International Journal of Mathematical Archive-3(9), 2012, 3422-3426
Available online through www.ijma.info ISSN 2229 – 5046

CPCN OF HYPOTRACEABLE GRAPHS

G. Rajasekar*

**P G& Research Department of Mathematics, Jawahar Science College, Neyveli -607 803, India*

(Received on: 20-08-12; Revised & Accepted on: 10-09-12)

ABSTRACT

In this paper the cpcn of Hypohamiltonian and Hypotraceble graph is being found. For this the cpcn of t-hypohamiltonian graph are being used. To find the Cyclic Path Covering Number of hypohamiltonian graphs and hypotraceable graph, two theorems which have been been developed previously are being used.

AMS MCS2010 code: *05C38, 05C45, 05C65, 05C70.*

Keywords: *cyclic path covering, cyclic path covering number, hypohamiltonian graph, T-hypohamiltoniangraph, hypohamiltoniangraph, hypotraceable graph.*

---

1. INTRODUCTION

The cyclic path covering number of graphs is developed with the motivation of road traffic with “roads as edges, junctions as nodes and traffic flow as path. These concepts were well discussed by A. Solairaju and G. Rajasekar [3, 4, 5, 6, 7, 9, 10, 11]. Already the methods of finding the cpcn of Hamiltonian graphs, cyclic cyclomatic graphs, tress and union of trees and Cartesian product of Hamiltonian graphs were developed. In the mathematical field of graph theory, a graph G is said to be hypohamiltonian if G does not itself have a Hamiltonian cycle but every graph formed by removing a single vertex from G is Hamiltonian. Hypohamiltonian graphs were first studied by Sousselier [12]. In this paper we develop a method to find the cpcn of Hypohamiltonian graphs. We follow the notations and terminology of Harary[1,2]. All graph considered in this paper are assumed to be connected graphs without isolated points. Let G = (V, E) be a graph. We denote the number of vertices if G by n and the number of edges in G by e.

2. CYCLIC PATH COVERING

2.1 Definition of Cyclic Path covering [3, 4, 5]

A Cyclic Path covering of a graph G is a collection Γ of paths in G whose union is G satisfying the conditions for distinct paths P_i and P_j with terminal vertices *u, v* and *w, z* respectively,

$$P_i \cap P_j = \begin{cases} A, \text{ A is the subset of the set } \{u, v, w, z\} \\ \phi, \text{ if } P_i \text{ and } P_j \text{ are cyclic.} \end{cases}$$

2.2. Definition of Cyclic Path covering number γ [3, 4, 5]

The Cyclic Path covering number of G is defined to be the minimum cardinality taken over all Cyclic Path covers of G.

Any Cyclic Path cover Γ of G with |Γ| = γ is called a minimum Cyclic Path cover of G.

2.3. Definition [11]. Let G be any graph and H be the sub graph of G. then the sub graph H_G is defined as $H_G = (V(H_G), E(H_G))$, where $E(H_G) = E(G) - E(H)$ and

$$V(H_G) = (V(G) - V(H)) \cap (V(G) \cup V(H)).$$

2.4. Definition [11]. The non-Hamiltonian graph G is said to be T-hypo Hamiltonian graph if for the sub tree T of G, T_G is a maximal Hamiltonian sub graph of G.

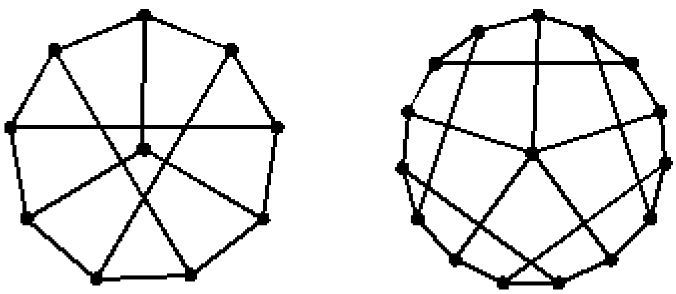
2.5. Theorem [11]. Let G be a T-hypo Hamiltonian graph then □(G) = □(T_G) + □(T).

Corresponding author: G. Rajasekar*
**P G& Research Department of Mathematics, Jawahar Science College, Neyveli -607 803, India*

2.6. Definition A graph G is hypohamiltonian if G is nonhamiltonian, but G –*v* is a Hamiltonian for every $v \in V(G)$ (Bondy and Murty [39].

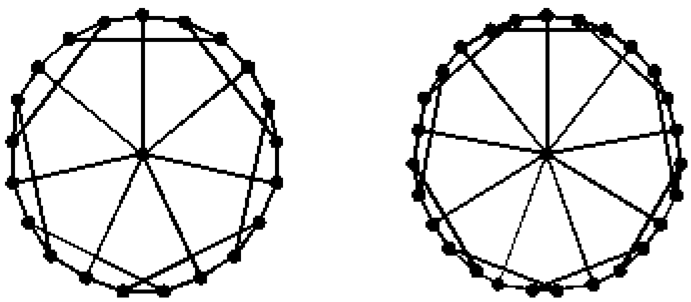
2.7. Theorem For any Hypo hamiltonian graph G with n vertices and e edges, $\gamma(G) = e - n + 1$, if G – *v* is a Cyclic Cyclomatic Graph, else γ(G) = e − n.

Proof: Let *v*∈V (G) be any vertex of G. Then G − *v*is the Hamiltonian graphand $[G - v]_G$ is a tree graph (actually a star graph) with d(v) number of pendentvertices. Then by our definition of T -hypo Hamiltonian graph we have G is a T -hypo Hamiltonian graph and hence we have $\gamma(G) = \gamma(G - v) + \gamma([G - v]_G)$. Now there aretwo cases.



Hypo Hamiltonian graphs

Figure 1



Hypo Hamiltonian graphs

Figure 2

Case (i). G − v is a cyclic cyclomatic graph. Then G − v will have e − d(*v*) edges and n−1 vertices.

Therefore $\gamma(G - v) = e - d(v) - (n-1) + 1 = e - n - d(v) + 2$ and $\gamma([G - v]_G) = d(v) - 1$.

Hence, $\gamma(G) = \gamma(G - v) + \gamma([G - v]_G) = e - n - d(v) + 2 + d(v) - 1 = e - n + 1$.

Case (ii). *G*−*v*is a non cycliccyclomatic graph. Then *G*−*v*will have *e*−*d(v)* edgesand *n*− 1 vertices.

Therefore $\gamma(G - v) = e - d(v) - (n-1) = e - n - d(v) + 1$ and $\gamma([G - v]_G) = d(v) - 1$.

Hence $\gamma(G) = \gamma(G - v) + \gamma([G - v]_G) = e - n - d(v) + 1 + d(v) - 1 = e - n$.

3. HYPOTRACEABLE GRAPH

3.1 Definition. A graph G is a hypotraceable graph if G has no Hamiltonian path (it is not a traceable graph), but G - *v* has a Hamiltonian path (it is a traceable graph) for every $v \in V(G)$ (Bondy and Murty [39]).

3.2 Definition. ***u* − *v*Traceable Graph:** A graph G is a *u* − *v*Traceable graphif for any two vertices *u*and *v*in V(G) there is a Hamiltonian path.

3.3 Theorem. If G is a $u - v$ Traceable graph then $\gamma(G) = e - n + 2$.

Proof: Let G be the $u-v$traceable graph. Let v_1 and v_2 be the terminal vertices of the Hamiltonian path in G. Here the graph G may be considered as $[(v_1, v_2)]_{G1}$,where G_1 is a Hamiltonian graph and (v_1, v_2) is a edge (subgraph) of the graph G_1.Here the graph G_1 may be Cyclic cyclomatic graph or otherwise.

Case 1: Assume G_1 is a Cyclic cyclomatic graph. Let v_1 and v_2 be the terminalvertices of the Hamiltonian path in G. Here the edge (v_1, v_2) may be the terminal orinterior edge of any of the path in the minimal cyclic path cover of G_1.

(a) If $d(v_1) > 1$ and $d(v_2) > 1$ in G

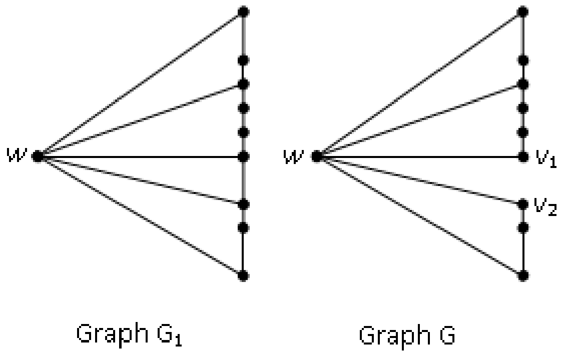


Figure 3

If (v_1, v_2) is the terminal edge or intermediate edge of any path P_i and v_2 is theterminal vertex of the path P_j in the minimal path cover Γ_2 of G_1, and suppose $v_1$is the terminal vertex of P_i , then correspondingly there is a path $P_i' = P_i - (v_1, v_2)$ and P_j' in the minimal path covering Γ of the graph G with v_2 as the terminal vertex of both the paths. As the $d(v_2) > 1$ in G, one may try to make this as internalvertex by joining the paths P_i' and P_j' , but this is impossible by the definition of Cyclic path covering, since $P_i' \cup P_j'$ is a closed path with terminal vertex w .

Therefore$G_1 = (n, e + 1)$ and $G = (n, e)$ have same cyclic path covering number.

(b) If $d(v_1) = 1$ and $d(v_2) > 1$

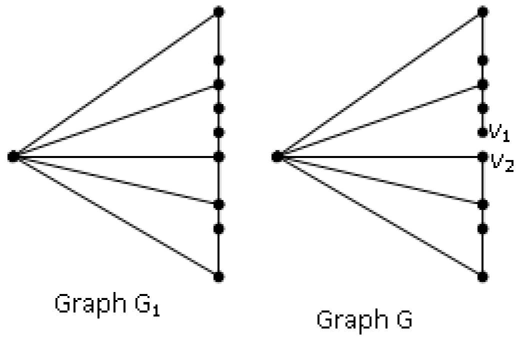


Figure 4

Here the graph G can be obtained in two different cases: They are (v_1, v_2) is theintermediate edge or the terminal edge of any path P_i in G_1.

(i) If (v_1, v_2) is the terminal edge of any path P_i with v_2 is the terminal vertex and interior vertex ofthe path in the minimal path cover of G_2 . Then correspondingly there is a path $P_i' = P_i - (v_1, v_2)$, with v_1 as the terminal vertex and P_j' with interior vertex v_2 in the minimal path covering Γ of the graph G. Now both G_2 and G have same numberof paths in the minimal cyclic path covers with the only difference that the path P_i' in that the length reduced by 1 than that of P_i in Γ_2. Thus $G_1 = (n, e + 1)$ and $G = (n, e)$ have same cyclic path covering number and hence $\gamma(G) = \gamma(G_1) = (e + 1) - n + 1 = e - n + 2$.

(ii) If (v_1, v_2) is the intermediate edge of any path P_i in the minimal path cover Γ_2 of G_1, then $P_i - (v_1, v_2)$ creates two paths $P_i'^{\,1}$ and $P_i'^{\,2}$ in Γ_2. Now by taking the path covering like the previous argument one can get same cyclic path covering number $\gamma(G) = e - n + 2$.

(c) If $d(v_1) = 1$ and $d(v_2) = 1$

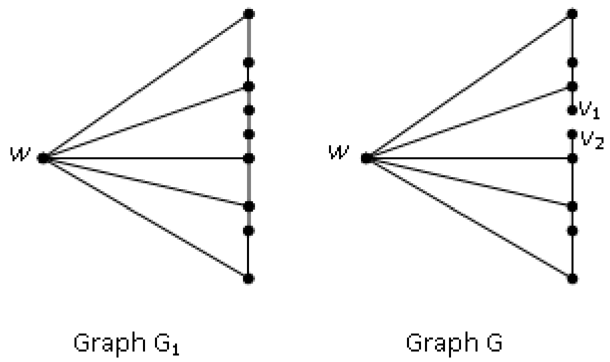


Figure 5

As G_1 is a Cyclic cyclomatic graph such that $G_1 = (n, e + 1)$ we have $\gamma(G_2) = e - n + 2$. If (v_1, v_2) is the intermediate edge of any path P_i in the minimal path cover of G_1, then correspondingly there arise two vertices of degree 1 in the graph G due to $P_i - (v_1, v_2)$ in G. Here one can get the minimal path cover of the graph G by taking a path from v_1 to $v_2$including w as one of the internal vertex of the path. The remaining paths can be taken by starting the paths from w treating w asterminal vertex of each path. As the $d(v_2) > 1$ in G, one may try to make this as internal vertex by joining the paths P_i' and P_j', but this is impossible since $P_i' \cup P_j'$ is a closed path with terminal vertex w. Therefore $G_1 = (n, e + 1)$ and $G = (n, e)$ have same cyclic path covering number $\gamma(G) = e - n + 2$.

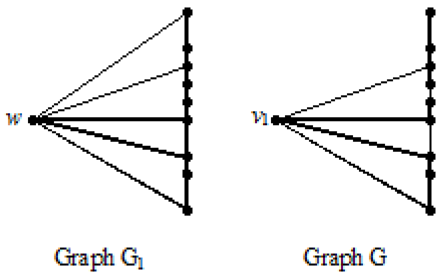


Figure 6

Case 2. G_1 is a non cycliccyclomatic graph. Here there are two cases (i) the number of edges exceeds than that of the number of vertices. (ii) The number of edgesequal to the number of vertices and so G_1 is a cycle.

(i) Let P be the Hamiltonian cycle in G_1 and Γ_1 be the minimal cyclic path cover of G_1. Then the edges of P will become part of two Paths in Γ_1. Then correspondingly two path covers in Γ of G, due to the absence of single edge and hence there anincrease of 1 in the number of path covers in Γ_1. Therefore the minimal number of cyclic path covers in G is minimal number of path covers for $G_1 + 1$.

Thus we have $\gamma(G) = \gamma(G_1) + 1 = (e + 1) - n + 1 = e - n + 2$. Thus in all cases $\gamma(G) = e - n + 2$, where G is the u - $v$traceable graph.

3.4 Theorem. If G is a Hypotraceable graph with n vertices and e edges then $\gamma(G) = e - n + 2$.

Proof: Let $v \in V(G)$ be any vetex. Then G-v has a Hamiltonian path. If d (v) =k, then G-v has n-1 vertices and e-k edges. Also the induced graph formed by k edges which are incident at the vertex v is a star graph G_{G-v} with k+1 vertices and k edges and hence $\gamma(G_{G-v}) = k-1$. Then by theorem 3.3 we have γ(G-v) = (e-k) – (n-1) + 2. Hence by the algorithm to find Cyclic path covering number [3],

We have $\gamma(G) = \gamma(G-v) + \gamma(G_{G-v}) = \left((e-k) - (n-1) + 2\right) + (k-1) = e - n + 2$.

REFERENCES

[1] F. Harary, *covering and packing in graphs I*, Ann. N. Y. Acad. Sci., 175(1970), 198-205.

[2] F. Harary and A.J Schwenk, *Evolution of the path number of a graph, covering and packing in graphs II*, Graph Theory and Computing, Eds. R.C. Road, Academic Press, New York,(1972),39-45.

[3] A. Solairaju and G. Rajasekar, *An algorithm to find Cyclic Path Covering Number*, Electronic Notes in Discrete Mathematics, Elsevier (2009) pp. 21-28.

[4] A. Solairaju and G. Rajasekar, On $\{e_m, v_k\}$ - Cyclic Path covering number, *Antartica Journal of Mathematics*.

[5] A. Solairaju and G. Rajasekar, On Cyclic Path Covering Number of Hamiltonian graphs, *Indian Journal of Mathematics and Mathematical Sciences*, 5(2) (2009) 257-265.

[6] A. Solairaju and G. Rajasekar, Categorization of Hamiltonian graph on the basis of Cyclic Path Covering Number, *International Review of Pure and Applied Mathematics*,5(2) (2009) 419-426.

[7] A. Solairaju and G. Rajasekar, On cyclic path covering number of Cartesian product of graphs, *Pacific Asian Journal of Mathematics*, 3 No. 1-2 (2009).

[8] Katerinis, P. "Some Properties of Hypohamiltonian Graphs." *Aeq. Math.***72**, 139-151, 2006.

[9] A. Solairaju and G. Rajasekar, Cyclic Cyclomatic Graphs, *Proc. International conf. on Mathematics and Computer Science* (2008), 212-220.

[10] A. Solairaju and G. Rajasekar, Cyclic Path Covering Number, *Proc. International conf. on Mathematics and Computer Science* (2008), 223-231.

[11] A. Solairaju and G. Rajasekar, Cyclic Path Covering Number of T-Hypohamiltoniangaphs, *Proc. International conf. on Mathematics and Computer Science* (2011).

[12] R. Sousselier, Probleme no. 29: Le cercle des irascibles, in C. Berge, Problmesplaisants et delectables, Rev. Franc. Rech. Operationnelle 7: (1963) 405-406.

Source of support: Nil, Conflict of interest: None Declared

