduced and study some of their properties.

Keywords: *αgω -closed set, αgω -open set, αgω -continuous function, αgω -irresolute.*

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1. INTRODUCTION

The concept of generalized closed sets was introduced by Levine [4]. Veerakumar introduced several generalized closed sets [15,16,17].Sheik John [14] defined ω-closed sets. We introduced αgω-closed sets and studied some of their properties by using ω-closed sets in this paper.

2. PRELIMINARIES

Throughout this paper we denote a topological space by (X, τ) or simply by X, when there is no possibilities of confusion. Let A be a subset of a space X. The closure of A (resp. preclosure and semiclosure) is the intersection of all closed sets that contain A and is denoted by cl(A)[resp.pcl(A) and scl(A)]. A^c denotes the complement of A in X. The following definitions are useful in the sequel.

Definitions 2.1: A subset of a space (X, τ) is called

1. semi open set [5] if A⊆cl (Int(A)) and a semiclosed set int(cl(A))⊆A.
2. preopen set [8] if A⊆ int (cl(A)) and a preclosed set if cl(int(A)) ⊆A (9).
3. α-open set [9] if A ⊆ int (cl(int(A)) and an α-closed set if cl(int(cl(A)) ⊆A.
4. semi preopen set[1](=β-open)if A⊆cl(int(A))and a semi pre closed set(=β-closed) if int(cl(int(A)) ⊆A.

Definition 2.2: A subset A of a space (X, τ) is called a generalized closed (briefly g-closed) [4] set if cl(A)⊆U whenever A⊆U and U is open in (x, τ); the complement of a g-closed set is called a g-open set.

i) An α-generalised closed (briefly αg-closed) [6] set if cl(A)⊆U whenever A⊆U and U is open in (X, τ); the complement of an αg-closed set is called and αg-open set.

ii) $g^{\#}$ closed set[18] if cl(A)⊆U whenever A⊆U and U is αg-open in (X, τ)

iii) g^* closed set [15] if cl(A)⊆U whenever A⊆U and U is g-open in (X, τ)

iv) gsp-closed set [2] if spcl(A)⊆U whenever A⊆U and U is open in (X, τ)

v) ή-closed set [12] if pcl(A)⊆U whenever A⊆U and U is ω-open in (X, τ)

vi) g^* p-closed set[16] if pcl(A)⊆U whenever A⊆U whenever A⊆U and U is g-open in (X, τ)

vii) ω-closed [17] if cl(A)⊆U whenever A⊆U and U is semi open in (X, τ).

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viii) gp-closed set [10] if αcl(A)⊆int (U) whenever A⊆U and U is α-open in (X, τ).

ix) αgp-closed set[11] if αcl(A)⊆U whenever A⊆U and U is pre-open in (X, τ).

3. BASIC PROPERTIES OF αgω -CLOSED SET

We introduce the following definition.

Definition 3.1: αgω-closed set: A subset A of a topological space (X, τ) is αgω-closed if αcl(A)⊆U whenever A⊆U and U is ω-open in (X, τ).

Theorem 3.2: If A is closed then A is αgω-closed.

Proof: Given A is closed. Let A⊆ U and U is ω-open. Since A is closed cl(A) =A.

Also we have αcl(A)⊆cl(A).Hence A⊆U ⇒cl(A)⊆U ⇒ αcl(A)⊆U. Thus A is αgω-closed.

The converse need not be true.

(eg) Let X= {a, b, c},τ={X, ø,{a}} clearly (X, τ) is a topology. The subsets {b}, {c}, {a, b} are αgω- closed but they are not closed.

Theorem 3.3: If A is α-closed then A is αgω-closed.

Proof: Let A be α-closed. Thus αcl(A) =A. Let A ⊆U,U is ω-open.

Then αcl(A)⊆U. Thus whenever A⊆U, U is ω-open, αcl(A)⊆U.

Thus A is αgω-closed.

The converse need not be true always.

Let X= {a, b, c}, τ={X, ø, {a, b}} clearly (X, τ) is a topology. The subsets {a, c}, {b, c} are αgω-closed but they are not α-closed.

Theorem 3.4: Every g-closedness is independent of αgω-closedness.

Let us take X= {a, b, c} τ = {X, ø,{a},{a, b}}.The subset {b} is αgω-closed but not g-closed. The subset {a, b} is g-closed in τ = {X, ø, {a}, {a, c}}

But {a, b} is not αgω-closed.

Also consider τ = {X, ø, {a}, {b}, {a, b},{a, c}},the subset {a, c} is g-closed but not αgω- closed.

Thus g-closed sets and αgω-closed sets are independent of each other.

Theorem 3.5: Every αgω-closed set is αg-closed.

Proof: Let A be αgω-closed. Let A⊆U and U is open in (x, τ)

Thus A⊆U and U is ω-open in (X, τ), since every open set is ω-open.

Hence αcl(A)⊆U.

⇒A is αg-closed.

The converse need not be true.

Let X= {a, b, c} and τ={X, ø, {a, c}. Here A= {b} is αg-closed but not αgω-closed.

Theorem 3.6: Every αgp-closed set is αgω-closed.

Proof: Let A be αgp-closed. To prove A is αgω-closed.

Let A ⊆ U, U is ω-open in (X, τ)

Since every ω-open set is pre open, U is preopen.

Hence A⊆U; pre open in (X, τ)

⇒ αcl(A) ⊆U ,since A is αgp-closed.

Hence A is αgω-closed.

Conversely, every αgω-closed set need not be αgp-closed.

Let X= {a, b, c} τ = {X, ø,{a},{a, b}}

Here {a, c} is αgω-closed but {a, c} is not αgp-closed.

Theorem 3.7: Every αgω-closed set is gsp-closed.

Proof: Every αgω-closed set is αg-cld & Every αg-closed set is gsp-closed.

Hence every αgω-closed set is gsp-closed.

Theorem 3.8: If A and B are αgω-closed then AUB is αgω-closed in x

Proof: Let A and B be αgω-closed.

Let AUB⊆U, U is ω-open.⇒A⊆U and B⊆U. ⇒αclA⊆U and αclB⊆U

We have αcl(AUB)=(αclA)U(αclB)

Thus αcl(AUB)=(αclA)U(αclB)⊆U

⇒AUB is αgω-closed.

Remark 3.9: If A and B are αgω-closed then A∩B need not be αgω-closed.

Let X= {a, b, c}, τ={X, ø, {c}}. (X, τ) is a topology.

The subsets {a, c} and {b, c} are αgω-closed but {a, c}∩{b, c}={c} is not αgω-closed.

Theorem 3.10: Every $g^{\#}$ closed set is αgω-closed.

Proof: Let A be $g^{\#}$ closed set. To prove A is αgω-closed.

Let A⊆ U, U is ω-open ⇒U is αg-open, Since every ω-open set is αg-open.

⇒cl(A)⊆U, Since A is $g^{\#}$ closed.

⇒αcl(A)⊆U, Since αcl(A)⊆cl(A).

Hence A is αgω-closed.

Theorem 3.11: Every g^{*} closed set is αgω-closed.

Proof: Let A be a g^{*}closed set.

Let A⊆ U, U is ω-open. ⟹ U is g-open, Since every ω-open set is g-open.[14]

Hence cl(A) ⊆U, Since A is g^{*} closed. Thus αcl(A)⊆U

$\Rightarrow$ A is αgω-closed.

Conversely, Every αgω-closed set need not be g^* closed.

Let X= {a, b, c}, τ={X, ø, {a}}.Here {b} is αgω-closed.

But cl{b}={b, c} $\not\subset$ {b} which is g-open.

$\Rightarrow$ {b} is not g^* closed set.

Thus converse is not true.

Theorem 3.12: Every $\hat{\eta}$ closed is αgω-closed

Proof: Let A⊆ U, U is ω-open $\Rightarrow$ pcl(A)⊆U.

Thus αcl(A)⊆A , since αcl(A) $\subseteq$ pcl(A). Hence A is αgω-closed.

Thus every $\hat{\eta}$ closed set is αgω-closed.

Converse need not be true. (eg) Let X= {a, b, c}, τ={X, ø, {a}}

Here the set {a, b} is αgω-closed but not $\hat{\eta}$ closed.

Hence every αgω-closed set need not be $\hat{\eta}$ closed.

Theorem 3.13: A set A is αgω-closed and A⊆B ⊆ αcl(A) then B is αgω-closed.

Proof: Let B⊆U, U is ω-open. $\Rightarrow$ A⊆U. $\Rightarrow$ αcl(A)⊆U, Since A is αgω-closed.

Also B⊆ αcl(A). $\Rightarrow$ αcl(B) ⊆ αcl(A) ⊆ U, Since αcl(B) $\subseteq$ B.

$\Rightarrow$ αcl(B)⊆U. Hence B is αgω-closed.

Conversely, any subset of a αgω-closed need not be αgω-closed

Let X= {a, b, c}, τ={X, ø, {b}, {a, b}, {b, c}}. Here {b, c} is αgω closed but {b} is not αgω closed.

Hence the subset of a αgω-closed set need not be αgω-closed.

Theorem 3.14: Every αgω-closed set is gp-closed.

Proof: Let A be αgω-closed set. Let A⊆U, U is open $\Rightarrow$ U is ω-open. Since every open set is ω-open.

Thus αcl(A)⊆U, Since A is αgω closed.

$\Rightarrow$ Pcl(A) ⊆αcl(A) ⊆U

(ie) Pcl(A) ⊆U $\Rightarrow$ A is gp-closed

Converse need not be true, every gp-closed need not be αgω-closed. Let τ={X, ø,{a}},{a} is gp -closed but not αgω-closed.

Theorem 3.15: If A is ω-open and αgω-closed subset of a topological space x, then A is α-closed.

Proof: Let A be ω-open and αgω-closed.

Since A⊆A, αclA⊆A Since A is αgω-cld.

$\Rightarrow$ A is α-closed, Since $A \subseteq \alpha clA$.

4. PROPERTIES OF αgω -CLOSED SETS

Definition 4.1: Let A be a subset of a topological space (X, τ).Then the pre kernel of A (briefly pker(A)) is the intersection of all open supersets of A.

Theorem 4.2: A subset A of a topological space X is αgω closed if and only if $\alpha cl(A) \subseteq pker(A)$.

Proof: Let A be a subset of X which is αgω-closed.

To Prove $\alpha cl(A) \subseteq pker(A)$.If $\alpha cl(A) \not\subseteq pker(A)$.

Let $x \in \alpha cl(A) \Rightarrow x \notin pker(A)$.Then there exist an open set $U \supseteq A$ such that $x \notin U$.

Since every open set is ω open, U is ω-open.

$\Rightarrow \alpha cl(A) \subseteq U$, Since A is αgω-closed. Hence $x \notin \alpha clA$, Since $x \notin U$.

This is $\Rightarrow\Leftarrow$ to $x \in \alpha cl(A)$.

Thus $\alpha cl(A) \subseteq pker(A)$.

Conversely, et A be a subset of X such that $A \subseteq U$, U is ω-open.Clearly $\alpha cl(A) \subseteq pker(A) \subseteq A$. Hence the result follows.

Theorem 4.3: A subset A of X is αgω-open if and only if $\alpha\ Int(A) \supseteq F$ whenever $A \supseteq F$ and F is ω-closed.

Proof: Given A is αgω-open. Let $A \supseteq F$ and F is ω-closed.(ie) $A^c \subseteq F^c$ and F^c is ω-open

$\Rightarrow \alpha cl(A^c) \subseteq F^c$ Since A^c is αgω closed. $\Rightarrow (\alpha\ int\ A)^c \subseteq F^c. \Rightarrow \alpha\ int(A) \supseteq F$.

Conversly, let $\alpha\text{-}int(A) \supseteq F$ whenever $A \supseteq F$ and F is ω-closed. Let $A^c \subseteq U$, U is ω-open.

$\Rightarrow A \supseteq U^c, U^c$ is ω-closed. $\Rightarrow \alpha\text{-}int(A) \supseteq U^c. \Rightarrow (\alpha\text{-}int(A))^c \subseteq U$ (ie)$\alpha cl(A^c) \subseteq U. \Rightarrow A^c$ is αgω-closed.

Hence A is αgω-open.

5. Properties of R-closed sets

Definition 5.1: αgω-continuous functions: A function $f:(X, \tau) \rightarrow (Y, \sigma)$ is αgω-continuous if

$f^{-1}(B)=A$ Where B is open in Y and A is αgω-open in X.

(ie) Inverse image of every open set in Y is αgω-open X.

Definition 5.2: *αgω*- irresolute: A function $f :(X, \tau) \rightarrow (Y, \sigma)$ is said to be αgω-irresolute if the inverse image of every αgω-open set in Y is αgω-open in X.

Theorem 5.3: Every αgω- irresolute function is αgω-continuous.

Since every closed set is αgω-closed, the proof follows.

Conversly, Every αgω-continuous functions need not be αgω-irresolute.

Let X= {a, b, c} and Y={x, y, z},τ={X, ø,{b},{a, b},{b, c}} and σ={Y, ø,{z},{x, z},{y, z}}

Consider f: $X \rightarrow Y$ defined by f(a) = y, f(b) = x, f(c) = z

Here $f^{-1}(\{x\}) =\{b\}$ which is not αgω-closed in X. But f is αgω-continuous.

Theorem 5.4: If $f :(X, \tau) \rightarrow (Y, \sigma)$ is αgω-irresolute and $g:(Y, \sigma) \rightarrow (Z, \delta)$ is αgω-continuous then gof is αgω-continuous.

Proof: Let C is closed in Z $\Rightarrow g^{-1}(C) =B$ is αgω-closed in Y $\Rightarrow f^{-1}(B) =A$ is αgω-closed in X.

Hence $f^{-1}(g^{-1}(C))$ =A is αgω-closed in X.

Thus $(gof)^{-1}$ C =A is αgω-closed.

Thus gof is αgω-continuous.

Theorem 5.5: Let A $\subseteq$ Y $\subseteq$ X and A is αgω-closed in X, then A is αgω-closed relative to Y.

Proof: Let A$\subseteq$ U, U is ω-closed

Then U=G $\cap$ Y,G is ω-open in x.

$\therefore$ A $\subseteq$G $\Rightarrow$ αclA$\subseteq$G

$\Rightarrow$ αclA $\cap$ Y$\subseteq$G $\cap$ Y=U.Thus αclA $\cap$ Y$\subseteq$U

$\Rightarrow$ A is αgω-closed relative to Y, Since A is an ω-closed set and F is a closed set then

A $\cap$ F is an ω-closed set.

Theorem 5.6: Let A and B be open subsets of X and αgω-closed subsets in X such that X=A $\cup$ B. Let f :(A, τ/A) $\rightarrow$ (Y, σ) and g : (B,τ/B) $\rightarrow$ (Y, σ) be compatible functions. If f is an αgω-continuous function and g is an αgω-continuous function then its combination f ∇g: (X, τ) $\rightarrow$ (Y, σ) is an αgω- continuous function.

Proof: Let C be a closed subset of (Y, σ). Thus f^{-1} (C) and g^{-1} (C) are αgω-closed subsets (A, τ/A) and (B, τ/B) respectively.

$\Rightarrow$ f^{-1} (C) and g^{-1}(C) are αgω closed in X. Hence $f^{-1}(C) \cup g^{-1}(C)=(f \nabla g)^{-1}(C)$ is αgω-closed in X.

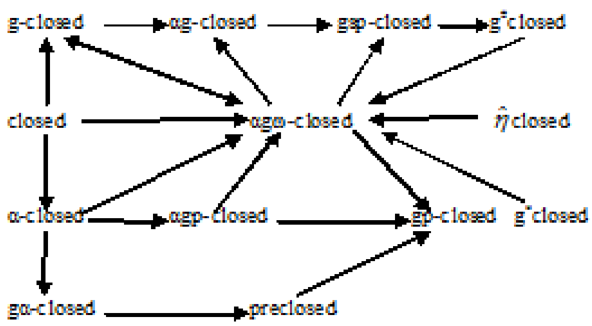
Hence f ∇ g is αgω-continuous.

Theorem 5.7: Every {x} is ω-closed (or) $\{x\}^c$ is αgω-closed.

Proof: Let {x} is not ω-closed in X. $\Rightarrow$ $\{x\}^c$ is not ω-open. $\Rightarrow$ The only ω-open set containing $\{x\}^c$ is X itself. Thus αcl$\{x\}^c$$\subseteq$ X.

$\Rightarrow$ $\{x\}^c$ is αgω-closed.

Remark 5.8: From the above discussions and known results we have the following implications A→B (A↔B) represents A implies B but not conversely.(A and B implies each other).



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