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A FUZZY LOGIC BASED APPROACH FOR BICRITERIA IN N JOBS, 2 MACHINE FLOW SHOP PRODUCTION SCHEDULING

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ABSTRACT

In this paper we consider bicriteria scheduling problem which minimizes the total rental cost of machines with the condition of minimum makespan for n-jobs, 2-machine flowshop scheduling problem. The processing times and setup times of jobs are uncertain that is not known exactly and only estimated values are given. The fuzzy triangular membership function is used to describe uncertain processing times and setup times. Further, the restrictions on job processing are also considered. The objective of the paper is to develop a new heuristic algorithm for minimizing the makespan as well as the rental cost of machines which is simple and straight forward. A numerical example demonstrating the computational process of the projected algorithm is also given.

Keywords: *Fuzzy Processing Time, Fuzzy Setup Time, Rental Cost, Average High Ranking, Utilization Time and Job Block.*

1. INTRODUCTION

The scheduling of jobs and the control of their flow through a production process is essential to modern production/manufacturing companies. Ever since the first results of modern scheduling theory appeared some 50 years ago, scheduling has attracted a lot of attention from both academia and industry. In a general flowshop scheduling problem, n jobs are to be scheduled on m machines in order to optimize some measures of performance. Two machine flow shop scheduling problem has been considered as a major subproblem due to its application in real life. There are the cases where setup times are negligible and therefore could be included in the processing times of jobs. However, in some applications, setups have major impact on the performance measure considered for the scheduling problem so they need to be considered separately. Information about production/ manufacturing processes can be both imprecise and/ or incomplete, or sometimes does not even exist. In these cases, application of standard methods of probability theory becomes difficult and often inappropriate. Fuzzy sets provide an appropriate tool for handling imprecise information. The research work on multi-objective scheduling is a subset, is limited in comparison to single objective. This is despite the fact that in real life the bicriteria and multi-criteria objective problems oftenly occurred. Johnson (1954) whose work is one of the earliest developed a polynomial time algorithm to minimize the makespan in two stage flowshop scheduling problem. Dileepan and Sen (1988) have extensively surveyed the bi-criterion static scheduling research for a single machine. MacCahon and Lee (1990) discussed the job sequencing with fuzzy processing time. Ishibuchi and Lee (1996) addressed the formulation of fuzzy flow shop scheduling problem with fuzzy processing time. Some of the noteworthy approaches are due to Zadeh (1965), Gupta (1975), Maggu and Das (1977), Yager (1981), Marin and Roberto (2001), Yao and Lin (2002), Singh and Gupta (2005), Singh, Sunita and Allawalia (2008).

Gupta and Sharma (2011) studied bicriteria in n×2 flow shop scheduling under specified rental policy in which processing time and setup time are associated with probabilities. As the fuzzy approach seems much more natural, we investigate its potential by solving the flowshop problem in real life situations. Our study recommends the use of triangular fuzzy membership functions to represent the uncertainty involved processing times and setup times.

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2. ROLE OF FUZZY LOGIC IN SCHEDULING

A fuzzy system can be thought of an attempt to understand a system for which no model exists, and it does so with the information that can be uncertain in a sense of being vague, or fuzzy, or imprecise, or altogether lacking. From this angle, fuzzy logic is a method to formalize the human capacity of imprecise reasoning. Such reasoning represents the human ability to reason approximately and judge under uncertainity. In fuzzy logic all truths are partial or approximate. In this sense the reasoning has also been termed interpolative reasoning, where the process of interpolating between the binary extremes of truth and false is represented by the ability of fuzzy logic to encapsulate partial truths.

Scheduling is a enduring process where the existence of real time information frequently forces the review and modification of pre-established schedules. The real world is complex; complexity in the world generally arises from uncertainty. From this prospective, the concept of fuzzy environment is introduced in the theory of scheduling.

All information contained in a fuzzy set is described by its membership function. The triangular membership functions are used to represent fuzzy processing times and fuzzy setup times in our algorithm. The membership value of the x denoted by $\mu_x, x \in R^+$, can be calculated according to the formula

$$\mu_x = \begin{cases} 0; x \le a \\ \dfrac{x-a}{b-a}; a \le x \le b \\ \dfrac{c-x}{c-b}; b < x < c \\ o; x \ge c \end{cases}$$

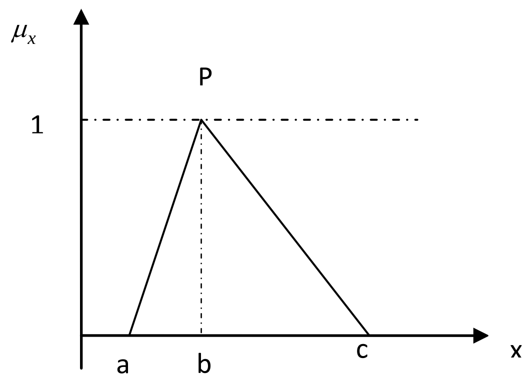

Fig. 1: Triangular membership function

Figure 1 shows the triangular membership function of a fuzzy set $\tilde{P}$, $\tilde{P}$ =(a, b, c). The membership value reaches the highest point at '*b*', while '*a*' and '*c*' denote the lower bound and upper bound of the set $\tilde{P}$ respectively.

To find the optimal sequence, the expected processing time of the jobs are calculated by using Yager's(1981) average high ranking formula (AHR) = $h(A) = \dfrac{3b+c-a}{3}$.

The following are the basic operations that can be performed on triangular fuzzy numbers:

If $A_1 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1})$ and $A_2 = (m_{A_2}, \alpha_{A_2}, \beta_{A_2})$ be the two triangular fuzzy numbers, then

(i) $A_1 + A_2 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1}) + (m_{A_2}, \alpha_{A_2}, \beta_{A_2}) = (m_{A_1} + m_{A_2}, \alpha_{A_1} + \alpha_{A_2}, \beta_{A_1} + \beta_{A_1})$

(ii) $A_1 - A_2 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1}) - (m_{A_2}, \alpha_{A_2}, \beta_{A_2}) = (m_{A_1} - m_{A_2}, \alpha_{A_1} - \alpha_{A_2}, \beta_{A_1} - \beta_{A_2})$

If the following condition is satisfied $DP\left(\tilde{A}_1\right) \geq DP\left(\tilde{A}_2\right)$, where $DP(\tilde{A}_1) = \dfrac{\beta_{A_1} - m_{A_1}}{2}$ and $DP(\tilde{A}_2) = \dfrac{\beta_{A_2} - m_{A_2}}{2}$. H, say ere DP denotes difference point of a Triangular fuzzy number.

Otherwise; $A_1 - A_2 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1}) - (m_{A_2}, \alpha_{A_2}, \beta_{A_2}) = (m_{A_1} - \beta_{A_2}, \alpha_{A_1} - \alpha_{A_2}, \beta_{A_1} - m_{A_2})$
(iii) $kA_1 = k(m_{A_1}, \alpha_{A_1}, \beta_{A_1}) = (km_{A_1}, k\alpha_{A_1}, k\beta_{A_1})$; if k>0.
(iv) $kA_1 = k(m_{A_1}, \alpha_{A_1}, \beta_{A_1}) = (k\beta_{A_1}, k\alpha_{A_1}, km_{A_1})$; if k<0.

3. PROBLEM FORMULATION

The following notations will be used through the present paper:

S : Sequence of jobs 1, 2, 3,…., n
S_k : Sequence obtained by applying Johnson's procedure, k = 1, 2, 3, -----
M_j : Machine j, j= 1,2
a_{ij} : Fuzzy Processing time of i^{th} job on machine M_j
s_{ij} : Fuzzy Setup time of i^{th} job on machine M_j
A'_{ij} : AHR of expected flow time of i^{th} job on machine M_j
Lj(Sk) : The latest time when machine Mjis taken on rent for sequence Sk
$t_{ij}(S_k)$: Completion time of i^{th} job of sequence S_k on machine M_j
t'_{ij} : Completion time of i^{th} job of sequence S_k on machine M_j when machine M_j start processing jobs at time $L_j(S_k)$
$I_{ij}(S_k)$: Idle time of machine M_j for job i in the sequence S_k
$U_j(S_k)$: Utilization time for which machine M_j is required, when M_j starts processing jobs at time $L_j(S_k)$
$R(S_k)$:Total rental cost for the sequence S_k of all machine
β : Equivalent job for job – block
C_i : Rental cost of i^{th} machine
$CT(S_i)$:Total completion time of the jobs for sequence S_i

Definition 1. Completion time of i^{th} job on machine M_j is denoted by t_{ij} and is defined as:

$$t_{ij} = \max(t_{i-1,j}, t_{i,j-1}) + a_{ij} + s_{(i-1),j} \text{ for } j \geq 2.$$

where $a_{i,,j}$ = Fuzzy processing time of i^{th} job on j^{th} machine
$s_{i,j}$ = Fuzzy setup time of i^{th} job on j^{th} machine

Definition 2. Completion time of i^{th} job on machine M_j when M_j starts processing jobs at time L_j is denoted by t'_{ij} and is defined as

$$t'_{i,j} = L_j + \sum_{k=1}^{i} a_{k,j} + \sum_{k=1}^{i-1} s_{k,j} = \sum_{k=1}^{i} I_{k,j} + \sum_{k=1}^{i} a_{k,j} + \sum_{k=1}^{i-1} s_{k,j}$$

Also $t'_{i,j} = \max(t_{i,j-1}, t'_{i-1,j}) + a_{i,j} + s_{i-1,j}$.

Further, The machines will be taken on rent as and when they are required and are returned as and when they are no longer required. .i.e. the first machine will be taken on rent in the starting of the processing the jobs, 2^{nd} machine will be taken on rent at time when 1^{st} job is completed on 1^{st} machine.

Let some job i (i = 1, 2,……..,n) are to be processed on two machines M_j (j = 1,2) under the specified rental policy. Let a_{ij} be the fuzzy processing time of i^{th} job on j^{th} machine and s_{ij} be the setup time of i^{th} job on j^{th} machine which are described by triangular fuzzy numbers. Let $A'_{i,j}; i = 1, 2, ..., n,\ j = 1, 2$ be the average high ranking (AHR) of the expected flow times on two machines M_1 and M_2. Our aim is to find the sequence $\{S_k\}$of the jobs which minimize the rental cost of the machines while minimizing total elapsed time.

The mathematical model of the problem in matrix form can be stated as:

Jobs	Machine M_1		Machine M_2	
i	a_{i1}	s_{i1}	a_{i2}	s_{i2}
1	a_{11}	s_{11}	a_{12}	s_{12}
2	a_{21}	s_{21}	a_{22}	s_{22}
3	a_{31}	s_{31}	a_{32}	s_{32}
-	-	-	-	-
n	a_{n1}	s_{n1}	a_{n2}	s_{n2}

Table: 1

Mathematically, the problem is stated as:

Minimize $U_j\left(S_k\right)$ and Minimize $R\left(S_k\right)=t_{n,1}(S_k)\times C_1+U_j\left(S_k\right)\times C_2$

Subject to constraint: Rental Policy (P)

Our objective is to minimize rental cost of machines while minimizing the total elapsed time.

4. THEOREM

The processing of jobs on M_2 at time $L_2=\sum_{i=1}^{n} I_{i,2}$ keeps $t_{n,2}$ unaltered:

Proof. Let $t'_{i,2}$ be the completion time of i^{th} job on machine M_2 when M_2 starts processing of jobs at L_2. We shall prove the theorem with the help of mathematical induction.

Let P(n) : $t'_{n,2}=t_{n,2}$

Basic step: For n = 1, j =2;

$$t'_{1,2}=L_2+\sum_{k=1}^{1} a_{k,2}+\sum_{k=1}^{1-1} s_{k,2}=\sum_{k=1}^{1} I_{k,2}+\sum_{k=1}^{1} a_{k,2}+\sum_{k=1}^{1-1} s_{k,2}$$

$$=\sum_{k=1}^{1} I_{k,2}+a_{1,2}=I_{1,2}+a_{1,2}=a_{1,1}+a_{1,2}=t_{1,2}$$

$\therefore$ P(1) is true.

Induction Step: Let P(m) be true, i.e., $t'_{m,2}=\text{t}_{m,2}$

Now we shall show that P (m+1) is also true, i.e., $t'_{m+1,2}=t_{m+1,2}$

$$\begin{aligned}\text{Since } t'_{m+1,2}&=\max(t_{m+1,1},t'_{m,2})+a_{m+1,2}+s_{m,2}\\&=\max\left(t_{m+1,1},L_2+\sum_{i=1}^{m} a_{i,2}+\sum_{i=1}^{m-1} s_{i,2}\right)+a_{m+1,2}+s_{m,2}\\&=\max\left(t_{m+1,1},\left(\sum_{i=1}^{m} I_{i,2}+\sum_{i=1}^{m} a_{i,2}+\sum_{i=1}^{m-1} s_{i,2}\right)+I_{m+1}\right)+a_{m+1,2}+s_{m,2}\end{aligned}$$

$$\begin{aligned}t_{n,2}=L_2+\sum_{i=1}^{n} a_{i,2}+\sum_{i=1}^{n-1} s_{i,2}&=\max\left(t_{m+1,1},t_{m,2}+I_{m+1}\right)+a_{m+1,2}+s_{m,2}\\&=\max\left(t_{m+1,1},t'_{m,2}+\max\left(\left(t_{m+1,1}-t_{m,2}\right),0\right)\right)+a_{m+1,2}+s_{m,2}\text{ (By Assumption)}\\&=\max\left(t_{m+1,1},t_{m,2}\right)+a_{m+1,2}+s_{m,2}\\&=t_{m+1,2}\end{aligned}$$

Therefore, *P (m+1)* is true whenever *P(m)* is true.

Hence by Principle of Mathematical Induction P (n) is true for all n. i.e. $t'_{n,2} = t_{n,2}$ for all n.

5. ALGORITHM

The following algorithm is proposed is to optimize the bicriteria taken as minimization of the rental cost of the machine taken on rent under a specified rental policy with minimum makespan.

Step 1: Define the two fictitious machines G and H with processing time A_{i1} and A_{i2} defined as follows: $A_{i1} = a_{i1} - s_{i2}; A_{i2} = a_{i2} - s_{i1}$

Step 2: Find the average high ranking (AHR) $A'_{ij}, i = 1, 2, ..., n, j = 1, 2$ of expected flow time for all jobs on two machines M_1 and M_2

Step 3: Take equivalent job β(*k*,*m*) and calculate the processing time $A'_{\beta 1}$ and $A'_{\beta 2}$ on the guide lines of Maggu and Das (1977) as follows

$$A'_{\beta 1} = A'_{k1} + A'_{m1} - \min(A'_{m1}, A'_{k2})$$
$$A'_{\beta 2} = A'_{k2} + A'_{m2} - \min(A'_{m1}, A'_{k2})$$

Step 4: Define a new reduced problem with the processing time A'_{i1} and A'_{i2} as defined in step 2 and jobs (k,m) are replaced by single equivalent job β with processing time $A'_{\beta 1}$ and $A'_{\beta 2}$ as defined in step 3.

Step 5: Using Johnson's technique [1954] obtain all the sequences S_k having minimum elapsed time. Let these be S_1, S_2,.....

Step 6: Compute total elapsed time t_{n2} (S_k), k = 1, 2, 3,...., by preparing in-out tables for S_k.

Step 7: Compute $L_2(S_k)$ for each sequence S_k as follows

$$L_2(S_k) = t_{n,2}(S_k) - \sum_{i=1}^{n} a_{i,2}(S_k) - \sum_{i=1}^{n-1} s_{i,2}(S_k)$$

Step 8: Find utilization time of 2nd machine for each sequence S_k as

$$U_2(S_k) = t_{n,2}(S_k) - L_2(S_k)$$

Step 9: Find minimum of $\{U_2(S_k)\}; k = 1, 2, 3, ...$ Let it be for sequence S_p. Then S_p is the optimal sequence and minimum rental cost for the sequence S_p is $R(S_p) = t_{n,1}(S_p) \times C_1 + U_2(S_p) \times C_2$,where C_1 and C_2 are the rental cost per unit time of 1st and 2nd machine respectively.

6. NUMERICAL ILLUSTRATION

Consider 5 jobs, 2 machine flow shop problem with processing time and setup time described by triangular fuzzy numbers as given in the following table and jobs 2,5 are to be processed as a group job (2,5). The rental cost per unit time for machines M_1 and M_2 are 4 units and 3 units respectively. Our objective is to obtain optimal schedule to minimize the total rental cost of the machines, under the rental policy P.

Jobs	**Machine M_1**		**Machine M_2**	
i	a_{i1}	s_{i1}	a_{i2}	s_{i2}
1	(12,14,16)	(4,5,6)	(5,7,9)	(4,5,6)

2	(14,16,18)	(2,3,4)	(4,6,8)	(3,4,5)
3	(8,10,25)	(4,5,6)	(5,7,9)	(2,3,4)
4	(11,13,16)	(2,3,4)	(3,5,7)	(3,4,5)
5	(9,11,14)	(4,6,7)	(5,8,10)	(4,5,6)

Table: 2

Solution: As per step 1: The expected flow times for the two machines G and H are

Jobs	**Machine G**	**Machine H**
I	A_{i1}	A_{i2}
1	(8,9,10)	(1,2,3)
2	(11,12,13)	(2,3,4)
3	(6,7,21)	(1,2,3)
4	(8,9,11)	(1,2,3)
5	(5,6,8)	(1,2,3)

Table: 3

As per step 2: AHR's of expected flow times are

Jobs	**Machine M_1**	**Machine M_2**
i	A'_{i1}	A'_{i2}
1	29/3	8/3
2	38/3	11/3
3	36/3	8/3
4	30/3	8/3
5	21/3	8/3

Table: 4

As per step 3: Here β = (2,5)
]

$$A'_{\beta 1} = 38/3 + 21/3 - 11/3 = 48/3$$

$$A'_{\beta 2} = 11/3 + 8/3 - 11/3 = 8/3$$

Jobs	**Machine M_1**	**Machine M_2**
i	A'_{i1}	A'_{i2}
1	29/3	8/3
β	48/3	8/3
3	36/3	8/3
4	30/3	8/3

Table: 5

As per step 5: Using Johnson's method optimal sequence is

S = 1– 4 – 3 – β i.e. 1– 4 – 3– 2 – 5

As per step 6: The In – Out table for the sequence S is as shown in table 6

Jobs	Machine M_1	Machine M_2
	In- out	In – out
1	(0,0,0) – (12,14,16)	(12,14,16) – (17,21,25)
4	(16,19,22) – (27,32,38)	(27,32,38) – (30,37,45)
3	(29,35,42) – (37,45,67)	(37,45,67) – (42,52,76)
2	(41,50,73) – (55,66,91)	(55,66,91) – (59,72,99)
5	(57,69,95) – (66,80,109)	(66,80,109) – (71,88,119)

Table: 6

Total elapsed time $t_{n,2}(S)$ = (71,88,119)

As per step 7: The latest time at which machine M_2 is taken on rent

$$L_2(S) = t_{n,2}(S) - \sum_{i=1}^{n} a_{i,2}(S) - \sum_{i=1}^{n-1} s_{i,2}(S) = (37, 39, 56)$$

As per step 8: The utilization time of machine M_2 is

$$U_2(S) = t_{n,2}(S) - L_2(S) = (34, 49, 63)$$

The Bi-objective In – Out table is as follows

Jobs	Machine M_1	Machine M_2
i	In – Out	In – Out
1	(0,0,0) – (12,14,16)	(37,39,56) – (42,46,65)
4	(16,19,22) – (27,32,38)	(46,51,71) – (49,56,78)
3	(29,35,42) – (37,45,67)	(52,60,83) – (57,67,92)
2	(41,50,73) – (55,66,91)	(59,70,96) – (63,76,104)
5	(57,69,95) – (66,80,109)	(66,80,109) – (71,88,119)

Table: 7

Total Minimum Rental Cost = $R(S) = t_{n,1}(S) \times C_1 + U_2(S) \times C_2$ = (366,467,625)

7. CONCLUSION

If the machine M_2 is taken on rent when it is required and is returned as soon as it completes the last job, the starting of processing of jobs at time $L_2(S) = t_{n,2}(S) - \sum_{i=1}^{n} a_{i,2}(S) - \sum_{i=1}^{n-1} s_{i,2}(S)$ on M_2 will, reduce the idle time of all jobs on it. Therefore total rental cost of M_2 will be minimum. Also rental cost of M_1 will always be minimum as idle time of M_1 is always zero. The study may further be extending by introducing the concept of transportation time, Weightage of jobs, Breakdown Interval etc.

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