International Journal of Mathematical Archive-3(9), 2012, 3427-3436
Available online through www.ijma.info ISSN 2229 – 5046

FREE CONVECTIVE VISCO-ELASTIC BOUNDARY LAYER FLOW PAST AN INCLINED PERMEABLE PLATE IN SLIP FLOW REGIME

Rita Choudhury[1*], Hillol Kanti Bhattacharjee[2] and Debasish Dey[3]

[1, 2]Department of Mathematics, Gauhati University, Guwahati-781 014, India

[3]Department of Mathematics, Girijananda Chowdhury Institute of Management & Technology, Guwahati-781 017, India

(Received on: 14-08-12; Revised & Accepted on: 03-09-12)

ABSTRACT

An analytical study of two dimensional free convective boundary layer flow of an electrically conducting visco-elastic fluid past an inclined porous plate in slip flow regime has been investigated. The visco-elastic fluid flow is characterized by Walters liquid (Model B′). The suction of the plate is assumed to be constant. A magnetic field of strength B_0 is applied normal to the plate. Let x-axis be taken along the inclined plate and y-axis be taken perpendicular to the plate. Navier's generalized boundary conditions for slip flow regime have been used. The perturbation scheme has been used to solve the governing equations of the fluid motion. The approximate solutions for velocity and temperature fields have been derived. Results are discussed for the flow past a heated plate (G_r < 0) and flow past a cooled plate (G_r > 0). The velocity profiles have been shown graphically and the shearing stress at the plate is given in tabular form for various values of the visco-elastic parameter with the combination of other flow parameters involved in the solution.

Keywords: *Walters liquid (Model B′), slip-flow regime, inclined plate, Grahof number, perturbation scheme.*

2000 Mathematics Subject Classification: *76A05, 76A10.*

1. INTRODUCTION:

The analysis of visco-elastic fluid flow is one of the recent interdisciplinary activities concerning engineering and technology developments. The complex stress-strain relationships of visco-elastic fluid flow mechanisms are used in geophysics, chemical engineering (absorption, filtration), petroleum engineering, hydrology, soil-physics, bio-physics, paper and pulp technology. The viscosity of the visco-elastic fluid enables the physics of the energy dissipated during the flow and its elasticity analyses the energy stored during the flow. The mechanisms of visco-elastic boundary layer flow are used in various manufacturing processes such as fabrication of adhesive tapes, extrusion of plastic sheets, coating layers into rigid surfaces etc. Various blood flow problems are also explained by using the visco-elastic boundary layer theory.

Analytical studies of forced, free and mixed convection flow of a viscous incompressible fluid along a vertical surface in the absence of magnetic field have been conducted by Sparrow and Gregg [1], Merkin [2], Loyed and Sparrow [3]. A group of continuous transformations computation for the boundary layer equations between the similarity regimes for mixed convection flow has been introduced by Hunt and Wilks [4]. They have recognized $\zeta(= G_r/Re_x,$ where G_r is the local Grashoff number and Re_x is the local Reynolds number), a governing parameter for the flow from a vertical plate. Forced convection exists when ζ goes to zero, which occurs at the leading edge and the free convection limit, can be reached at large values of ζ. Perturbation solutions have been developed in both the cases, since both the forced convection and free convection limits admit similarity solution. Empirical patching of two perturbation solutions has also been carried out to provide a uniformly valid solution by Raju *et al.* [5] which covers the whole range of the values of ζ. They have obtained a finite difference solution applying an algebraic transformation $z = 1/(1+\zeta^2)$. Tingwi *et al.* [6] have also studied the effect of forced and free convection along a vertical flat plate with uniform heat flux by considering that the buoyancy parameter ζ_p to be $G_r/Re_x^{5/2}$. The solutions were obtained for the small buoyancy parameter by considering the perturbation technique.

Corresponding author: *Rita Choudhury[1*]*
[1, 2]Department of Mathematics, Gauhati University, Guwahati-781 014, India

The influence of magnetic field on an electrically conducting viscous incompressible fluid is extensively used in many applications such as extrusion of plastics in the manufacture of Rayon and Nylon, purification of crude oil, textile industry etc. Because of its application for MHD natural convection flow in the nuclear engineering where convection aids the cooling of reactors, the natural convection boundary layer flow of an electrically conducting fluid up a hot vertical wall in the presence of strong magnetic field has been studied by several authors such as Sparrow and Cess [7], Riley [8] and Kuiken [9]. Simultaneous occurrence of buoyancy and magnetic field forces in the flow of an electrically conducting fluid up a hot vertical flat plate in the presence of a strong cross magnetic field was studied by Singh and Cowling [10] who have shown that regardless of strength of applied magnetic field there will always be a region in the neighbourhood of the leading edge of the plate where electromagnetic forces are unimportant. Crammer and Pai [11] have presented a similarity solution for the above problem with uniform heat flux by formulating it in terms of both a regular and inverse series expansions of characterizing co-ordinate that provided a link between the similarity state closed to and far from the leading edge. The combined effect of free and forced convection with uniform heat flux in the presence of strong magnetic field has been studied by Hossain and Ahmed [12]. Hossain *et al.* [13] have studied the convection flow from an isothermal plate inclined at a small angle to the horizontal. Wilks [14] has investigated the MHD free convection flow about a semi infinite vertical plate in a strong cross magnetic field by using only series solution method. Ahmed and Sarkar [15] have analyzed the MHD natural convection flow of viscous incompressible fluid from a vertical flat plate. In the discussion of Ahmed and Sarkar, the governing equations are reduced to ordinary differential equations by introducing appropriate co-ordinate transformation.

In this study, an analysis is carried out to study the heat transfer by free convection steady flow of a visco-elastic fluid past an inclined permeable plate under the action of transverse magnetic field. The velocity field and the shearing stress at the plate are obtained and illustrated graphically to observe the visco-elastic effects in combination with other flow parameters.

The constitutive equation for Walters liquid (Model B') is

$$\sigma_{ik} = -pg_{ik} + \sigma'_{ik}\,, \quad \sigma'^{ik} = 2\eta_0 e^{ik} - 2k_0 e'^{ik} \tag{1.1}$$

where σ^{ik} is the stress tensor, p is isotropic pressure, g_{ik} is the metric tensor of a fixed co-ordinate system x^i, v_i is the velocity vector, the contravarient form of e'^{ik} is given by

$$e'^{ik} = \frac{\partial e^{ik}}{\partial t} + v^m e^{ik}{}_{,m} - v^k{}_{,m}\, e^{im} - v^i{}_{,m}\, e^{mk}, \tag{1.2}$$

It is the convected derivative of the deformation rate tensor e^{ik} defined by

$$2e^{ik} = v_{i,k} + v_{k,i} \tag{1.3}$$

Here η_0 is the limiting viscosity at the small rate of shear which is given by

$$\eta_0 = \int_0^\infty N(\tau)d\tau \ \text{ and } \ k_0 = \int_0^\infty \tau N(\tau)d\tau \tag{1.4}$$

$N(\tau)$ being the relaxation spectrum . This idealized model is a valid approximation of Walters liquid (Model B) taking very short memories into account so that terms involving

$$\int_0^\infty t^n N(\tau)d\tau, n \geq 2 \tag{1.5}$$

have been neglected.

Walters [16] reported that the mixture of polymethyl metha crylate and pyridine at 25^0 C containing 30.5 gm of polymer per litre and having density 0.98 gm/ml fits very nearly to this model. For this mixture, the relaxation spectrum as given by Walters is

$$N(\lambda) = \sigma\eta_0\delta(\lambda) + \frac{1-\sigma}{\beta}\eta_0 \qquad (0 \le \lambda \le \beta)$$
$$= 0 \qquad \lambda > \beta$$

where $\sigma = 0.13, \eta_0 = 7.9$ poises, $\beta = 0.18$ sec and $\delta(\lambda)$ is the Dirac's delta function (Kapur et al.[17]).

Polymers are used in the manufacture of space crafts, aeroplanes, tyres, belt conveyers, ropes, cushions, seats, foams, plastic, engineering equipments, contact lens etc. Walters liquid (Model B') forms the basis for the manufacture of many such important and useful products.

2. MATHEMATICAL FORMULATION:

The steady two dimensional free convective boundary layer flow of an electrically conducting visco-elastic fluid past an inclined porous plate is analyzed. A magnetic field of uniform strength B_0 is applied in the normal to the plate. It is also assumed that the interaction of induced magnetic field with the flow is of negligible order in comparison with the interaction of the imposed magnetic field. The electrical conductivity of the fluid is also assumed to be of smaller order of magnitude. The suction of the plate is assumed to be constant. Let x'-axis be taken along the inclined plate and y'-axis is taken normal to the plate. Let T_0 be the temperature of the fluid at the plate and T_∞ be the equilibrium temperature of the fluid.. Let α be the small angle made by the plate with the vertical so that $sin\alpha = 0$

The governing equations of the fluid motion are as follows:

$$\frac{\partial v'}{\partial y'} = 0 \tag{2.1}$$

$$v'\frac{\partial u'}{\partial y'} = v\frac{\partial^2 u'}{\partial y'^2} - \frac{k_0}{\rho}v'\frac{\partial^3 u'}{\partial y'^3} + g\{\beta(T' - T_\infty)\}cos\alpha - \frac{\sigma B_0^2 u'}{\rho} - \frac{u'}{K'} \tag{2.2}$$

$$v'\frac{\partial T'}{\partial y'} = \frac{\bar{k}}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} \tag{2.3}$$

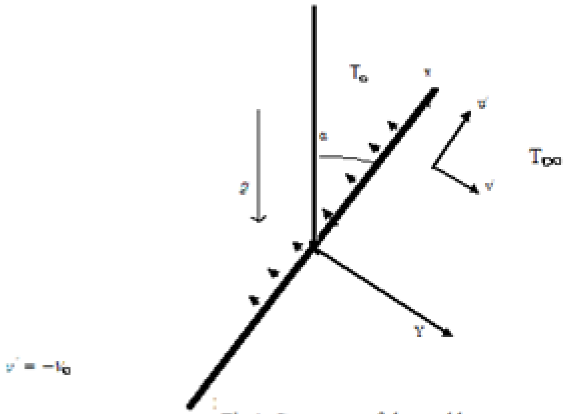


Fig 1: Geometry of the problem

where u' and v' are the velocity components along x' axis and y' axis respectively. Also g,$\beta, \rho, \sigma, K', \bar{k}, c_p, k_0$ are respectively the acceleration due to gravity, co-efficient of volume expansion, density of the fluid, electrical conductivity of the fluid, permeability of the plate, thermal conductivity of the fluid, specific heat at constant pressure and visco-elastic parameter

The corresponding boundary conditions of the problem are

$$u' = L'\sigma'_{xy'}, \qquad T' \to T_0 \ \ at\ y = 0$$

$$u' \to 0,\ \ T' \to T_\infty \ \ as\ \ y \to \infty \tag{2.4}$$

where, σ'_{xy} is the dimensional form of shearing stress and it is defined as

$$\sigma'_{xy} = \mu\frac{\partial u'}{\partial y'} - k_0\left(v'\frac{\partial^2 u'}{\partial y'^2}\right) \tag{2.5}$$

From the equation of continuity, we get,

$$v' = -V_0$$

where V_0 is a constant and its negative value indicates that the suction is towards the plate.

Let us introduce the following non-dimensional quantities

$$u = \frac{u'}{V_0}\ ,\quad y = y'\frac{V_0}{\nu}\ ,\quad \theta = \frac{T' - T_\infty}{T_0 - T_\infty}\ ,\qquad h = \rho L' V_0$$

$$G_r = \frac{g\beta(T_0 - T_\infty)\nu}{V_0^3}\ ,\qquad \nu = \frac{\mu}{\rho}\ ,\ k = \frac{k_0 V_0^2}{\rho \nu^2}\ ,\ M = \frac{\sigma B_0^2 \nu}{\rho V_0^2}$$

$$K = \frac{k' V_0^2}{\nu}\ ,\qquad v_0' = -V_0\ ,\ P_r = \frac{\mu c_p}{\bar{k}} \tag{2.6}$$

Then the non-dimensional form of the governing equation

$$-\frac{du}{dy} = \frac{d^2u}{dy^2} + k_1\frac{d^3u}{dy^3} + G_r cos\alpha\,\theta - Mu - \frac{u}{K} \tag{2.7}$$

$$\frac{d^2\theta}{dy^2} + P_r\frac{d\theta}{dy} = 0 \tag{2.8}$$

Where k is the non-dimensional visco-elastic parameter, K is the permeability parameter, M is the magnetic parameter, P_r prandtl number, G_r is the Grashof number for heat transfer.

The relevant boundary conditions of the problem are

$$u = h\sigma_{xy}\ ,\qquad \theta = 1\qquad at\ \ y = 0$$

$$u \to 0\ ,\quad \theta \to 0\qquad at\ \ y \to \infty \tag{2.9}$$

3. METHOD OF SOLUTION :

The solution of the equation (2.8) subject to the boundary conditions (2.9) is given as

$$\theta = e^{-P_r y} \tag{3.1}$$

To solve the equation (2.7), we use the perturbation technique, where the velocity profile can be expressed as

$$u = u_0 + ku_1 + o(k^2) \tag{3.2}$$

For small shear rate, the visco-elastic parameter is assumed to be small (k<<1).

Then substituting the equation (3.2) into the equation (2.7) and equating the like powers of the parameter k, we get the following equations:

$$u_0'' + u_0' - \left(M + \frac{1}{K}\right)u_0 = -G_r cos\alpha\, e^{-P_r y} \tag{3.3}$$

$$u_1'' + u_1' - \left(M + \frac{1}{K}\right)u_1 = -u_0''' \tag{3.4}$$

Modified boundary conditions for solving the equations (3.3) and (3.4) are

$$u_0 = hu_0',\ \ u_1 = h\left(u_1' + u_0''\right)\qquad at\ \ y = 0$$

$$u_0 \to 0,\ u_1 \to 0\ \ as\ \ y \to \infty \tag{3.5}$$

The solutions of equations (3.3) and (3.4), by using the condition (3.5) is given as follows

$$u_0 = c_2 e^{-\alpha_2 y} + c_3 e^{-P_r y}$$

$$u_1 = A_2 e^{-\alpha_2 y} + A_3 y e^{-\alpha_2 y} + A_4 e^{-P_r y}$$

The constants of the solutions are:

$$\alpha_1 = \frac{-1+\sqrt{1+4(M+\frac{1}{K})}}{2}, \quad \alpha_2 = \frac{-1-\sqrt{1+4(M+\frac{1}{K})}}{2}, \quad A_2 = -\frac{c_3 P_r^3}{P_r^2 - P_r - (M+\frac{1}{K})}$$

$$A_3 = \frac{c_2\alpha_2^3}{1-2\alpha_2}, A_4 = \frac{c_3 P_r^3}{P_r^2 - P_r - (M+\frac{1}{K})}, c_2 = -\frac{c_3(1+hP_r)}{1+h\alpha_2}, \; c_3 = -\frac{G_r cos\alpha}{P_r^2 - P_r - (M+\frac{1}{K})}$$

4. RESULTS AND DISCUSSIONS:

The velocity profile of the fluid flow is given as

$$u = c_2 e^{-\alpha_2 y} + c_3 e^{-P_r y} + k(A_2 e^{-\alpha_2 y} + A_3 y e^{-\alpha_2 y} + A_4 e^{-P_r y}) \qquad (4.1)$$

The non-dimensional shearing stress at the plate is given by

$$\sigma = -c_2\alpha_2 - c_3 P_r + k(c_2\alpha_2^2 + c_3 P_r^2 - A_2\alpha_2 + A_3 - A_4 P_r) \qquad (4.2)$$

The purpose of this study is to bring out the effects of the non-Newtonian parameter on the governing fluid flow with the combinations of the other flow parameters. The visco-elastic effect is exhibited through the non-dimensional parameter k . The corresponding results for Newtonian fluid are obtained by setting $k = 0$.

Figures 2 to 6 represent the velocity profiles u against y to observe the visco-elastic effects for various sets of values of the Grashof number for heat transfer G_r, Prandtl number P_r, magnetic parameter M, permeability parameter K. In this paper, we have investigated the difference in flow patterns of Newtonian fluid and visco-elastic fluid.

The figures reveal that in the neighbourhood of the plate the speed of fluid flow increases and after attaining the *peak*, its speed slows down as one moves away from the plate. Also, it can be concluded that the enlargement of elasticity factor present in the Walters liquid (Model B$^{/}$) accelerates the fluid flow in-comparison to the simple Newtonian fluid.

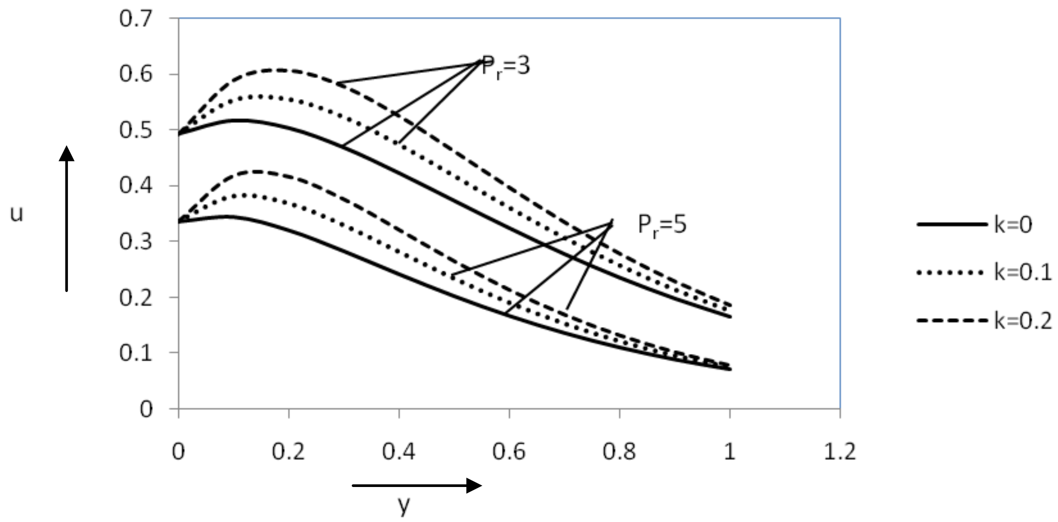


Figure 2: Variation of u against y for Gr=7, M=2, K=1,h=1

Prandtl number (Pr) analyses the simultaneous behaviour of momentum and thermal diffusion and is defined as the ratio of momentum diffusion to the thermal diffusion. Figure 2 depicts the flow behaviour under the influence of Prandtl number. The increasing values of Prandtl number lead to raise the viscosity of the fluid which in turn reduces the speed of the fluid. This physical phenomenon is experienced in both Newtonian and non-Newtonian fluid flows.

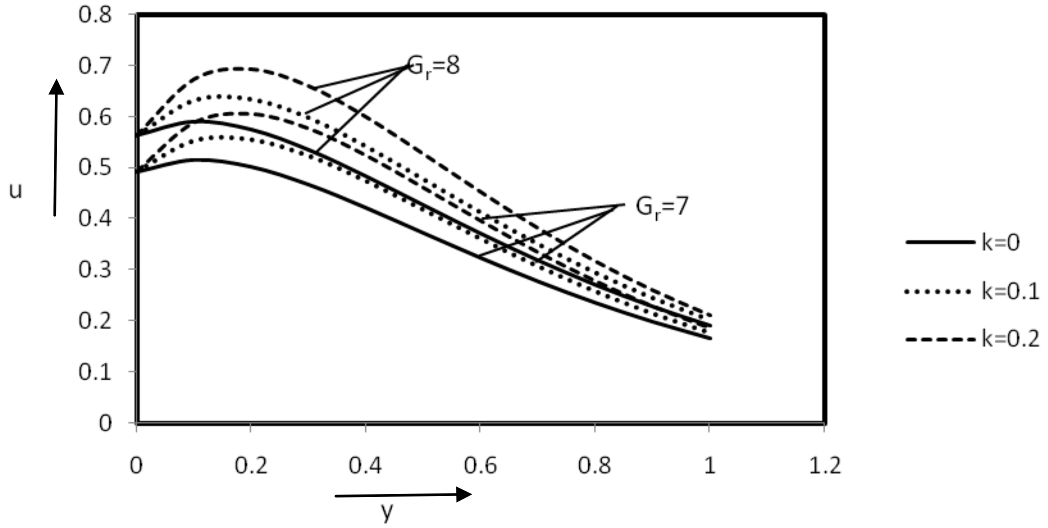


Figure 3: Variation of u against y Pr=3,M=2,K=1,h=1

Grashof number studies the behaviour of free convection and it is defined as the ratio of buoyancy force to viscous force. It plays an important role in free convection heat transfer mechanisms. G_r characterizes the free convection parameter for heat transfer. We have computed the result for G_r < 0 corresponds to an externally heated plate and G_r > *0* corresponds to an externally cooled plate. Concept of cooling (G_r > 0) is extensively used in many engineering applications like as cooling of various electronic components. Effects of Grshof number past an externally cooled plate are shown in figure 3. The figure states that rising value of Grshof number declines the viscosity of both Newtonian and non-Newtonian fluids, which in turn boost up the speeds of both Newtonian and visco-elastic fluids.

Figure 4 discusses the variations of flow past an externally heated plate and flow past a cooled plate against the displacement variable *y*. Gr = 7, which indicates the flow past a cooled plate and in case of cooling problem the visco-elasticity will enhance the speed of the fluid in comparison to the Newtonian fluid but during the flow past a heated plate (Gr = -7) a reverse behaviour is experienced. Also, we can conclude that the velocity profiles experienced in case of cooling problem is the mirror image of the velocity profiles experienced by the flow past a heated plate.

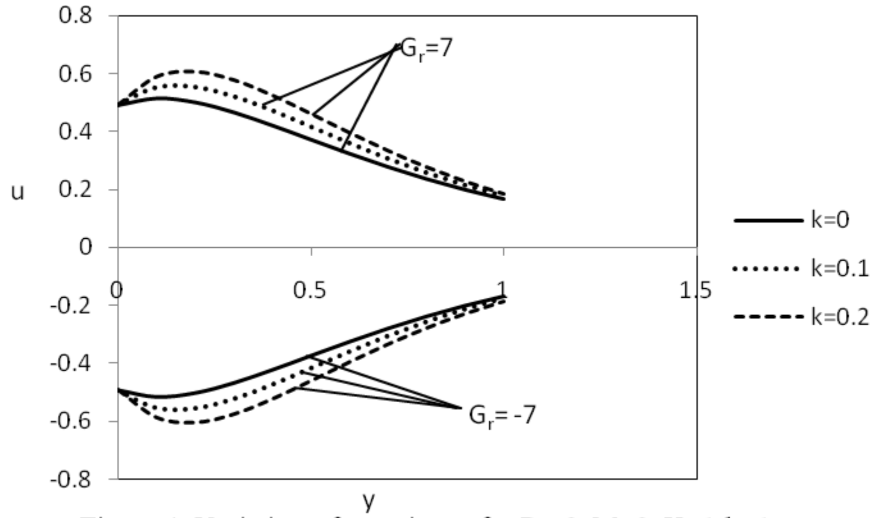


Figure 4: Variation of u against y for Pr=3, M=2, K=1,h=1

Figure 5 illustrates the behaviour of fluid flow for various values Hartmann number. Hartmann number depicts the strength of transverse magnetic field. The application of transverse magnetic field produces Lorentz force and the Lorentz force has a retarding effect on the velocity. As a consequence, the thickness of the fluid will be enlarged and the speed will go down. This diminishing trend in speed is observed in visco-elastic fluid characterized by Walters liquid (Model B$^{/}$).

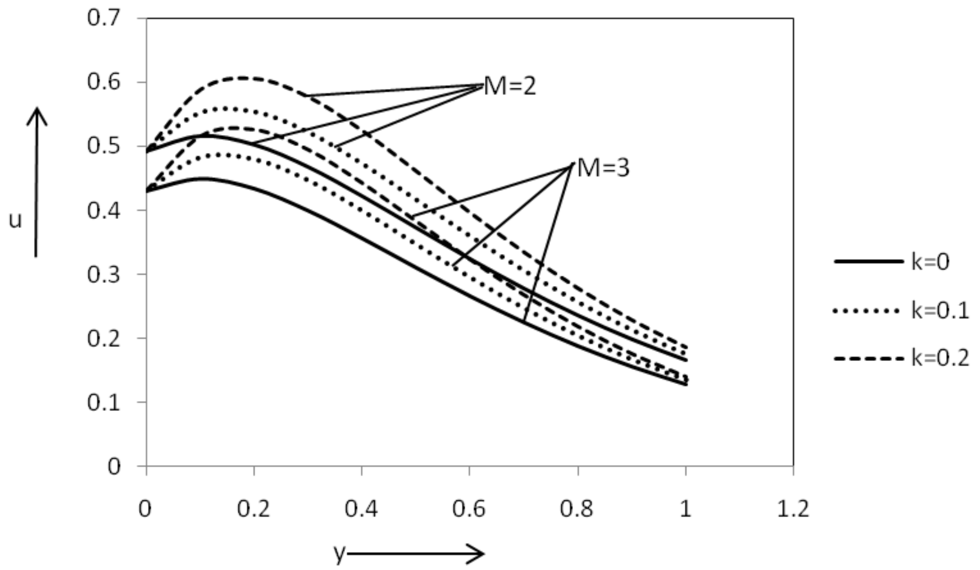


Figure 5 : Variation of u against y for Pr=3,Gr=7,K=1,h=1

The effect of permeability on the fluid flow is shown in figure 6 and the figure characterizes that the increase of permeability parameter modifies the speed of the both complex and simple fluid mechanisms along with the increasing values of visco-elastic parameter.

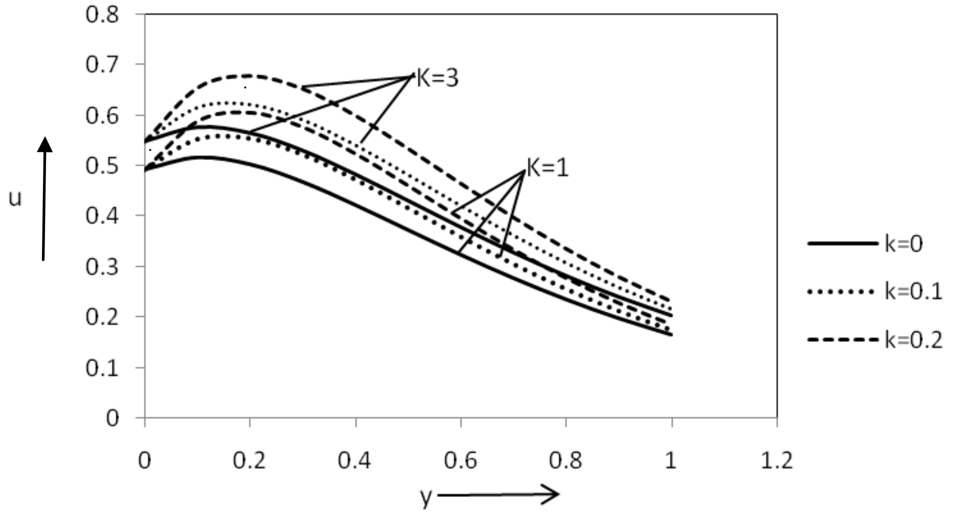


Figure 6: Variation of u against y for Pr=3,Gr=7 ,M=2,h=1

Knowing the velocity field, it is important from a practical point of view to know the effect of visco-elastic parameter on shearing stress or viscous drag. Figures 7 to 11 depict the shearing stress at the plate for the visco-elastic fluid in comparison with the Newtonian fluid for the various values of flow parameters involved in the solution. The analysis is given for both flow past an externally heated plate (G_r < 0) and externally cooled plate (G_r > 0).

The graphs enable that for the flow past an externally cooled plate, the shearing stress experienced by the governing fluid motion subdues its magnitude with the modification of visco-elasticity (k) of Walters liquid but an opposite phenomenon is noticed for the flow past a heated plate.

Figure 7 shows the nature of shearing stress formed at the plate during the changes created in the values of Prandtl number. It shows that the magnitude of viscous drag is reduced along with the amplified values of Prandtl number for both Newtonian and Non-Newtonian cases.

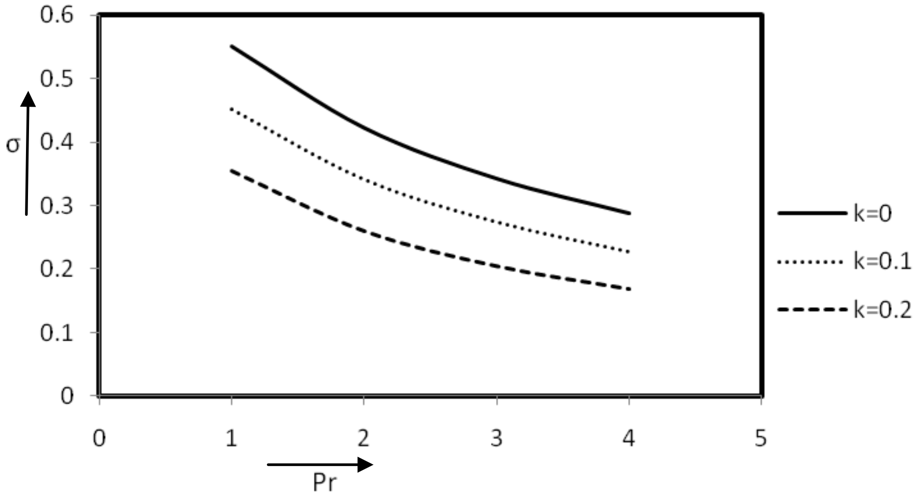


Figure 7: variation of σ against Pr for Gr=6,M=2,h=1,K=1

The creation of Lorentz force under the application of transverse magnetic field also marks an impact on the shearing stress formed at the plate and its effect is shown in figure 8. As the Hartmann number (M) increases, viscous drag declines for both Newtonian and non-Newtonian fluid flows.

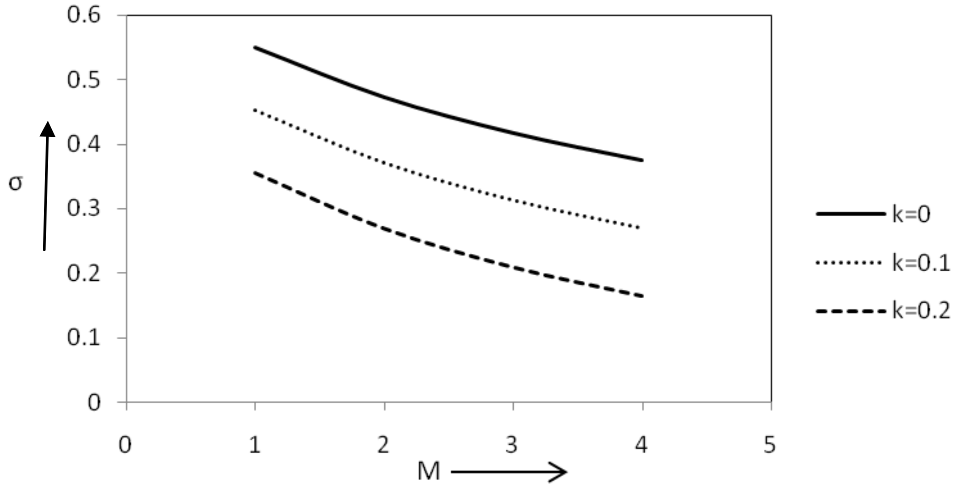


Figure 8: variation of σ against M for Gr=6,Pr=2,h=1,K=1

Figure 9 illustrates the appearance of shearing stress at the plate against permeability parameter during the fluid flow motions. Rising nature of permeability parameter enhances the magnitude of shearing stress of visco-elastic fluid as well Newtonian fluid.

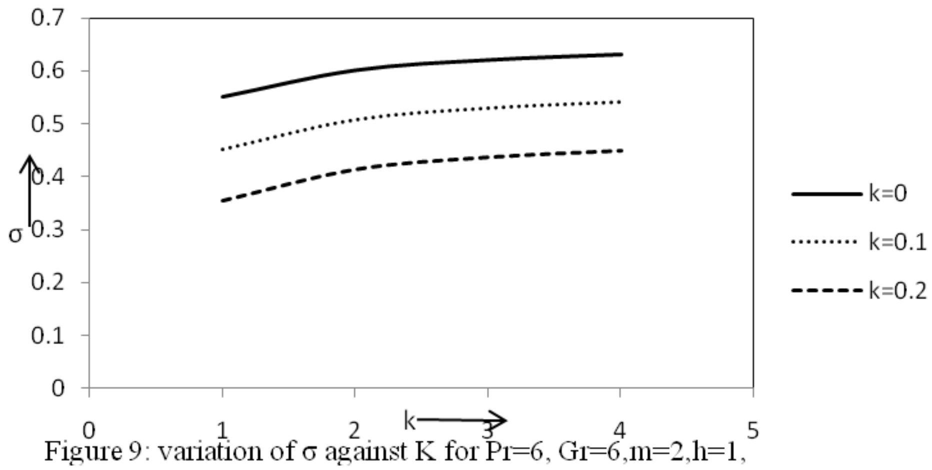


Figure 9: variation of σ against K for Pr=6, Gr=6,m=2,h=1,

Variations of shearing stress formed at externally heated plate (Gr < 0) and cooled plate (Gr > 0) are analysed in figure 9 and 10. The figure notifies that for the flow past a cooled plate, the increasing values of Grashof number amplifies the magnitude of shearing stress formed by both Newtonian as well as non-Newtonian fluid but it shows a downward trend for the flow past an externally heated plate.

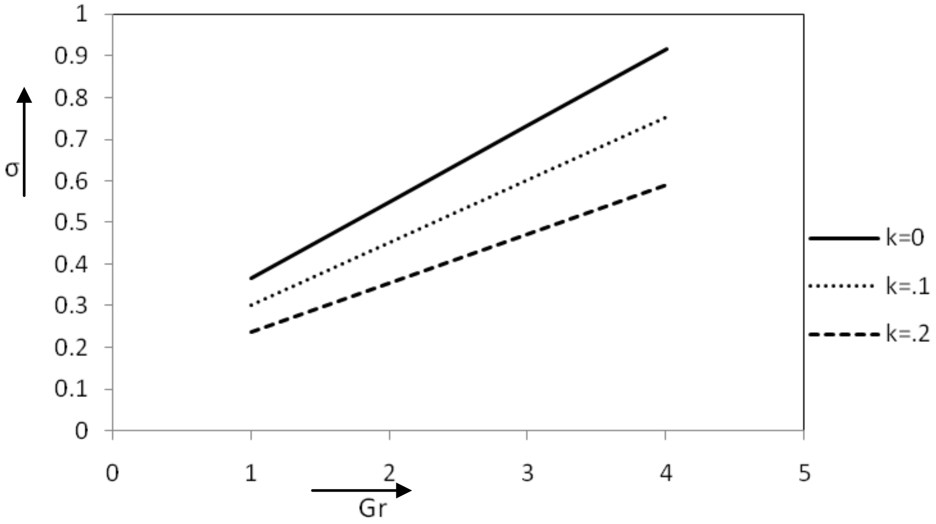
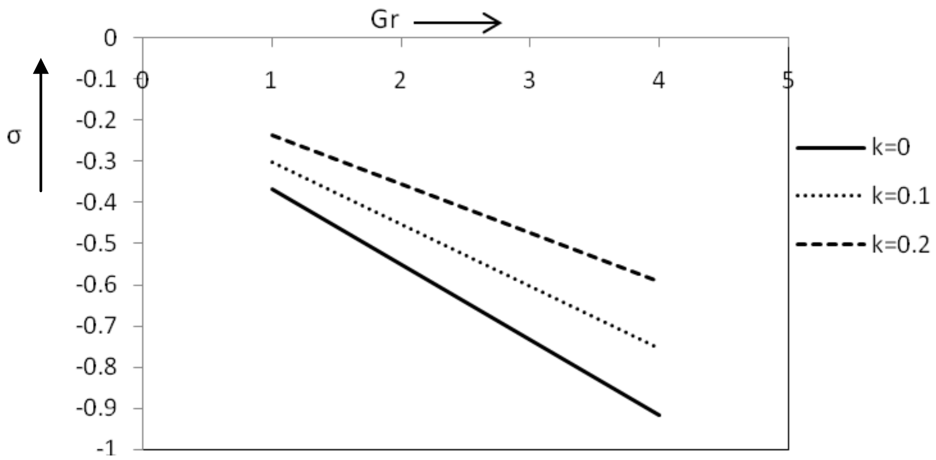


Figure10: Variation of σ against Gr for Pr=2, h=1, K=1,M=2

Figure 11: Variation of σ against Gr for Pr=2, h=1, K=1, M=2



The temperature field and rate of heat transfer are not significantly affected by visco-elastic parameter.

5. CONCLUSION

The effects of visco-elastic parameter on the free convective flow over an inclined porous plate in presence of magnetic field have been studied in this chapter. Some significant points of the present study are listed as below.

- The velocity field is considerably affected by the visco-elastic parameter.
- The velocity profiles increase with the increasing values of the visco-elastic parameter in comparison with the Newtonian fluid.
- The shear stress at the plate decreases with the increase of visco-elastic parameter.
- The temperature field is not significantly affected by the visco-elastic parameter.

6. REFERENCES

[1] Sparrow, E. M. and Gregg, J. L., 1959. Buoyancy effects in forced convection flow and heat transfer, Trans. Am. Soc. Mech. Engrs. E., J. Appl. Mech., 81, 133.

[2] Merkin, J. H., 1969. The effect of buoyancy forces on the boundary-layer over a semi infinite vertical flat plate in a uniform free stream, J. Fluid Mech., 35, 439.

[3] Loyed, J. R. and Sparrow, E. M., 1970. Combined forced and free convection flow on vertical surfaces, Int. J. Heat Mass Transfer, 13, 434.
[4] Wilks, G. and Hunt, R., 1981. Continuous transformation computation of boundary-layer equations between similarity regimes, J. Comp. Phys., 40, 478.
[5] Raju, M. S., Liu, X. R. and Law, C. K., 1984. A formulation of combined forced and free convection past horizontal and vertical surfaces, Int. J. Heat Mass Transfer, 27(9-12), 2215.
[6] Tingwei, G., Bachrun, P. and Daguent, M., 1982. Influence dela convection naturelle sur la convection forces au-dessus dune surface plane vertical soumise a un flux rayonnement, Int. J. Heat Mass Transfer, 25(5-8), 1061.
[7] Sparrow, E. M. and Cess, R. D., 1961. The effect of a magnetic field on free convection heat transfer, Int. J. Heat Mass Transfer, 3(4), 267.
[8] Riley, N., 1964. Magneto-hydrodynamic free convection, J. Fluid Mech., 18, 267.
[9] Kuiken, H. K., 1970. Magneto-hydrodynamic free convection in a strong cross field, J. Fluid Mech., 40, 21.
[10] Singh, K. R. and Cowling, T. G., 1963. Effect of magnetic field on free convective flow of electrically conducting fluids past a semi infinite flat plate, Quart. J. Mech. Appl. Math., 16, 1.
[11] Crammer, E. M. and Pai, S. I., 1974. Magneto-fluid Dynamics for Engineering and Applied Physicists, McGraw-Hill, New York.
[12] Hossain, M. A. and Ahmed, M., 1984. MHD forced and free convection boundary layer flow near the leading edge. Int. J. Heat Mass Transfer, 33, 571.
[13] Hossain, M. A., Pop, I. and Ahmed, M. 1996. MHD free convection flow from an isothermal plate inclined at a small angle to the horizontal, J. Theo. Appl. Fluid Mech., 1, 194.
[14] Wilks, G., 1976. Magneto-hydrodynamic free convection about a semi infinite vertical plate in a strong cross magnetic field, J. Appl. Math. Phys., 27, 621.
[15]. Ahmed, S. F. and Sarkar, A., 2009. MHD natural convection flow of viscous incompressible fluid from a vertical flat plate, J. Phy. Sc., 13, 77.
[16] Walters K., 1962. Non-Newtonian effects in some elastico-viscous liquids whose behaviour at small rates of shear is characterized by a general linear equation of state, Quart. J. Mech. Appl. Math., 15, 63.
[17] Kapur, J. N., Bhatt, B. S. and Sachbti, N. C. S. A., 1982. Non-Newtonain fluid flow, Pragati Prakashan, Meerut, 1st Ed.

Source of support: Nil, Conflict of interest: None Declared