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# WEYL TYPE THEOREMS FOR A-POLAROID OPERATOR

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**ABSTRACT**

*If a Banach space operator $T$ is a-polaroid then it satisfies a-Weyl's theorem iff $T$ has SVEP at $\lambda \notin \sigma_{uw}(T)$. Also an a-polaroid operator can be described as a quasi-nilpotent part of an operator. For such an operator $T$, $f(T)$ satisfies a-Weyl's theorem for every non-constant function $f$ analytic on a neighbourhood of $\sigma(T)$ if and only if $f(\sigma_{uw}(T)) = \sigma_{uw}(f(T))$.*

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## 1. INTRODUCTION

Throughout this paper $X$ will denote an infinite-dimensional complex Banach space and $\mathcal{L}(X)$, the Banach algebra of bounded linear operators acting on $X$. For an operator $T \in \mathcal{L}(X)$, let $T^*$ denote the adjoint, $\mathcal{N}(T)$, the kernel, $R(T)$, its range. $\sigma(T)$, $\sigma_a(T)$, $\sigma_s(T)$, $\sigma_p(T)$ denote respectively, the spectrum, approximate spectrum, surjective spectrum and point spectrum. Let $\alpha(T)$ and $\beta(T)$ denote the dimensions of $\mathcal{N}(T)$ and $N(T^*)$, respectively.

Let $\Phi_+(X) := \{T \in \mathcal{L}(X) : R(T)$ is closed, $\alpha(T) < \infty\}$ be the class of all upper semi-Fredholm operators and let $\Phi_-(X) := \{T \in \mathcal{L}(X), \beta(T) < \infty\}$ be the class of all lower semi-Fredholm operators. Moreover, $\Phi(X) = \Phi_-(X) \cap \Phi_+(X)$ defines the class of all Fredholm operators. If $T \in \Phi_\pm(X)$, the index of $T$ is defined by $\operatorname{ind} T = \alpha(T) - \beta(T)$.

Define

$$W(X) := \{T \in \mathcal{L}(X) : T \text{ is Fredholm operator of index zero}\},$$

$$W_+(X) := \{T \in \Phi_+(X), indT \le 0\},$$

$$W_-(X) := \{T \in \Phi_-(X), indT \ge 0\}.$$

Thus,

$$W(X) = W_+(X) \cap W_-(X).$$

The classes of operators defined above generate the following spectra. Denote by

$$\sigma_w(T) := \{\lambda \in \mathbb{C} : \lambda I - T \notin W(X)\}$$

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The Weyl spectrum. The Weyl essential approximate point spectrum is defined by

$$\sigma_{uw}(T) := \{\lambda \in \mathbb{C} \ : \ \lambda I - T \notin W_+(X)\}$$

while the Weyl essential surjective spectrum by

$$\sigma_{sw}(T) := \{\lambda \in \mathbb{C} : \ \lambda I - T \notin W_-(X)\}$$

Obviously, $\sigma_w(T) = \sigma_{uw}(T) \cup \sigma_{sw}(T)$ and by Fredholm theory we know
$\sigma_{uw}(T) = \sigma_{sw}(T^*)$, $\sigma_{sw}(T) = \sigma_{uw}(T^*)$.

The ascent of an operator $T \in \mathcal{L}(X)$ is defined as the smallest non-negative integer $p := p(T)$ such that $\operatorname{Ker} T^p = \operatorname{Ker} T^{p+1}$, the descent of $T$ is defined as the smallest non-negative integer $q := q(T)$ such that $T^q(X) = T^{q+1}(X)$, and if such an integer does not exist we put $q(T) = \infty$. If $p(T)$ and $q(T)$ are both finite then $p(T) = q(T)$. A bounded operator $T \in \mathcal{L}(X)$ is said to be Browder (resp., upper semi-Browder, lower semi-Browder) if $T \in \Phi(X)$ and $p(T) = q(T) < \infty$ (resp., $T \in \Phi_+(X)$ and $p(T) < \infty$, $T \in \Phi_-(X)$ and $q(T) < \infty$ )

Let $\mathcal{B}(X)$, $\mathcal{B}_+(X)$, $\mathcal{B}_-(X)$ denote respectively, the classes of Browder operators, upper semi-Browder operators and lower semi-Browder operators. Obviously,

$$\mathcal{B}(X) \subseteq W(X), \ \ \mathcal{B}_+(X) \subseteq W_+(X), \ \ \mathcal{B}_-(X) \subseteq W_-(X).$$

Let
$\sigma_b(T) := \{\lambda \in \mathbb{C} \ : \ \lambda I - T \text{ is not Browder}\}$ denote the Browder spectrum and $\sigma_{ub}(T)$ denote the upper semi-Browder spectrum of $T$ and is defined by

$$\sigma_{ub}(T) := \{\lambda \in \mathbb{C} \ : \ \lambda I - T \text{ is not upper semi-Browder}\}.$$

Clearly, $\sigma_w(T) \subseteq \sigma_b(T), \sigma_{uw}(T) \subseteq \sigma_{ub}(T)$.

In the year 2001,the concept of semi-Fredholm operator was generalized by Berkani [7] in the following way: For every $T \in \mathcal{L}(X)$ and a non-negative integer $n$, $T_{[n]}$, the restriction of $T$ to $R[T^n]$ is viewed as a map from the space $T^n(X)$ into itself (we set $T_{[0]} = T$ ). $T$ is said to be semi B-Fredholm (resp., B. Fredholm, upper semi B-Fredholm, lower semi-B-Fredholm), if for some integer $n \geq 0$ and $R(T^n)$ is closed and $T_{[n]}$ is a semi-Fredholm operator (resp., Fredholm, upper semi-Fredholm, lower semi-Fredholm). In this case $T_{[m]}$ is a semi-Fredholm operator for all $m \geq n$.

We can define index of semi B-Fredholm as $\mathbf{ind}\, T = \mathbf{ind}\, T_{[n]}$. A bounded linear operator $T \in \mathcal{L}(X)$ is said to be B-Weyl (resp., upper semi B-Weyl, lower semi B-Weyl) if $T$ is B-Fredholm of index zero (resp., upper semi B-Fredholm with negative index and lower semi B-Frefholm with positive index).

In this paper we deal with a-polaroid operators, for that purpose we need to define Drazin invertibility in a more abstract way. $T \in \mathcal{L}(X)$ is said to be Drazin invertible if and only if $p(T) = q(T) < \infty$ or $T = T_1 \oplus T_1$ where $T_0$ is nilpotent and $T_1$ is invertible [13], so every Drazin invertible operator is B-Fredholm. The Drazin spectrum is defined as

$$\sigma_d(T) := \{\lambda \in \mathbb{C} \ : \ \lambda I - T \text{ is not Drazin invertible}\}$$

Also, $\lambda \in \mathbb{C}$ is a pole of $T \in \mathcal{L}(X)$ if $\lambda I - T$ is Drazin invertible, or equivalently if $\lambda \in \mathbb{C}$ is a pole of the resolvent of $T$ if $0 < p(\lambda I - T) = q(\lambda I - T) < \infty$. In this case $\lambda$ is an eigenvalue of $T$ and an isolated point of spectrum $\sigma(T)$ [12, Proposition 50.2]. An operator $T \in \mathcal{L}(X)$ is said to be polaroid if every isolated point of $\sigma(T)$ is a pole of the resolvent of $T$.

A bounded operator $T \in \mathcal{L}(X)$ is said to be a-polaroid if every $\lambda \in \text{iso}\, \sigma_a(T)$ is a pole of the resolvent of $T$. Trivially $T$ is a-polaroid $\Rightarrow T$ is polaroid.

The following property has relevant role in local spectral Theory and Fredholm Theory [1].

An operator $T$ is said to have single-valued extension property at $\lambda_0 \in \mathbb{C}$ (abbreviated SVEP at $\lambda_0$ ), if for every open disc $D$ of $\lambda_0$, the only analytic function $f : \mathcal{D} \to X$ which satisfies the equation $(\lambda I - T) f(\lambda) = 0$ for all $\lambda \in \mathcal{D}$, is the function $f \equiv 0$. An operator $T \in \mathcal{L}(X)$ is said to have SVEP if $T$ has SVEP at every point $\lambda \in \mathbb{C}$. Evidently, $T \in \mathcal{L}(X)$ has SVEP at every isolated point of the spectrum.

Also, if

$$p(\lambda I - T) < \infty \text{ then } T \text{ has SVEP at } \lambda. \tag{1}$$

$$q(\lambda I - T) < \infty \text{ then } T^* \text{ has SVEP at } \lambda. \tag{2}$$

From the definition of localized SVEP, it is evident

$$\sigma_a(T) \text{ does not cluster } \lambda \Rightarrow T \text{ has SVEP at } \lambda \text{ and dually} \tag{3}$$

$$\sigma_s(T) \text{ does not cluster } \lambda \Rightarrow T^* \text{ has SVEP at } \lambda. \tag{4}$$

In particular, if $\sigma_p(T) = \phi$, then $T$ has SVEP.

An important subspace in local spectral theory is the quasi-nilpotent part of $T$ defined by

$$H_0(T) := \{x \in X : \lim_{n \to \infty} \| T^n X \|^{1/n} = 0\}.$$

Obviously, $\ker T^n \subseteq H_0(T)$ for all n. We also have [1, Theorem 2.31].

$$H_0(\lambda I - T) \text{ is closed } \Rightarrow T \text{ has SVEP at } \lambda. \tag{5}$$

**Remark 1.1.** *All implications* (1), (2), (3), (4) *and* (5) *are equivalent if* $\lambda I - T$ *is semi-Fredholm* [1, Chapter 3].

Another important subspace in local spectral theory is analytic core $K(T)$ defined as $K(T) = \{x \in X$ : there exist a constant $c > 0$ and a sequence $\{x_n\} \subset X$ such that $x_0 = x$, $Tx_n = x_{n-1}$ and $\| x_n \| \le c^n \| x \|$ for all natural no.n} (see [1]).

## 2. A-WEYL'S THEOREM AND A-POLAROID OPERATORS

In this section we show the equivalence between a-Weyl theorem and Kato type operator in case $T \in \mathcal{L}(X)$ is a-polaroid operator. If $T \in \mathcal{L}(X)$ define

$$E(T) := \{\lambda \in \text{iso}\, \sigma(T) : \ 0 < \alpha(\lambda I - T)\}$$

and

$$E^a(T) := \{\lambda \in \text{iso}\, \sigma_a(T) : \ 0 < \alpha(\lambda I - T)\}.$$

Obviously, $E(T) \subseteq E^a(T)$ for every $T \in \mathcal{L}(X)$.

Define

$\pi_{00}(T) := \{\lambda \in \text{iso}\,\sigma(T) : 0 < \alpha(\lambda I - T) < \infty\}$

$\pi_{00}^{a}(T) := \{\lambda \in \text{iso}\,\sigma_a(T) : 0 < \alpha(\lambda I - T) < \infty\}.$

Let $p_{00}(T) = \sigma(T) \sim \sigma_b(T)$ or $p_{00}(T)$ is the set of all poles of the resolvent of $T$ having finite rank for every $T \in \mathcal{L}(X)$, we have

$p_{00}(T) \subseteq \pi_{00}(T) \subseteq \pi_{00}^{a}(T).$

For every a-polaroid operator [4, Lemma 3.1]

$p_{00}(T) = \pi_{00}(T) = \pi_{00}^{a}(T)$ and $E(T) = E^{a}(T).$

We now give Weyl theorem and its various variants in the following form:

A bounded operator $T \in \mathcal{L}(X)$ is said to satisfy Weyl's theorem, if

$\sigma(T) \sim \sigma_w(T) = \pi_{00}(T)$.

$T \in \mathcal{L}(X)$ is said to satisfy a-Weyl's theorem if

$\sigma_a(T) \sim \sigma_{uw}(T) = \pi_{00}^{a}(T)$

Weyl's theorem for $T$ entails Browder's theorem if $\sigma_w(T) = \sigma_b(T)$ .

A bounded linear operator $T \in \mathcal{L}(X)$ is said to satisfy property $(w)$ if

$\sigma_a(T) \sim \sigma_{uw}(T) = \pi_{00}(T).$

$T \in \mathcal{L}(X)$ is said to satisfy property (b) if

$\sigma_a(T) \sim \sigma_{uw}(T) = p_{00}(T).$

The preceding result is reminiscent of the equivalence established in[10, Theorem 2.2] under the assumption that is T a-polaroid.

**Theorem 2.1.** *Let $T \in \mathcal{L}(X)$ be a-polaroid. Then the following are equivalent:*

(i) $T$ *satisfies a-Weyl's theorem.*

(ii) $T$ *has SVEP at* $\lambda \notin \sigma_{uw}(T)$.

(iii) $T$ *has SVEP at* $\lambda \notin \sigma_{uw}(T)$ *and* $T$ *is Kato type at* $\lambda \in iso\,\sigma(T)$.

(iv) $T$ *satisfies a-Browder's theorem and* $dsc(T - \lambda I) < \infty$ .

(v) $\sigma_a(T) = \sigma_{uw}(T) \cup iso\,\sigma_a(T)$ .

**Proof..** (i) $\Rightarrow$ (ii). Let $T$ satisfy a-Weyl's Theorem, then $\sigma_a(T) \sim \sigma_{uw}(T) = \pi_{00}^{a}(T)$. Let $\lambda \in \sigma_a(T) \sim \sigma_{uw}(T)$, $\lambda \notin \sigma_{uw}(T)$ and $\lambda \in \pi_{00}^{a}(T)$ then $\lambda \in \text{iso}\,\sigma_a(T)$. Thus, $T$ has SVEP at $\lambda \notin \sigma_{uw}(T)$.

(ii) $\Rightarrow$ (i). If $T$ has SVEP at $\lambda \notin \sigma_{uw}(T)$ then a-Browder's theorem holds for $T$. Then $T$ satisfies a-Weyl's theorem if $p_{00}^{a}(T) = \pi_{00}^{a}(T)$ [1, Theorem 2.14]. We know $p_{00}^{a}(T) \subseteq \pi_{00}^{a}(T)$ for every $T \in \mathcal{L}(X)$. We need to prove the reverse inclusion.

For that if $\lambda \in \pi_{00}^{a}(T)$ then $\lambda \in \text{iso}\,\sigma_a(T)$ and $0 < p(\lambda I - T) = q(\lambda I - T) < \infty$ as $T$ is a-polaroid.

Also $\alpha(\lambda I - T) < \infty$ therefore, using [1, Theorem 3.4] we get $\beta(\lambda I - T) < \infty$ and hence $\lambda \in p_{00}^{a}(T)$.

 

(i) $\Rightarrow$ (v). If $T$ satisfies a-Weyl's theorem $\lambda \in \sigma_a(T) \sim \sigma_{uw}(T) = \pi_{00}^a(T)$. Then $\lambda \in \text{iso}\,\sigma_a(T)$. Thus, $\sigma_a(T) \subseteq \sigma_{uw}(T) \cup \text{iso}\,\sigma_a(T)$. Also $\sigma_{uw}(T) \cup \text{iso}\,\sigma_a(T) \subseteq \sigma_a(T)$ for every operator $T \in \mathcal{L}(X)$. Hence, $\sigma_a(T) = \sigma_{uw}(T) \cup \text{iso}\,\sigma_a(T)$.

(v) $\Rightarrow$ (iii). Suppose $\sigma_a(T) = \sigma_{uw}(T) \cup \text{iso}\,\sigma_a(T)$. Let $\lambda \notin \sigma_{uw}(T)$ then either $\lambda \in \text{iso}\,\sigma_a(T)$ or $\lambda \notin \text{iso}\,\sigma_a(T)$. If $\lambda \in \text{iso}\,\sigma_a(T)$ then $T$ has SVEP at $\lambda$. If $\lambda \notin \text{iso}\,\sigma_a(T)$, then $\lambda \notin \sigma_{uw}(T) \cup \text{iso}\,\sigma_a(T) = \sigma_a(T)$ and hence $T$ has SVEP at $\lambda$ then using [11, Theorem 3.6] $T$ satisfies a-Browder's theorem and $p_{00}^a(T) = \pi_{00}^a(T)$ (see (ii) $\Rightarrow$ (i)). Thus, a-Weyl's theorem holds for $T$. Since $T$ is a-polaroid, therefore $\pi_{00}^a(T) = p_{00}(T) = \pi_{00}(T)$. Thus $\lambda \in \sigma_a(T) \sim \sigma_{uw}(T) = \pi_{00}^a(T) = \pi_{00}(T)$. Also $\lambda \in p_{00}(T)$. Thus, $\lambda$ is a pole of the resolvent. Thus $(\lambda I - T)$ is kato type [17].

(iii) $\Rightarrow$ (iv). $T$ has SVEP and is kato type at $\lambda \in \text{iso}\,\sigma_a(T)$ then $T$ satisfies a-Browder's theorem and since $T$ is Kato type at $\lambda \in \text{iso}\,\sigma_a(T)$. Thus $dsc(\lambda I - T) < \infty$ [17].

(iv) $\Rightarrow$ (v). Since $T$ satisfies a-Browder's theorem therefore $T$ has SVEP at $\lambda \notin \sigma_{uw}(T)$.

Therefore $\lambda \in \text{iso}\,\sigma_a(T)$. Then $\sigma_a(T) \subseteq \sigma_{uw}(T) \cup \text{iso}\,\sigma(T)$.
Thus, $\sigma_a(T) = \sigma_{uw}(T) \cup \text{iso}\,\sigma_a(T)$.

The a-Weyl's theorem and property (b) for $T \in \mathcal{L}(X)$ are independent, but the next result shows that if $T$ is a-polaroid then property (b) and property (w) are equivalent to a-Weyl's theorem.

**Theorem 2.2.** *Let $T \in \mathcal{L}(X)$. If $T$ is a-polaroid then the following are equivalent:*

(i) $T$ *satisfies property* $(w)$.
(ii) $T$ *satisfies a-Weyl's theorem.*
(iii) $T$ *satisfies property* $(b)$.

*We know if $T^*$ has SVEP then* $\sigma(T) = \sigma_a(T)$ [1, Corollary 2.45], *so that $T^*$ has SVEP and $T$ is polaroid then $T$ is a-polaroid. The following result shows that condition may be described in terms of quasi-nilpotent part of an operator.*

**Theorem 2.3.** *Let $T \in \mathcal{L}(X)$. Then $T$ is a-polaroid if and only if there exist $p := p(\lambda) \in \mathbb{N}$ such that*

$$H_0(\lambda I - T) = \ker(\lambda I - T)^p \text{ for all } \lambda \in \text{iso}\,\sigma(T) \tag{6}$$

**Proof.** Suppose $T$ satisfies (1) and $\lambda \in \text{iso}\,\sigma(T) \subseteq \text{iso}\,\sigma_a(T)$. Then there exists $p \in \mathbb{N}$ such that $H_0(\lambda I - T) = \text{k r}(\lambda I - eT)^p$.
Since $\lambda \in \text{iso}\,\sigma(T)$. Then

$$X = H_0(\lambda I - T) \oplus K(\lambda I - T)$$

$$= \ker(\lambda I - T)^p \oplus K(\lambda I - T).$$

Therefore, $X = \text{k r}(\lambda I - eT)^p \oplus (\lambda I - T)^p X$ which implies $p(\lambda I - T) = q(\lambda I - T) < p$ and hence $\lambda$ is a pole of the resolvent so that $T$ is polaroid. Since $q(\lambda I - T) < \infty$ thus $T^*$ has SVEP at $\lambda$ therefore, $T$ is a-polaroid.

Conversely, $T$ is a-polaroid and hence polaroid. Then by [2, Theorem 2.9] there exists $p \in \mathbb{N}$ such that $H_0(\lambda I - T) = \text{k r}(\lambda I - eT)^p$.

 

An operator $U \in \mathcal{L}(Y, X)$ between two Banach spaces $Y$ and $X$ is said to be a quasi-affinity if $U$ is injective and has dense range. The operator $S \in \mathcal{L}(Y)$ is said to be a quasi-affine transform of $T \in \mathcal{L}(X)$ (notation $S \prec T$), if there is a quasi-affinity $U \in \mathcal{L}(Y, X)$ such that $TU = US$. If both $S \prec T$ and $T \prec S$ hold then $S$, $T$ are said to be quasi-similar.

**Theorem 2.4.** *Suppose that $T \in \mathcal{L}(X)$ and $S \prec T$. If $T$ is a-polaroid then property $(b)$, property $(w)$ and a-Weyl's theorem equivalently hold for $S$.*

**Proof.** Since $T$ is a-polaroid, we show that $S$ is a -polaroid. Let $\lambda \in \text{iso}\,\sigma(T) \subseteq \text{iso}\,\sigma_a(T)$. Then there exists $p \in \mathbb{N}$ such that $H_0(\lambda I - T) = \ker(\lambda I - T)^p$. Let $x \in H_0(\lambda I - S)$. Then

$$\begin{aligned}\lim_{n\to\infty} \|(\lambda I - T)^n Ux\|^{1/n} &= \lim_{n\to\infty} \|U(\lambda I - S)^n x\|^{1/n} \\ &\le \lim_{n\to\infty} \|(\lambda I - S)^n x\|^{1/n} = 0.\end{aligned}$$

So $Ux \in H_0(\lambda I - T) = \ker(\lambda I - T)^p$.

Therefore,

$$U(\lambda I - S)^p x = (\lambda I - T)^p Ux = 0.$$

and since $U$ is injective, therefore $x \in \ker(\lambda I - S)^p$. This gives $H_0(\lambda I - S) \subseteq \ker(\lambda I - S)^p$. Since opposite inclusion is true we get $H_0(\lambda I - S) = \ker(\lambda I - S)^p$. By Theorem 2.2, the result follows.

Let $\mathcal{H}(\sigma(T))$ denote the set of analytic functions which are defined on an open neighbourhood $\mathcal{U}$ of $\sigma(T)$.

**Theorem 2.5.** *Let $T \in \mathcal{L}(X)$ is a-polaroid, $f \in \mathcal{H}(\sigma(T))$. Then*

$f(\sigma_a(T) \sim \pi_{00}^a(T)) = \sigma_a(f(T)) \sim \pi_{00}^a(f(T)).$ For any $T \in \mathcal{L}(X)$

$$\sigma_a(f(T)) \sim \pi_{00}^a(f(T)) \subset f(\sigma_a(T) \sim \pi_{00}^a(T)).$$

Let $\lambda \in f(\sigma_a(T) \sim \pi_{00}^a(T)) \subset f(\sigma_a(T))$. Suppose $\lambda \in \pi_{00}^a(f(T))$ then

$$f(T) - \lambda = (T - \mu_1)(T - \mu_2)\ldots(T - \mu_n)g(T)$$

for some $\mu_1, \mu_2 \ldots \mu_n \in \sigma_a(T)$ and $g(T)$ is invertible. If some $\mu_i \in \sigma_a(T)$ then $\mu_i \in \text{iso}\,\sigma_a(T) \subset \pi(T)$ as $T$ is a-polaroid Since $\lambda$ is an isolated point of $\sigma_a(T)$ of finite multiplicity, then $\mu_i \in \sigma_a(T)$ are poles of finite multiplicity. Since $T$ is a-polaroid, therefore, poles of finite multiplicity are in $\pi_{00}^a(T)$, which is a contradiction.

Thus,

$$\lambda \in \sigma_a(f(T)) \sim \pi_{00}^a(f(T)).$$

It is known that $\sigma_{uw}(f(T)) \subset f(\sigma_{uw}(T))$. The next theorem gives us some sufficient conditions such that the equality holds.

**Theorem 2.6.** *Let $T \in \mathcal{L}(X)$ be a-polaroid then $T$ obeys a-Weyl's theorem for $f \in \mathcal{H}(\sigma(T))$. Then $f(T)$ obeys a-Weyl's theorem if and only if*

$$f(\sigma_{uw}(T)) = \sigma_{uw}(f(T)).$$

**Proof.** By Theorem 2.5 we have

$$\begin{aligned}f(\sigma_{uw}(T)) &= f(\sigma_a(T) \sim \pi_{00}^a(T)) \\ &= \sigma_a(f(T)) \sim \pi_{00}^a(f(T))\end{aligned}$$

The right hand side is equal to $\sigma_{uw}(f(T))$ if and only if $f(\sigma_{uw}(T)) = \sigma_{uw}(f(T))$.

 

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