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Somewhat almost rg-continuous functions and Somewhat almost rg-open functions

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ABSTRACT

In this paper we tried to introduce a new variety of continuous and open functions called Somewhat almost rg-continuous functions and Somewhat almost rg-open functions. Its basic properties are discussed.

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1. INTRODUCTION

b-open sets are introduced by Andrijevic in 1996. K.R. Gentry introduced somewhat continuous functions in the year 1971. V.K. Sharma and the present authors of this paper defined and studied basic properties of *v*-open sets and *v*-continuous functions in the year 2006 and 2010 respectively. T. Noiri and N. Rajesh introduced somewhat b-continuous functions in the year 2011. Inspired with these developments we introduce in this paper somewhat almost *rg*-continuous functions, somewhat almost *rg*-open functions and study its basic properties and interrelation with other type of such functions available in the literature. Throughout the paper (X,τ) and (Y,σ) (or simply X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For $A\subset(X;\tau)$, $cl\{A\}$ and A^o denote the closure of A and the interior of A in X, respectively.

2. PRELIMINARIES

Definition 2.1: A subset *A* of X is said to be
(i) b-open [1] if $A\subset(cl\{A\})^o\cap cl\{A^o\}$.
(ii) rg-dense in X if there is no proper rg-closed set C in X such that $M\subset C\subset X$.

Definition 2.2: A function f is said to be
(i) somewhat continuous[20][resp: somewhat b-continuous; somewhat rg-continuous] if for $U\in\sigma$ and $f^{-1}(U)\neq\varphi$, there exists an open[resp: b-open; rg-open] set V in X such that $V\neq\varphi$ and $V\subset f^{-1}(U)$.

(ii) somewhat open [resp: somewhat b-open; somewhat rg-open] provided that if $U\in\tau$ and $U\neq\varphi$, then there exists an open[resp: b-open; rg-open] set V in Y such that $V\neq\varphi$ and $V\subset f(U)$.

Definition 2.3: (X,τ) is said to be resolvable [b-resolvable] if there exists a set A in (X,τ) such that both A and X - A are dense[b-dense] in (X,τ). Otherwise, (X,τ) is called irresolvable.

Definition 2.4: If X is a set and τ and σ are topologies on X, then τ is said to be equivalent[resp: rg- equivalent] to σ provided if $U\in\tau$ and $U\neq\varphi$, then there is an open[resp:rg-open] set V in *X* such that $V\neq\varphi$ and $V\subset U$ and if $U\in\sigma$ and $U\neq\varphi$, then there is an open[resp:rg-open] set V in (X,τ) such that $V\neq\varphi$ and $U\supset V$.

3. SOMEWHAT ALMOST RG-CONTINUOUS FUNCTION

Definition 3.1: A function f is said to be somewhat almost rg-continuous if for $U\in RO(\sigma)$ and $f^{-1}(U)\neq\varphi$, there exists a non-empty rg-open set V in X such that $V\subset f^{-1}(U)$.

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It is clear that every almost continuous function is somewhat almost continuous and every somewhat almost continuous is somewhat almost rg-continuous. But the converses are not true.

Example 1: Let X = {a, b, c}, τ = {φ, {a}, {b, c}, X} and σ = {φ, {a}, X}. The function f :(X,τ)$\to$ (X,σ) defined by f(a) = c, f(b) = a and f(c) = b is somewhat almost rg-continuous, somewhat rg-continuous but not somewhat continuous.

Example 2: Let X = {a, b, c}, τ = {φ, {b, c}, X}, σ = {φ, {b}, {a, c}, X} and η = {φ, {a}, X}. Then the identity functions f :(X,τ)$\to$ (X, σ) and g:(X, σ)$\to$ (X; η) and g•f are somewhat almost rg-continuous.

However, we have the following

Theorem 3.1: If f is somewhat almost rg-continuous and g is continuous[r-continuous; r-irresolute], then g•f is somewhat almost rg-continuous.

Theorem 3.2: For a surjective function f, the following statements are equivalent:
(i) f is somewhat almost rg-continuous.
(ii) If C is regular closed in Y such that $f^{-1}(C) \neq X$, then there is a proper rg-closed subset D of X such that $f^{-1}(C) \subset D$.
(iii)If M is a rg-dense subset of X, then f(M) is a dense subset of Y.

Proof: (i) ⇒(ii): Let C$\in$RC(Y) such that $f^{-1}(C) \neq X$. Then Y-C$\in$RO(Y) such that $f^{-1}(Y-C) = X - f^{-1}(C) \neq \varphi$. By (i), there exists V$\neq \varphi \in$GSO(X) and V$\subset f^{-1}(Y-C) = X - f^{-1}(C)$. Thus X-V$\supset f^{-1}(C)$ and X - V = D is a proper rg-closed set in X.

(ii) ⇒(i): Let U$\in$RO(σ) and $f^{-1}(U) \neq \varphi$ Then Y-U$\in$RC(σ) and $f^{-1}(Y-U) = X-f^{-1}(U) \neq X$. By (ii), there exists a proper D $\in$RGC(X) such that D$\supset f^{-1}(Y-U)$. This implies that X-D $\subset f^{-1}(U)$ and X-D is rg-open and X-D $\neq \varphi$.

(ii) ⇒(iii): Let M be a rg-dense set in X. If f (M) is not dense in Y. Then there exists a proper C$\in$RC(Y) such that f(M) $\subset$ C$\subset$ Y. Clearly $f^{-1}(C) \neq X$. By (ii), there exists a proper D$\in$RGC(X) such that M$\subset f^{-1}(C) \subset$ D$\subset$ X. This is a contradiction to the fact that M is rg-dense in X.

(iii) ⇒(ii): If (ii) is not true. there exists C$\in$RC(Y) such that $f^{-1}(C) \neq X$ but there is no proper D$\in$RGC(X) such that $f^{-1}(C) \subset D$. Thus $f^{-1}(C)$ is rg-dense in X. But by (iii), $f(f^{-1}(C)) = C$ is dense in Y, which contradicts the choice of C.

Theorem 3.3: Let f be a function and X = A$\cup$ B, where A, B$\in$ RO(X). If $f_{/A}$: (A; $\tau_{/A}$)$\to$(Y, σ) and $f_{/B}$:(B; $\tau_{/B}$)$\to$(Y, σ) are somewhat almost rg-continuous, then f is somewhat almost rg-continuous.

Proof: Let U$\in$ RO (σ) such that $f^{-1}(U) \neq \varphi$. Then $(f_{/A})^{-1}(U) \neq \varphi$ or $(f_{/B})^{-1}(U) \neq \varphi$ or both $(f_{/A})^{-1}(U) \neq \varphi$ and $(f_{/B})^{-1}(U) \neq \varphi$. Suppose $(f_{/A})^{-1}(U) \neq \varphi$, Since $f_{/A}$ is somewhat almost rg-continuous, there exists V$\neq \varphi \in$GSO(A) and V$\subset (f_{/A})^{-1}(U) \subset f^{-1}(U)$. Since V$\in$GSO (A) and A$\in$RO(X), V$\in$GSO(X). Thus f is somewhat almost rg-continuous. The proof of other cases are similar.

Theorem 3.4: Let f :(X, τ)$\to$ (Y, σ) be a somewhat almost rg-continuous surjection and τ* be a topology for X, which is rg-equivalent to τ. Then f:(X, τ*)$\to$ (Y, σ) is somewhat almost rg-continuous.

Proof: Let V$\in$RO (σ) such that $f^{-1}(V) \neq \varphi$. Since f is somewhat almost rg-continuous, there exists U$\neq \varphi \in$GSO(X,τ) such that U$\subset f^{-1}(V)$. But by hypothesis τ* is rg-equivalent to τ. Therefore, there exists U*$\neq \varphi \in$GSO(X; τ*) such that U*$\subset$ U. But U$\subset f^{-1}(V)$. Then U*$\subset f^{-1}(V)$; hence f:(X, τ*)$\to$ (Y, σ) is somewhat almost rg-continuous.

Theorem 3.5: Let f :(X, τ)$\to$ (Y, σ) be a somewhat almost rg-continuous surjection and σ* be a topology for Y, which is equivalent to σ. Then f :(X, τ)$\to$ (Y, σ*) is somewhat almost rg-continuous.

Proof: Let V*$\in$RO(σ*) such that $f^{-1}(V^*) \neq \varphi$. Since σ* is equivalent to σ, there exists V$\neq\phi\in$RO(Y, σ) such that V$\subset$ V*. Now $\varphi \neq f^{-1}(V) \subset f^{-1}(V^*)$. Since f is somewhat almost rg-continuous, there exists U$\neq \varphi \in$GSO(X,τ) such that U$\subset f^{-1}(V)$. Then U$\subset f^{-1}(V^*)$; hence f:(X, τ)$\to$ (Y, σ*) is somewhat almost rg-continuous.

4. SOMEWHAT RG-IRRESOLUTE FUNCTION

Definition 4.1: A function f is said to be somewhat rg-irresolute if for U$\in$GSO (σ) and $f^{-1}(U) \neq \varphi$, there exists a non-empty rg-open set V in X such that V$\subset f^{-1}(U)$.

Example3: Let X = {a, b, c}, $\tau = \{\varphi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\varphi, \{a\}, \{a, b\}, X\}$. The function $f:(X, \tau)\to (X, \sigma)$ defined by $f(a) = c$, $f(b) = a$ and $f(c) = b$ is somewhat rg-irresolute but not somewhat-irresolute.

Example 4: The identity functions *f*; *g* and *g•f* in Example 2 are somewhat rg-irresolute.

However, we have the following

Theorem 4.1: If *f* is somewhat rg-irresolute and *g* is irresolute, then *g•f* is somewhat rg-irresolute.

Theorem 4.2: For a surjective function *f*, the following statements are equivalent:
(i) *f* is somewhat rg-irresolute.
(ii) If C is rg-closed in Y such that $f^{-1}(C) \neq X$, then there is a proper rg-closed subset D of X such that $f^{-1}(C) \subset D$.
(iii) If M is a rg-dense subset of X, then $f(M)$ is a rg-dense subset of Y.

Proof: (i) ⇒(ii): Let $C\in RGC(Y)$ such that $f^{-1}(C) \neq X$. Then $Y\text{-}C\in GSO(Y)$ such that $f^{-1}(Y\text{-}C) = X - f^{-1}(C) \neq \varphi$ By (i), there exists $V\neq \varphi\in GSO(X)$ and $V\subset f^{-1}(Y\text{-}C) = X - f^{-1}(C)$. This means $X\text{-}V\supset f^{-1}(C)$ and X - V = D is proper rg-closed in X.

(ii) ⇒(i): Let $U\in GSO(\sigma)$ and $f^{-1}(U) \neq \varphi$ Then $Y\text{-}U\neq \varphi\in RGC(Y)$ and $f^{-1}(Y\text{-}U) = X\text{-}f^{-1}(U) \neq X$. By (ii), there exists $D\neq \varphi\in RGC(X)$ such that $D\supset f^{-1}(Y\text{-}U)$. This implies that $X\text{-}D \subset f^{-1}(U)$ and X-D is rg-open and $X\text{-}D \neq \varphi$.

(ii) ⇒(iii): Let M be a rg-dense set in X. If $f(M)$ is not rg-dense in Y. Then there exists a proper $C\in RGC(Y)$ such that $f(M) \subset C\subset Y$. Clearly $f^{-1}(C) \neq X$. By (ii), there exists a proper $D\in RGC(X)$ such that $M\subset f^{-1}(C) \subset D\subset X$. This is a contradiction to the fact that M is rg-dense in X.

(iii) ⇒(ii): Suppose (ii) is not true. there exists $C\in RGC(Y)$ such that $f^{-1}(C) \neq X$ but there is no proper $D\neq \varphi\in RGC(X)$ such that $f^{-1}(C) \subset D$. This means that $f^{-1}(C)$ is rg-dense in X. But by (iii), $f(f^{-1}(C)) = C$ must be rg-dense in Y, which is a contradiction to the choice of C.

Theorem 4.3: Let *f* be a function and $X = A\cup B$, where $A, B\in RO(X)$. If $f_{/A}: (A; \tau_{/A}) \to (Y, \sigma)$ and $f_{/B}:(B; \tau_{/B}) \to (Y, \sigma)$ are somewhat rg-irresolute, then *f* is somewhat rg-irresolute.

Proof: Let $U\in GSO(\sigma)$ such that $f^{-1}(U) \neq \varphi$. Then $(f_{/A})^{-1}(U) \neq \varphi$ or $(f_{/B})^{-1}(U) \neq \varphi$ or both $(f_{/A})^{-1}(U) \neq \varphi$ and $(f_{/B})^{-1}(U) \neq \varphi$. If $(f_{/A})^{-1}(U) \neq \varphi$, Since $f_{/A}$ is somewhat rg-irresolute, there exists $V\neq \varphi\in GSO(A)$ and $V\subset (f_{/A})^{-1}(U) \subset f^{-1}(U)$. Since $V\in GSO(A)$ and $A\in RO(X)$, $V\in GSO(X)$. Thus *f* is somewhat rg-irresolute.

The proof of other cases are similar.

If *f* is the identity function and τ and σ are rg-equivalent. Then *f* and f^{-1} are somewhat rg-irresolute. Conversely, if the identity function *f* is somewhat rg-irresolute in both directions, then τ and σ are rg-equivalent.

Theorem 4.4: Let $f:(X, \tau)\to (Y, \sigma)$ be a somewhat rg-irresolute surjection and τ^* be a topology for X, which is rg-equivalent to τ. Then $f:(X, \tau^*)\to (Y, \sigma)$ is somewhat rg-irresolute.

Proof: Let $V\in GSO(\sigma)$ such that $f^{-1}(V) \neq \varphi$. Since *f* is somewhat rg-irresolute, there exists $U\neq \varphi\in GSO(X, \tau)$ with $U\subset f^{-1}(V)$. But for τ^* is rg-equivalent to τ, there exists $U^*\neq \varphi\in GSO(X; \tau^*)$ such that $U^*\subset U$. But $U\subset f^{-1}(V)$. Then $U^*\subset f^{-1}(V)$; hence $f:(X, \tau^*)\to (Y, \sigma)$ is somewhat rg-irresolute.

Theorem 4.5: Let $f:(X, \tau)\to (Y, \sigma)$ be a somewhat rg-irresolute surjection and σ^* be a topology for Y, which is equivalent to σ. Then $f:(X, \tau)\to (Y, \sigma^*)$ is somewhat rg-irresolute.

Proof: Let $V^*\in\sigma^*$ such that $f^{-1}(V^*) \neq \varphi$. Since σ^* is equivalent to σ, there exists $V\neq \varphi\in(Y, \sigma)$ such that $V\subset V^*$. Now $\varphi \neq f^{-1}(V) \subset f^{-1}(V^*)$. Since *f* is somewhat rg-irresolute, there exists $U\neq \varphi\in GSO(X, \tau)$ such that $U\subset f^{-1}(V)$. Then $U\subset f^{-1}(V^*)$; hence $f:(X, \tau)\to (Y, \sigma^*)$ is somewhat rg-irresolute.

5. SOMEWHAT ALMOST RG-OPEN FUNCTION

Definition 5.1: A function *f* is said to be somewhat almost rg-open provided that if $U\in RO(\tau)$ and $U\neq \varphi$, then there exists a non-empty rg-open set V in Y such that $V\subset f(U)$.

Example 5: Let X = {a, b, c}, τ = {φ, {a}, X} and σ = {φ, {a}, {b, c}, X}. The function *f*: (X, τ)$\to$ (X, σ) defined by *f*(a) = a, *f*(b) = c and *f*(c) = b is somewhat almost rg-open, somewhat rg-open and somewhat open.

Theorem 5.1: Let *f* be r-open and *g* be somewhat almost rg-open. Then *g*• *f* is somewhat almost rg-open.

Theorem 5.2: For a bijective function *f*, the following are equivalent:
(i) *f* is somewhat almost rg-open.
(ii) If C is regular closed in X, such that *f*(C) $\neq$ Y, then there is a rg-closed subset D of Y such that D$\neq$ Y and D$\supset$ *f*(C).

Proof: (i) $\Rightarrow$(ii): Let C$\in$RC(X) such that *f*(C) $\neq$ Y. Then X-C$\neq$ $\varphi\in$RO(X). Since *f* is somewhat almost rg-open, there exists V$\neq$ $\varphi\in$GSO(Y) such that V$\subset$ *f*(X-C). Put D = Y-V. Clearly D$\neq$ $\varphi\in$RGC(Y). If D = Y, then V = φ, which is a contradiction. Since V$\subset$ *f*(X-C), D = Y-V $\supset$ (Y- *f*(X-C)) = *f*(C).

(ii) $\Rightarrow$(i): Let U$\neq$ $\varphi\in$RO(X). Then C = X-U$\in$RC(X) and *f*(X-U) = *f*(C) = Y- *f*(U) implies *f*(C) $\neq$ Y. By (ii), there is D $\neq$ $\varphi\in$RGC(Y) and *f*(C) $\subset$ D. Clearly V = Y-D$\neq$ $\varphi\in$GSO(Y). Also, V = Y-D $\subset$ Y- *f*(C) = Y- *f*(X-U) = *f*(U).

Theorem 5.3: The following statements are equivalent:
(i) *f* is somewhat almost rg-open.
(ii)If A is a rg-dense subset of Y, then f^{-1}(A) is a dense subset of X.

Proof: (i) $\Rightarrow$(ii): Let A be a rg-dense set in Y. If f^{-1}(A) is not dense in X, then there exists B$\in$RC(X) such that f^{-1}(A) $\subset$ B$\subset$ X. Since *f* is somewhat almost rg-open and X-B$\in$RO(X), there exists C $\neq$ $\varphi\in$GSO(Y) such that C$\subset$ *f*(X-B). Therefore, C$\subset$ *f*(X-B) $\subset$ $f(f^{-1}$(Y-A)) $\subset$ Y-A. That is, A$\subset$ Y-C $\subset$ Y. Now, Y-C is a rg-closed set and A$\subset$ Y-C $\subset$ Y. This implies that A is not a rg-dense set in Y, which is a contradiction. Therefore, f^{-1}(A) is a dense set in X.

(ii) $\Rightarrow$(i): If A$\neq$ $\varphi\in$RO(X). We want to show that gs(f(A))$^{o}\neq$ φ. Suppose gs(f(A))^{o} = φ. Then, RGC*l*{(*f*(A))} = Y. Then by (ii), f^{-1}(Y - *f*(A)) is dense in X. But f^{-1}(Y- *f* (A)) $\subset$ X-A. Now, X-A$\in$RC(X). Therefore, f^{-1}(Y- *f*(A)) $\subset$ X-A gives X = *cl*{(f^{-1}(Y- *f*(A)))} $\subset$ X-A. Thus A = φ, which contradicts A $\neq$ φ. Therefore, gs(f(A))$^{o}\neq$ φ. Hence *f* is somewhat almost rg-open.

Theorem 5.4: Let *f* be somewhat almost rg-open and A$\in$RO(X). Then $f_{/A}$ is somewhat almost rg-open.

Proof: Let U$\neq$ $\varphi\in$RO ($\tau_{/A}$). Since U$\in$RO(A) and A$\in$RO(X), U$\in$RO(X) and since *f* is somewhat almost rg-open, there exists V$\in$GSO(Y), such that V$\subset$ *f*(U). Thus $f_{/A}$ is a somewhat almost rg-open function.

Theorem 5.5: Let *f* be a function and X = A$\cup$ B, where A, B$\in$RO(X). If $f_{/A}$ and $f_{/B}$ are somewhat almost rg-open, then *f* is somewhat almost rg-open.

Proof: Let U$\neq$ $\varphi\in$RO(X). Since X = A$\cup$ B, either A$\cap$ U $\neq$ φ or B$\cap$ U $\neq$ φ or both A$\cap$U $\neq$ φ and B$\cap$U $\neq$ φ. Since U is regular open in X, U is regular open in both A and B.

Case (i): If A$\cap$U $\neq$ $\varphi\in$RO (A). Since $f_{/A}$ is somewhat almost rg-open there exists a rg-open set V of Y such that V$\subset$*f*(U$\cap$A) $\subset$ *f*(U), which implies that *f* is somewhat almost rg-open.

Case (ii): If B$\cap$U $\neq$ $\varphi\in$RO (B). Since $f_{/B}$ is somewhat almost rg-open, there exists a rg-open set V in Y such that V$\subset$ *f* (U$\cap$B) $\subset$ *f* (U), which implies that *f* is somewhat almost rg-open.

Case (iii): Suppose that both A$\cap$U $\neq$ φ and B$\cap$U $\neq$ φ. Then by case (i) and (ii) *f* is somewhat almost rg-open.

Remark 1: Two topologies τ and σ for X are said to be rg-equivalent if and only if the identity function *f*: (X,τ)$\to$ (Y,σ) is somewhat almost rg-open in both directions.

Theorem 5.6: If *f* :(X, τ)$\to$(Y, σ) is somewhat almost open. Let τ* and σ* be topologies for X and Y, respectively such that τ* is equivalent to τ and σ* is rg-equivalent to σ. Then *f*: (X; τ*)$\to$ (Y; σ*) is somewhat almost rg-open.

6. SOMEWHAT M-RG-OPEN FUNCTION

Definition 6.1: A function *f* is said to be somewhat M-rg-open provided that if U$\in$GSO (τ) and U$\neq$ φ, then there exists a non-empty rg-open set V in Y such that V$\subset$ *f* (U).

Example 6: *f* as in Example 5 is somewhat M-rg-open.

Theorem 6.1: Let f be r-open and g be somewhat M-rg-open. Then $g\bullet f$ is somewhat M-rg-open.

Theorem 6.2: For a bijective function f, the following are equivalent:
(i) f is somewhat M-rg-open.
(ii) If $C\in RGC(X)$, such that $f(C) \neq Y$, then there is a $D\in RGC(Y)$ such that $D\neq Y$ and $D\supset f(C)$.

Proof: (i) $\Rightarrow$(ii): Let $C\in RGC(X)$ such that $f(C) \neq Y$. Then $X-C\neq \varphi\in GSO(X)$. Since f is somewhat M-rg-open, there exists $V\neq \varphi\in GSO(Y)$ such that $V\subset f(X-C)$. Put $D = Y-V$. Clearly $D\neq \varphi\in RGC(Y)$. If $D = Y$, then $V = \varphi$, which is a contradiction. Since $V\subset f(X-C)$, $D = Y-V \supset (Y- f(X-C)) = f(C)$.

(ii) $\Rightarrow$(i): Let $U\neq \varphi\in RO(X)$. Then $C = X-U\in RGC(X)$ and $f(X-U) = f(C) = Y- f(U)$ implies $f(C) \neq Y$. By (ii), there is $D\in RGC(Y)$ such that $D\neq Y$ and $f(C) \subset D$. Clearly $V = Y-D\neq \varphi\in GSO(Y)$. Also, $V = Y-D \subset Y- f(C) = Y- f(X-U) = f(U)$.

Theorem 6.3: The following statements are equivalent:
(i) f is somewhat M-rg-open.
(ii)If A is a rg-dense subset of Y, then $f^{-1}(A)$ is a rg-dense subset of X.

Proof: (i) $\Rightarrow$(ii): Let A be a rg-dense set in Y. If $f^{-1}(A)$ is not rg-dense in X, then there exists $B\in RGC(X)$ in X such that $f^{-1}(A) \subset B\subset X$. Since f is somewhat M-rg-open and X-B is rg-open, there exists a $C\neq \varphi\in GSO(Y)$ such that $C\subset f(X-B)$. Therefore, $C\subset f(X-B) \subset f(f^{-1}(Y-A)) \subset Y-A$. That is, $A\subset Y-C \subset Y$. Now, Y-C is a rg-closed set and $A\subset Y-C \subset Y$. This implies that A is not a rg-dense set in Y, which is a contradiction. Therefore, $f^{-1}(A)$ is a rg-dense set in X.

(ii) $\Rightarrow$(i): Let $A\neq \varphi\in GSO(X)$. We want to show that $gs(f(A))^{o}\neq \varphi$. Suppose $gs(f(A))^{o} = \varphi$. Then, $RGCl(f(A)) = Y$. Then by (ii), $f^{-1}(Y - f(A))$ is rg-dense in X. But $f^{-1}(Y- f(A)) \subset X-A$. Now, $X-A\in RGC(X)$. Therefore, $f^{-1}(Y- f(A)) \subset X-A$ gives $X = cl(f^{-1}(Y- f(A))) \subset X-A$. Thus $A = \varphi$, which contradicts $A\neq \varphi$. Therefore, $gs(f(A))^{o}\neq \varphi$. Hence f is somewhat M-rg-open.

Theorem 6.4: If f is somewhat M-rg-open and A is r-open in X. Then $f_{/A}:(A; \tau_{/A}) \rightarrow (Y, \sigma)$ is somewhat M-rg-open.

Proof: Let $U \neq \varphi\in GSO(\tau_{/A})$. Since $U\in GSO(A)$ and $A\in RO(X)$, $U\in RO(X)$ and since f is somewhat M-rg-open, there exists $V\in GSO(Y)$, such that $V\subset f(U)$. Thus $f_{/A}$ is somewhat M-rg-open.

Theorem 6.5: Let f be a function and $X = A\cup B$, where $A, B\in GSO(X)$. If $f_{/A}$ and $f_{/B}$ are somewhat M-rg-open, then f is somewhat M-rg-open.

Proof: Let $U\neq \varphi\in RO(X)$. Since $X = A\cup B$, either $A\cap U \neq \varphi$ or $B\cap U \neq \varphi$ or both $A\cap U \neq \varphi$ and $B\cap U \neq \varphi$. Since U is regular open in X, U is regular open in both A and B.

Case (i): If $A\cap U \neq \varphi\in RO(A)$. Since $f_{/A}$ is somewhat M-rg-open, there exists a rg-open set V of Y such that $V\subset f(U\cap A) \subset f(U)$, which implies that f is a somewhat M-rg-open.

Case (ii): If $B\cap U \neq\varphi\in RO(B)$. Since $f_{/B}$ is somewhat M-rg-open, there exists a rg-open set V in Y such that $V\subset f(U\cap B) \subset f(U)$, which implies that f is somewhat M-rg-open.

Case (iii): If both $A\cap U \neq \varphi$ and $B\cap U \neq \varphi$. Then by case (i) and (ii) f is somewhat M-rg-open.

Remark 2: Two topologies τ and σ for X are said to be rg-equivalent if and only if the identity function f is somewhat M-rg-open in both directions.

Theorem 6.6: If $f:(X, \tau)\rightarrow(Y, \sigma)$ is somewhat M-open. Let τ^* and σ^* be topologies for X and Y, respectively such that τ^* is equivalent to τ and σ^* is rg-equivalent to σ. Then $f: (X; \tau^*)\rightarrow (Y; \sigma^*)$ is somewhat M-rg-open.

CONCLUSION: In this paper we defined Somewhat-rg-continuous functions, studied its properties and their interrelations with other types of Somewhat-continuous functions.

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