

WEYL TYPE THEOREMS FOR A-POLAROID OPERATOR

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ABSTRACT

If a Banach space operator T is a -polaroid then it satisfies a -Weyl's theorem iff T has SVEP at $\lambda \notin \sigma_{uw}(T)$. Also an a -polaroid operator can be described as a quasi-nilpotent part of an operator. For such an operator T , $f(T)$ satisfies a -Weyl's theorem for every non-constant function f analytic on a neighbourhood of $\sigma(T)$ if and only if $f(\sigma_{uw}(T)) = \sigma_{uw}(f(T))$.

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1. INTRODUCTION

Throughout this paper X will denote an infinite-dimensional complex Banach space and $\mathcal{L}(X)$, the Banach algebra of bounded linear operators acting on X . For an operator $T \in \mathcal{L}(X)$, let T^* denote the adjoint, $\mathcal{N}(T)$, the kernel, $R(T)$, its range. $\sigma(T)$, $\sigma_a(T)$, $\sigma_s(T)$, $\sigma_p(T)$ denote respectively, the spectrum, approximate spectrum, surjective spectrum and point spectrum. Let $\alpha(T)$ and $\beta(T)$ denote the dimensions of $\mathcal{N}(T)$ and $N(T^*)$, respectively.

Let $\Phi_+(X) := \{T \in \mathcal{L}(X) : R(T) \text{ is closed, } \alpha(T) < \infty\}$ be the class of all upper semi-Fredholm operators and let $\Phi_-(X) := \{T \in \mathcal{L}(X), \beta(T) < \infty\}$ be the class of all lower semi-Fredholm operators. Moreover, $\Phi(X) = \Phi_-(X) \cap \Phi_+(X)$ defines the class of all Fredholm operators. If $T \in \Phi_{\pm}(X)$, the index of T is defined by $\text{in } T = \alpha(T) - \beta(T)$.

Define

$$W(X) := \{T \in \mathcal{L}(X) : T \text{ is Fredholm operator of index zero}\},$$

$$W_+(X) := \{T \in \Phi_+(X), \text{ind} T \leq 0\},$$

$$W_-(X) := \{T \in \Phi_-(X), \text{ind} T \geq 0\}.$$

Thus,

$$W(X) = W_+(X) \cap W_-(X).$$

The classes of operators defined above generate the following spectra. Denote by

$$\sigma_w(T) := \{\lambda \in \mathbb{C} : \lambda I - T \notin W(X)\}$$

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The Weyl spectrum. The Weyl essential approximate point spectrum is defined by

$$\sigma_{iw}(T) := \{\lambda \in \mathbb{C} : \lambda I - T \notin W_+(X)\}$$

while the Weyl essential surjective spectrum by

$$\sigma_{sw}(T) := \{\lambda \in \mathbb{C} : \lambda I - T \notin W_-(X)\}$$

Obviously, $\sigma_w(T) = \sigma_{iw}(T) \cup \sigma_{sw}(T)$ and by Fredholm theory we know

$$\sigma_{iw}(T) = \sigma_{sw}(T^*), \quad \sigma_{sw}(T) = \sigma_{iw}(T^*).$$

The ascent of an operator $T \in \mathcal{L}(X)$ is defined as the smallest non-negative integer $p := p(T)$ such that $\text{Ker } T^p = \text{Ker } T^{p+1}$, the descent of T is defined as the smallest non-negative integer $q := q(T)$ such that $T^q(X) = T^{q+1}(X)$, and if such an integer does not exist we put $q(T) = \infty$. If $p(T)$ and $q(T)$ are both finite then $p(T) = q(T)$. A bounded operator $T \in \mathcal{L}(X)$ is said to be Browder (resp., upper semi-Browder, lower semi-Browder) if $T \in \Phi(X)$ and $p(T) = q(T) < \infty$ (resp., $T \in \Phi_+(X)$ and $p(T) < \infty$, $T \in \Phi_-(X)$ and $q(T) < \infty$)

Let $\mathcal{B}(X)$, $\mathcal{B}_+(X)$, $\mathcal{B}_-(X)$ denote respectively, the classes of Browder operators, upper semi-Browder operators and lower semi-Browder operators. Obviously,

$$\mathcal{B}(X) \subseteq W(X), \quad \mathcal{B}_+(X) \subseteq W_+(X), \quad \mathcal{B}_-(X) \subseteq W_-(X).$$

Let

$\sigma_b(T) := \{\lambda \in \mathbb{C} : \lambda I - T \text{ is not Browder}\}$ denote the Browder spectrum and $\sigma_{ub}(T)$ denote the upper semi-Browder spectrum of T and is defined by

$$\sigma_{ub}(T) := \{\lambda \in \mathbb{C} : \lambda I - T \text{ is not upper semi-Browder}\}.$$

Clearly, $\sigma_w(T) \subseteq \sigma_b(T), \sigma_{iw}(T) \subseteq \sigma_{ub}(T)$.

In the year 2001, the concept of semi-Fredholm operator was generalized by Berkani [7] in the following way: For every $T \in \mathcal{L}(X)$ and a non-negative integer n , $T_{[n]}$, the restriction of T to $R[T^n]$ is viewed as a map from the space $T^n(X)$ into itself (we set $T_{[0]} = T$). T is said to be semi B-Fredholm (resp., B. Fredholm, upper semi B-Fredholm, lower semi-B-Fredholm), if for some integer $n \geq 0$ and $R(T^n)$ is closed and $T_{[n]}$ is a semi-Fredholm operator (resp., Fredholm, upper semi-Fredholm, lower semi-Fredholm). In this case $T_{[m]}$ is a semi-Fredholm operator for all $m \geq n$.

We can define index of semi B-Fredholm as $\text{ind } T = \text{ind } T_{[n]}$. A bounded linear operator $T \in \mathcal{L}(X)$ is said to be B-Weyl (resp., upper semi B-Weyl, lower semi B-Weyl) if T is B-Fredholm of index zero (resp., upper semi B-Fredholm with negative index and lower semi B-Fredholm with positive index).

In this paper we deal with a-polaroid operators, for that purpose we need to define Drazin invertibility in a more abstract way. $T \in \mathcal{L}(X)$ is said to be Drazin invertible if and only if $p(T) = q(T) < \infty$ or $T = T_1 \oplus T_0$ where T_0 is nilpotent and T_1 is invertible [13], so every Drazin invertible operator is B-Fredholm. The Drazin spectrum is defined as

$$\sigma_d(T) := \{\lambda \in \mathbb{C} : \lambda I - T \text{ is not Drazin invertible}\}$$

Also, $\lambda \in \mathbb{C}$ is a pole of $T \in \mathcal{L}(X)$ if $\lambda I - T$ is Drazin invertible, or equivalently if $\lambda \in \mathbb{C}$ is a pole of the

resolvent of T if $0 < p(\lambda I - T) = q(\lambda I - T) < \infty$. In this case λ is an eigenvalue of T and an isolated point of spectrum $\sigma(T)$ [12, Proposition 50.2]. An operator $T \in \mathcal{L}(X)$ is said to be polaroid if every isolated point of $\sigma(T)$ is a pole of the resolvent of T .

A bounded operator $T \in \mathcal{L}(X)$ is said to be a-polaroid if every $\lambda \in \text{iso } \sigma_a(T)$ is a pole of the resolvent of T . Trivially T is a-polaroid $\Rightarrow T$ is polaroid.

The following property has relevant role in local spectral Theory and Fredholm Theory [1].

An operator T is said to have single-valued extension property at $\lambda_0 \in \mathbb{C}$ (abbreviated SVEP at λ_0), if for every open disc D of λ_0 , the only analytic function $f : \mathcal{D} \rightarrow X$ which satisfies the equation $(\lambda I - T)f(\lambda) = 0$ for all $\lambda \in \mathcal{D}$, is the function $f \equiv 0$. An operator $T \in \mathcal{L}(X)$ is said to have SVEP if T has SVEP at every point $\lambda \in \mathbb{C}$. Evidently, $T \in \mathcal{L}(X)$ has SVEP at every isolated point of the spectrum.

Also, if

$$p(\lambda I - T) < \infty \text{ then } T \text{ has SVEP at } \lambda. \tag{1}$$

$$q(\lambda I - T) < \infty \text{ then } T^* \text{ has SVEP at } \lambda. \tag{2}$$

From the definition of localized SVEP, it is evident

$$\sigma_a(T) \text{ does not cluster } \lambda \Rightarrow T \text{ has SVEP at } \lambda \text{ and dually} \tag{3}$$

$$\sigma_s(T) \text{ does not cluster } \lambda \Rightarrow T^* \text{ has SVEP at } \lambda. \tag{4}$$

In particular, if $\sigma_p(T) = \emptyset$, then T has SVEP.

An important subspace in local spectral theory is the quasi-nilpotent part of T defined by

$$H_0(T) := \{x \in X : \lim_{n \rightarrow \infty} \|T^n x\|^{1/n} = 0\}.$$

Obviously, $\ker T^n \subseteq H_0(T)$ for all n . We also have [1, Theorem 2.31].

$$H_0(\lambda I - T) \text{ is closed } \Rightarrow T \text{ has SVEP at } \lambda. \tag{5}$$

Remark 1.1. All implications (1), (2), (3), (4) and (5) are equivalent if $\lambda I - T$ is semi-Fredholm [1, Chapter 3].

Another important subspace in local spectral theory is analytic core $K(T)$ defined as $K(T) = \{x \in X : \text{there exist a constant } c > 0 \text{ and a sequence } \{x_n\} \subset X \text{ such that } x_0 = x, Tx_n = x_{n-1} \text{ and } \|x_n\| \leq c^n \|x\| \text{ for all natural no. } n\}$ (see [1]).

2. A-WEYL'S THEOREM AND A-POLAROID OPERATORS

In this section we show the equivalence between a-Weyl theorem and Kato type operator in case $T \in \mathcal{L}(X)$ is a-polaroid operator. If $T \in \mathcal{L}(X)$ define

$$E(T) := \{\lambda \in \text{iso } \sigma(T) : 0 < \alpha(\lambda I - T)\}$$

and

$$E^a(T) := \{\lambda \in \text{iso } \sigma_a(T) : 0 < \alpha(\lambda I - T)\}.$$

Obviously, $E(T) \subseteq E^a(T)$ for every $T \in \mathcal{L}(X)$.

Define

$$\pi_{00}(T) := \{\lambda \in \text{iso } \sigma(T) : 0 < \alpha(\lambda I - T) < \infty\}$$

$$\pi_{00}^a(T) := \{\lambda \in \text{iso } \sigma_a(T) : 0 < \alpha(\lambda I - T) < \infty\}.$$

Let $p_{00}(T) = \sigma(T) \sim \sigma_b(T)$ or $p_{00}(T)$ is the set of all poles of the resolvent of T having finite rank for every $T \in \mathcal{L}(X)$, we have

$$p_{00}(T) \subseteq \pi_{00}(T) \subseteq \pi_{00}^a(T).$$

For every a-polaroid operator [4, Lemma 3.1]

$$p_{00}(T) = \pi_{00}(T) = \pi_{00}^a(T) \text{ and } E(T) = E^a(T).$$

We now give Weyl theorem and its various variants in the following form:

A bounded operator $T \in \mathcal{L}(X)$ is said to satisfy Weyl's theorem, if

$$\sigma(T) \sim \sigma_w(T) = \pi_{00}(T).$$

$T \in \mathcal{L}(X)$ is said to satisfy a-Weyl's theorem if

$$\sigma_a(T) \sim \sigma_{iw}(T) = \pi_{00}^a(T)$$

Weyl's theorem for T entails Browder's theorem if $\sigma_w(T) = \sigma_b(T)$.

A bounded linear operator $T \in \mathcal{L}(X)$ is said to satisfy property (w) if

$$\sigma_a(T) \sim \sigma_{iw}(T) = \pi_{00}(T).$$

$T \in \mathcal{L}(X)$ is said to satisfy property (b) if

$$\sigma_a(T) \sim \sigma_{iw}(T) = p_{00}(T).$$

The preceding result is reminiscent of the equivalence established in [10, Theorem 2.2] under the assumption that T is a-polaroid.

Theorem 2.1. Let $T \in \mathcal{L}(X)$ be a-polaroid. Then the following are equivalent:

- (i) T satisfies a-Weyl's theorem.
- (ii) T has SVEP at $\lambda \notin \sigma_{iw}(T)$.
- (iii) T has SVEP at $\lambda \notin \sigma_{iw}(T)$ and T is Kato type at $\lambda \in \text{iso } \sigma(T)$.
- (iv) T satisfies a-Browder's theorem and $\text{dsc}(T - \lambda I) < \infty$.
- (v) $\sigma_a(T) = \sigma_{iw}(T) \cup \text{iso } \sigma_a(T)$.

Proof. (i) \Rightarrow (ii). Let T satisfy a-Weyl's Theorem, then $\sigma_a(T) \sim \sigma_{iw}(T) = \pi_{00}^a(T)$. Let $\lambda \in \sigma_a(T) \sim \sigma_{iw}(T)$, $\lambda \notin \sigma_{iw}(T)$ and $\lambda \in \pi_{00}^a(T)$ then $\lambda \in \text{iso } \sigma_a(T)$. Thus, T has SVEP at $\lambda \notin \sigma_{iw}(T)$.

(ii) \Rightarrow (i). If T has SVEP at $\lambda \notin \sigma_{iw}(T)$ then a-Browder's theorem holds for T . Then T satisfies a-Weyl's theorem if $p_{00}^a(T) = \pi_{00}^a(T)$ [1, Theorem 2.14]. We know $p_{00}^a(T) \subseteq \pi_{00}^a(T)$ for every $T \in \mathcal{L}(X)$. We need to prove the reverse inclusion.

For that if $\lambda \in \pi_{00}^a(T)$ then $\lambda \in \text{iso } \sigma_a(T)$ and $0 < p(\lambda I - T) = q(\lambda I - T) < \infty$ as T is a-polaroid.

Also $\alpha(\lambda I - T) < \infty$ therefore, using [1, Theorem 3.4] we get $\beta(\lambda I - T) < \infty$ and hence $\lambda \in p_{00}^a(T)$.

(i) \Rightarrow (v). If T satisfies a-Weyl's theorem $\lambda \in \sigma_a(T) \sim \sigma_{uw}(T) = \pi_{00}^a(T)$. Then $\lambda \in \text{iso } \sigma_a(T)$. Thus, $\sigma_a(T) \subseteq \sigma_{uw}(T) \cup \text{iso } \sigma_a(T)$. Also $\sigma_{uw}(T) \cup \text{iso } \sigma_a(T) \subseteq \sigma_a(T)$ for every operator $T \in \mathcal{L}(X)$. Hence, $\sigma_a(T) = \sigma_{uw}(T) \cup \text{iso } \sigma_a(T)$.

(v) \Rightarrow (iii). Suppose $\sigma_a(T) = \sigma_{uw}(T) \cup \text{iso } \sigma_a(T)$. Let $\lambda \notin \sigma_{uw}(T)$ then either $\lambda \in \text{iso } \sigma_a(T)$ or $\lambda \notin \text{iso } \sigma_a(T)$. If $\lambda \in \text{iso } \sigma_a(T)$ then T has SVEP at λ . If $\lambda \notin \text{iso } \sigma_a(T)$, then $\lambda \notin \sigma_{uw}(T) \cup \text{iso } \sigma_a(T) = \sigma_a(T)$ and hence T has SVEP at λ then using [11, Theorem 3.6] T satisfies a-Browder's theorem and $p_{00}^a(T) = \pi_{00}^a(T)$ (see (ii) \Rightarrow (i)). Thus, a-Weyl's theorem holds for T . Since T is a-polaroid, therefore $\pi_{00}^a(T) = p_{00}(T) = \pi_{00}(T)$. Thus $\lambda \in \sigma_a(T) \sim \sigma_{uw}(T) = \pi_{00}^a(T) = \pi_{00}(T)$. Also $\lambda \in p_{00}(T)$. Thus, λ is a pole of the resolvent. Thus $(\lambda I - T)$ is kato type [17].

(iii) \Rightarrow (iv). T has SVEP and is kato type at $\lambda \in \text{iso } \sigma_a(T)$ then T satisfies a-Browder's theorem and since T is Kato type at $\lambda \in \text{iso } \sigma_a(T)$. Thus $dsc(\lambda I - T) < \infty$ [17].

(iv) \Rightarrow (v). Since T satisfies a-Browder's theorem therefore T has SVEP at $\lambda \notin \sigma_{uw}(T)$.

Therefore $\lambda \in \text{iso } \sigma_a(T)$. Then $\sigma_a(T) \subseteq \sigma_{uw}(T) \cup \text{iso } \sigma(T)$.

Thus, $\sigma_a(T) = \sigma_{uw}(T) \cup \text{iso } \sigma_a(T)$.

The a-Weyl's theorem and property (b) for $T \in \mathcal{L}(X)$ are independent, but the next result shows that if T is a-polaroid then property (b) and property (w) are equivalent to a-Weyl's theorem.

Theorem 2.2. Let $T \in \mathcal{L}(X)$. If T is a-polaroid then the following are equivalent:

- (i) T satisfies property (w).
- (ii) T satisfies a-Weyl's theorem.
- (iii) T satisfies property (b).

We know if T^* has SVEP then $\sigma(T) = \sigma_a(T)$ [1, Corollary 2.45], so that T^* has SVEP and T is polaroid then T is a-polaroid. The following result shows that condition may be described in terms of quasi-nilpotent part of an operator.

Theorem 2.3. Let $T \in \mathcal{L}(X)$. Then T is a-polaroid if and only if there exist $p := p(\lambda) \in \mathbb{N}$ such that

$$H_0(\lambda I - T) = \ker(\lambda I - T)^p \text{ for all } \lambda \in \text{iso } \sigma(T) \tag{6}$$

Proof. Suppose T satisfies (1) and $\lambda \in \text{iso } \sigma(T) \subseteq \text{iso } \sigma_a(T)$. Then there exists $p \in \mathbb{N}$ such that $H_0(\lambda I - T) = \ker(\lambda I - T)^p$.

Since $\lambda \in \text{iso } \sigma(T)$. Then

$$\begin{aligned} X &= H_0(\lambda I - T) \oplus K(\lambda I - T) \\ &= \ker(\lambda I - T)^p \oplus K(\lambda I - T). \end{aligned}$$

Therefore, $X = \ker(\lambda I - T)^p \oplus (\lambda I - T)^p X$ which implies $p(\lambda I - T) = q(\lambda I - T) < p$ and hence λ is a pole of the resolvent so that T is polaroid. Since $q(\lambda I - T) < \infty$ thus T^* has SVEP at λ therefore, T is a-polaroid.

Conversely, T is a-polaroid and hence polaroid. Then by [2, Theorem 2.9] there exists $p \in \mathbb{N}$ such that $H_0(\lambda I - T) = \ker(\lambda I - T)^p$.

An operator $U \in \mathcal{L}(Y, X)$ between two Banach spaces Y and X is said to be a quasi-affinity if U is injective and has dense range. The operator $S \in \mathcal{L}(Y)$ is said to be a quasi-affine transform of $T \in \mathcal{L}(X)$ (notation $S \prec T$), if there is a quasi-affinity $U \in \mathcal{L}(Y, X)$ such that $TU = US$. If both $S \prec T$ and $T \prec S$ hold then S, T are said to be quasi-similar.

Theorem 2.4. Suppose that $T \in \mathcal{L}(X)$ and $S \prec T$. If T is a-polaroid then property (b), property (w) and a-Weyl's theorem equivalently hold for S .

Proof. Since T is a-polaroid, we show that S is a -polaroid. Let $\lambda \in \text{iso } \sigma(T) \subseteq \text{iso } \sigma_a(T)$. Then there exists $p \in \mathbb{N}$ such that $H_0(\lambda I - T) = \text{k r}(\lambda I - eT)^p$. Let $x \in H_0(\lambda I - S)$. Then

$$\begin{aligned} \lim_{n \rightarrow \infty} \|(\lambda I - T)^n Ux\|^{1/n} &= \lim_{n \rightarrow \infty} \|U(\lambda I - S)^n x\|^{1/n} \\ &\leq \lim_{n \rightarrow \infty} \|(\lambda I - S)^n x\|^{1/n} = 0. \end{aligned}$$

So $Ux \in H_0(\lambda I - T) = \text{k r}(\lambda I - eT)^p$.

Therefore,

$$U(\lambda I - S)^p x = (\lambda I - T)^p Ux = 0.$$

and since U is injective, therefore $x \in \text{ker}(\lambda I - S)^p$. This gives $H_0(\lambda I - S) \subseteq \text{k r}(\lambda I - eS)^p$. Since opposite inclusion is true we get $H_0(\lambda I - S) = \text{k r}(\lambda I - eS)^p$. By Theorem 2.2, the result follows.

Let $\mathcal{H}(\sigma(T))$ denote the set of analytic functions which are defined on an open neighbourhood \mathcal{U} of $\sigma(T)$.

Theorem 2.5. Let $T \in \mathcal{L}(X)$ is a-polaroid, $f \in \mathcal{H}(\sigma(T))$. Then

$$f(\sigma_a(T) \sim \pi_{00}^a(T)) = \sigma_a(f(T)) \sim \pi_{00}^a(f(T)).$$

$$\sigma_a(f(T)) \sim \pi_{00}^a(f(T)) \subset f(\sigma_a(T) \sim \pi_{00}^a(T)).$$

Let $\lambda \in f(\sigma_a(T) \sim \pi_{00}^a(T)) \subset f(\sigma_a(T))$. Suppose $\lambda \in \pi_{00}^a(f(T))$ then

$$f(T) - \lambda = (T - \mu_1)(T - \mu_2) \dots (T - \mu_n)g(T)$$

for some $\mu_1, \mu_2 \dots \mu_n \in \sigma_a(T)$ and $g(T)$ is invertible. If some $\mu_i \in \sigma_a(T)$ then $\mu_i \in \text{iso } \sigma_a(T) \subset \pi(T)$ as T is a-polaroid Since λ is an isolated point of $\sigma_a(T)$ of finite multiplicity, then $\mu_i \in \sigma_a(T)$ are poles of finite multiplicity. Since T is a-polaroid, therefore, poles of finite multiplicity are in $\pi_{00}^a(T)$, which is a contradiction.

Thus,

$$\lambda \in \sigma_a(f(T)) \sim \pi_{00}^a(f(T)).$$

It is known that $\sigma_{uw}(f(T)) \subset f(\sigma_{uw}(T))$. The next theorem gives us some sufficient conditions such that the equality holds.

Theorem 2.6. Let $T \in \mathcal{L}(X)$ be a-polaroid then T obeys a-Weyl's theorem for $f \in \mathcal{H}(\sigma(T))$. Then $f(T)$ obeys a-Weyl's theorem if and only if

$$f(\sigma_{uw}(T)) = \sigma_{uw}(f(T)).$$

Proof. By Theorem 2.5 we have

$$f(\sigma_{uw}(T)) = f(\sigma_a(T) \sim \pi_{00}^a(T))$$

$$= \sigma_a(f(T)) \sim \pi_{00}^a(f(T))$$

The right hand side is equal to $\sigma_{uw}(f(T))$ if and only if $f(\sigma_{uw}(T)) = \sigma_{uw}(f(T))$.

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