International Journal of Mathematical Archive-3(9), 2012, 3306-3314
Available online through www.ijma.info ISSN 2229 – 5046

STOCHASTIC MANPOWER SYSTEMS WITH GEOMETRIC RECRUITMENT PERIOD DEPENDING ON ATTRITION DURING THE BUSY PERIOD

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(Received on: 14-07-12; Revised & Accepted on: 11-09-12)

ABSTRACT

Manpower system is getting much more attention these days due to the absolute neccessity of manpower for any business. A system in which employees leave the organization during busy period and new employees are recruited after the busy period. The project completion time is busy period. Two models are discussed. In Model A, the busy period has general distribution and the inter departure time of employees has exponential distribution and in Model B the busy period has exponential distribution and the inter departure time of employees has general distribution. The recruitments are made with inter-recruitment times in accordance with Geometric process. Joint Laplace transforms of busy period and recruitment time, their means and co-variance are obtained. Numerical results are presented.

AMS subject classification: *30C45, 30C80, 90B05*

Keywords: *Manpower system, Loss of manpower and recruitment, Geometric process, Joint transform.*

INTRODUCTION

It is very common that business faces manpower availability and shortage periods. These two periods are liable to be dependent on each other. If more persons leave the organization due to several reasons then the business would be severely affected during busy period. As loss and shortcomings are inevitable and fund management is to be done during busy period and in the recruitment period, one may absolutely have to speed up recruitments using different strategies in order to start business early. The duration of busy period and the duration to recruit employees are random and they occur alternately in any business organization. When a busy period is long, one may like to speed up recruitment, so as to start the next busy period early. A systematic approach to manpower system was made as early as 1947 by Vajda [7] and others. For clear understanding of manpower planning, one can refer to Bartholomew [1], Grinold and Marshall [2] and Vajda [8]. Lesson [3] had given methods to compute wastages (Resignation, dismissal and death) and promotion intensities which produce the proportions corresponding to some desired planning proposals.

Stochastic models were designed for wastage and promotion in manpower system by Vassiliou [9]. V. Subramanian [6] had made an attempt to provide optimal policy for recruitment training, promotion, and wastages in manpower planning models, with special provisions such as time bound promotions, cost of training and voluntary retirement scheme. For other manpower models one may refer K. Setlhare [5]. For three characteristics system in manpower models one may refer to C. Mohan and R. Ramanarayanan [4]. So far no models in these areas have been studied in which recruitments occur in accordance with Geometric Processes. The random variables considered for them are usually independent and identically distributed. It is natural to consider that such stochastic times either increase or decrease but not identical. The recruitment times may decrease geometrically by continuous refinement and hiring more people and machines for recruitments. So far the non identical nature of such inter occurrence times, are not considered at any depth.

In this paper, we consider that the first recruitment time is general and subsequent such times are geometric. We consider two models. In model A the busy period of operation of an organization has general distribution and the inter-departure time of employees has exponential distribution and in model B the busy period of operation has exponential

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distribution and the inter-departure time of employees has general distribution. After the busy period, recruitment of employees starts to fill up the vacancies, due to employees leaving the organization during the busy period in accordance with Geometric process. The joint transform of busy period and total recruitment time, the expected values and co-variance are obtained. Numerical examples are also presented for illustration.

MODEL A: GENERAL BUSY PERIOD AND CONSTANT DEPARTURE RATE

The main assumptions of the model are given below.

1. The busy period T of an organization is a random variable with cumulative distribution function (cdf) F(x) and probability density function (pdf) f(x).

2. The inter-departure times of employees are independent random variables with exponential distribution with rate λ.

Let N be the number of employees left during the operation time T.

3. After the busy period T the recruitment starts. The recruitment time of the i- th employee is $Y_i = \left(\frac{W_i}{a^{i-1}}\right)$, where the Cdf of Y_i is $G_i(y)$ with pdf $g_i(y)$ for i = 1, 2 …., with a >1 and W_i for i = 1, 2, 3,… are independent and identically distributed random variables with cdf $G_1(y)$ with pdf $g_1(y)$.

Noting that the number of employees left during busy period has Poisson distribution, we may derive the joint distribution of T, N, and S as follows. The joint probability density function of T and S and probability function of N is

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}P(T \le x, N = n, S \le y)\right) = f(x,n,y)$$
$$= f(x)\left\{e^{-\lambda x}\frac{(\lambda x)^n}{n!}\right\}h_n(y) \tag{1}$$

for n = 0, 1, 2, 3…. The right side of equation (1) is the pdf that the busy period is x, the recruitment time is y, the number of employees left is 'n' during the busy period. Here

$$h_n(X) = g_1(y)\Theta g_2(y)\Theta g_3(y) \ldots . \Theta g_n(y) \tag{2}$$

for n = 1, 2, 3..., Θ denotes convolution. When a = 1, $h_n(y)$ n - fold convolution of g(y) with itself for n = 0, 1, 2, 3,.... Here $h_0(y) = 1$ for $y = 0$ and 0 for $y > 0$. The Laplace transform of equation is,

$$h_n^*(s) = \prod_1^n g_i^*(s)$$
$$= \prod_1^n g_1^*\left(\frac{s}{a^{i-1}}\right) \tag{3}$$

where * indicates Laplace transform. Let us define the joint Laplace transform as follows.

$$E(e^{-tT}e^{-sS}) = \int_0^\infty \int_0^\infty \sum_{n=0}^\infty f(x,n,y)\, e^{-tx} e^{-sy}\, dx dy.$$

$$= \int_0^\infty \int_0^\infty \sum_0^\infty f(x)\, e^{-tx} e^{-sy} e^{-\lambda x}\left(\frac{(\lambda x)^n}{n!}\right) h_n(y) dx dy.$$

This on simplification gives,

$$E(e^{-tT}e^{-sS}) = \sum_0^\infty \int_0^\infty f(x)\, e^{-tx} e^{-sy} e^{-\lambda x}\left(\frac{(\lambda x)^n}{n!}\right) h_n^*(y) dx.$$

Using equation (3),

$$E(e^{-tT}e^{-sS}) = f^*(t+\lambda) + \sum_1^\infty \int_0^\infty f(x)\, e^{-tx} e^{-sy} e^{-\lambda x}\frac{(\lambda x)^n}{n!}\prod_1^n g_1^*\left(\frac{s}{a^{i-1}}\right) dx \tag{4}$$

We may note from equation (4) for s = 0, $E(e^{-tT}) = f^*(t)$ and for $t = 0$,

$$E(e^{-sS}) = f^*(\lambda) + \sum_1^\infty \int_0^\infty f(x)\, e^{-tx}\, e^{-sy}\, e^{-\lambda x} \left(\frac{(\lambda x)^n}{n!}\right) \prod_1^n g_1^*\left(\frac{s}{a^{i-1}}\right) dx \tag{5}$$

For a = 1 we get simple result as follows.

$$E(e^{-tT}\, e^{-sS}) = f^*\left(t + \lambda\big(1 - g^*(s)\big)\right).$$
$$E(e^{-sS}) = f^*\left(\lambda\big(1 - g^*(s)\big)\right).$$

Using differentiations we get,

$$E(S) = \sum_{n=1}^{\infty} \int_0^\infty f(x) e^{-\lambda x} \left(\frac{(\lambda x)^n}{n!}\right) \sum_1^n \left(\frac{1}{a^{i-1}}\right) dx E(Y_1).$$
$$= \sum_{n=1}^{\infty} \int_0^\infty f(x) e^{-\lambda x} \left(\frac{(\lambda x)^n}{n!}\right) \left(\frac{1 - \frac{1}{a^n}}{1 - \frac{1}{a}}\right) dx E(Y_1).$$

On simplification we get for a > 1,

$$E(S) = \left(\frac{a}{a-1}\right) E(Y_1) \left[1 - f^*\left(\lambda\left(1 - \frac{1}{a}\right)\right)\right] \tag{6}$$

For a = 1, we obtain using L'Hospital rule,

$$E(S) = \lambda E(Y_1) E(T).$$

Using differentiation of equation of (5),

$$E(TS) = \sum_{n=1}^{\infty} \int_0^\infty x f(x) e^{-\lambda x} \left(\frac{(\lambda x)^n}{n!}\right) \sum_1^n \left(\frac{1}{a^{i-1}}\right) dx E(Y_1).$$

We get after calculation

$$E(TS) = \left(\frac{a}{a-1}\right) E(Y_1) \left[E(T) - \int_0^\infty x f(x) e^{-\lambda x}\, e^{\left(\frac{\lambda x}{a}\right)} dx\right] \tag{7}$$

and for a = 1, we obtain using L'Hospital rule

$$E(TS) = \lambda\, E(Y_1) E(T^2)$$

When T is exponential with parameter μ we get,

$$E(TS) = \lambda\, E(Y_1) \left(\frac{2\lambda}{\mu^2}\right) \tag{8}$$

$$Cov(T,S) = -\left(\frac{a}{a-1}\right)\, E(Y_1) \left[\int_0^\infty x f(x) e^{-\lambda x}\, e^{\left(\frac{\lambda x}{a}\right)} dX - E(Y_1) f^*\left(\lambda\left(1 - \frac{1}{a}\right)\right)\right] \tag{9}$$

and for a = 1, we obtain using L'Hospital rule,

$$Cov(T,S) = \lambda\, E(Y_1) Var(T).$$

When 'T' is exponential with parameter $'\mu'$, we get

$$Cov(T,S) = \lambda\, E(Y_1) \left(\frac{1}{\mu^2}\right).$$

MODEL B: GENERAL DEPARTURE TIME AND EXPONENTIAL OPERATION TIME

The main assumptions of the model are given below.

1. The operation time T is a random variable with exponential distribution function with parameter $'\mu'$.

2. The inter-departure times of employees are independent random variables with Cdf F(x) and pdf f(x). Let N be the number of employees left during the operation time T.

3. After the busy period T the recruitment starts. The recruitment time of the i- th employee is $Y_i = \left(\frac{W_i}{a^{i-1}}\right)$, where the Cdf of Y_i is $G_i(y)$ with pdf $g_i(y)$ for i = 1, 2 …., with a >1 and W_i for i = 1, 2, 3,… are independent and identically distributed random variables with cdf $G_1(y)$ with pdf $g_1(y)$.

We may derive the joint distribution of T, N and S as follows. The joint probability density function of T and S and probability function of N is

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}P(T \le x, N = n, S \le y)\right) = f(x,n,y)$$
$$= \mu e^{-\mu x}\,[F_n(x) - F_{n+1}(x)]h_n(y), \tag{10}$$

for n = 1, 2, 3..., The right side of equation (10) is the pdf that the busy period is x, the recruitment time is y, the number of employees left is n during the busy period. Here $h_n(y)$ and its Laplace transforms are seen earlier in equations (2) and (3). Further $F_k(x)$ is the k fold Cdf convolution and is the probability that the time for k departure of employees is less than x, for k = 1, 2, 3... Let us define the joint Laplace transform as follows.

$$E(e^{-tT}e^{-sS}) = \int_0^\infty \int_0^\infty \sum_{n=0}^{\infty} f(x,n,y)\, e^{-tx} e^{-sy}\, dxdy.$$
$$= \int_0^\infty \int_0^\infty \sum_{n=0}^{\infty} \mu e^{-\mu x}\,[F_n(x) - F_{n+1}(x)]h_n(y)\, e^{-tx} e^{-sy}\, dxdy.$$

This on simplification gives,

$$E(e^{-tT}e^{-sS}) = \left(\frac{\mu}{t+\mu}\right)[1 - f^*(t+\lambda)]\left[1 + \sum_1^\infty f^{*n}(t+\lambda)\prod_1^n g_1^*\left(\frac{s}{a^{i-1}}\right)\right] \tag{11}$$

$$E(e^{-tT}) = \left(\frac{\mu}{t+\mu}\right), E(e^{-sS}) = [1 - f^*(\mu)]\left[1 + \sum_1^\infty f^{*n}(\mu)\prod_1^n g_1^*\left(\frac{s}{a^{i-1}}\right)\right], \tag{12}$$

We may get by differentiation,

$$E(S) = [1 - f^*(\mu)]\left[\sum_1^\infty f^{*n}(\mu)\left(\frac{1-\frac{1}{a^n}}{1-\frac{1}{a}}\right)E(Y_1)\right].$$

This on calculation becomes

$$E(S) = \left(\frac{af^*(\mu)E(Y_1)}{a - f^*(\mu)}\right) \tag{13}$$

$$E(TS) = \frac{\partial}{\partial t \partial s}^2\left(\frac{\mu}{t+\mu}\right)[1 - f^*(t+\mu)]\left[1 + \sum_1^\infty f^{*n}(t+\mu)\prod_1^n g_1^*\left(\frac{s}{a^{i-1}}\right)\right]_{t=0,s=0}$$
$$= -\frac{\partial}{\partial t}\left(\frac{\mu}{t+\mu}\right)[1 - f^*(t+\mu)]\left[\sum_1^\infty f^{*n}(t+\mu)\left(\frac{1-\frac{1}{a^n}}{1-\frac{1}{a}}\right)E(Y_1)\right]_{t=0}$$

Simplifying further we find,

$$E(TS) = -aE(Y_1)\frac{\partial}{\partial t}\left\{\frac{\mu f^*(t+\mu)}{(t+\mu)\big(a - f^*(t+\mu)\big)}\right\}_{t=0}$$

This reduces to

$$E(TS) = -a\left(\frac{E(Y_1)}{\mu\big(1-f^*(\mu)\big)^2}\right)\{a\mu f^{*'}(\mu) - af^*(\mu) + f^{*2}(\mu)\}. \tag{14}$$

For a = 1,

$$E(TS) = -\left(\frac{E(Y_1)}{\mu\left(1-f^*(\mu)\right)^2}\right)\{\mu f^{*'}(\mu) - f^*(\mu) + f^{*2}(\mu)\}.$$

When inter departure time is exponential with parameter 'λ' we obtain as seen earlier in equation (8)

$$E(TS) = E(Y_1)\left(\frac{2\lambda}{\mu^2}\right) \tag{15}$$

Using equations (14) and (13) after simplification we obtain

$$Cov(T,S) = -\left(\frac{a^2 E(Y_1) f^{*'}(\mu)}{\left(a-f^*(\mu)\right)^2}\right). \tag{16}$$

For $a = 1$ we get,

$$Cov(T,S) = -\left(\frac{E(Y_1) f^{*'}(\mu)}{\left(1-f^*(\mu)\right)^2}\right). \tag{17}$$

When inter departure time is exponential with parameter 'λ' we obtain as seen earlier in model A.

$$Cov(T,S) = \lambda\, E(Y_1)\left(\frac{1}{\mu^2}\right).$$

NUMERICAL ILLUSTRATIONS

We consider the case of the busy period, the inter-departure time of employees, have exponential distributions with rate parameters 'μ' and 'λ'. The expected recruitment time and the co-variance are given below. From equations (6) and (13) we get

$$E(S) = \frac{a\,\lambda\, E(Y_1)}{a(\lambda+\mu)-\mu} \tag{18}$$

From Equation (9) and (16) we find

$$Cov(T,S) = \frac{a^2\,\lambda\, E(Y_1)}{[a(\lambda+\mu)-\mu]^2} \tag{19}$$

We vary the busy period parameter 'λ' is equal to 10, 20, 30, 40 and the inter-departure time of employees 'μ' is equal to 5, 10, 15, 20.

We fix $a = 2$ **and** $E(Y_1) = 5$

Table (i): The effect of variation of 'λ' and 'μ' on $E(S)$ and $Cov(T,S)$.

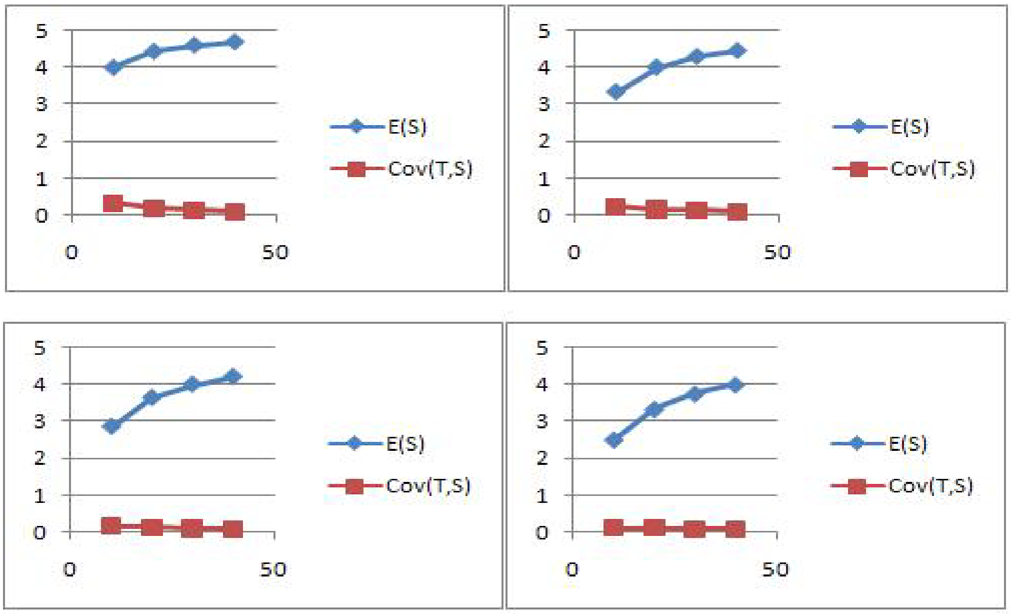
$E(S)$

λ / μ	5	10	15	20
10	4	3.3333	2.8571	2.5
20	4.4444	4	3.6364	3.3333
30	4.6154	4.2857	4	3.75
40	4.7059	4.4444	4.2105	4

$Cov(T,S)$

λ / μ	5	10	15	20
10	0.32	0.2222	0.1633	0.125
20	0.1975	0.16	0.1322	0.1111
30	0.1420	0.1224	0.1067	0.0938
40	0.1107	0.0988	0.0886	0.08

The effect of variation of 'λ' and 'μ' on $E(S)$ and $Cov(T,S)$



The increase in the values of 'μ' decreases expected recruitment time and the increase in the values of 'λ' increases expected recruitment time and When 'λ' and 'μ' increase $Cov(T,S)$ decreases.

We fix $a = 5$ **and** $E(Y_1) = 5$

Table (ii): The effect of variation of 'λ' and 'μ' on $E(S)$ and $Cov(T,S)$.

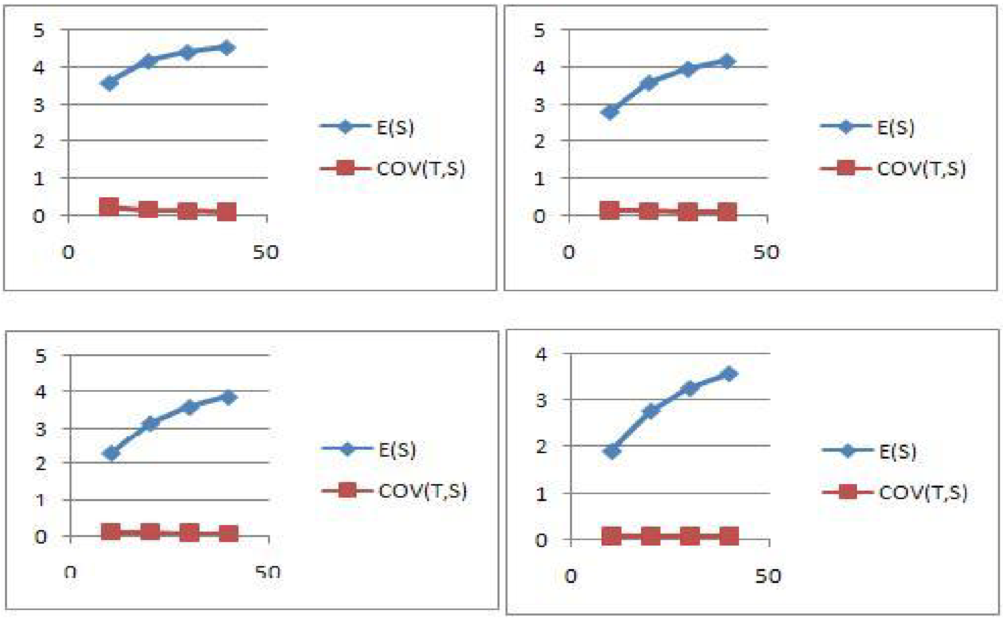
$E(S)$

λ / μ	5	10	15	20
10	3.5714	2.7778	2.2727	1.9231
20	4.1667	3.5714	3.125	2.7778
30	4.4118	3.9474	3.5714	3.2609
40	4.5455	4.1667	3.8462	3.5714

$Cov(T,S)$

λ / μ	5	10	15	20
10	0.2551	0.1543	0.1033	0.0740
20	0.1736	0.1276	0.0977	0.0772
30	0.1298	0.1039	0.0850	0.0709
40	0.1033	0.0868	0.0740	0.0638

The effect of variation of 'λ' and 'μ' on $E(S)$ and $Cov(T,S)$



The increase in the values of 'μ' decreases expected recruitment time and the in $Cov(T,S)$ decreases.

We fix $a = 12$ **and** $E(Y_1) = 5$

Table (iii): The effect of variation of 'λ' and 'μ' on $E(S)$ and $Cov(T,S)$.

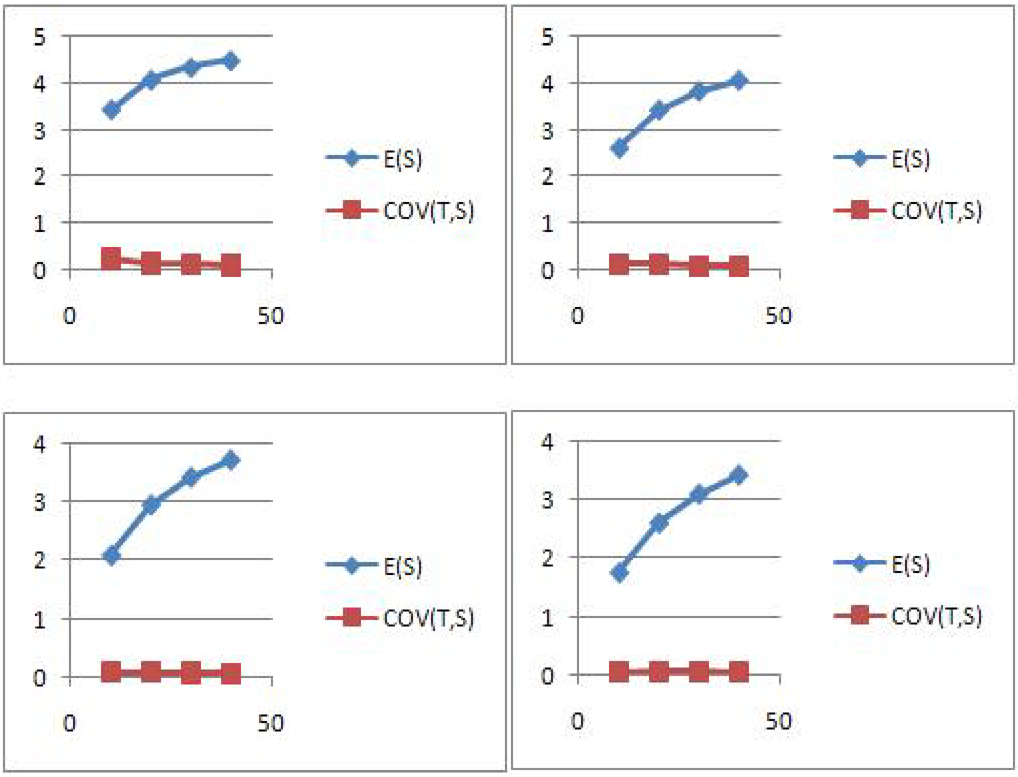
$E(S)$

λ \ μ	5	10	15	20
10	3.4286	2.6087	2.1052	1.7647
20	4.0678	3.4286	2.9630	2.6087
30	4.3374	3.8298	3.4286	3.1035
40	4.4860	4.0678	3.7209	3.4286

$Cov(T,S)$

λ \ μ	5	10	15	20
10	0.2351	0.1361	0.0886	0.0628
20	0.1655	0.1176	0.0878	0.0681
30	0.1254	0.0978	0.0784	0.0642
40	0.1006	0.0827	0.0692	0.0588

The effect of variation of 'λ' and 'μ' on $E(S)$ and $Cov(T,S)$



The increase in the values of 'μ' decreases expected recruitment time and the increase in the values of 'λ' increases expected recruitment time also when 'λ' and 'μ' increase $Cov(T,S)$ decreases.

We fix $a = 20$ **and** $E(Y_1) = 5$

Table (iv): The effect of variation of 'λ' and 'μ' on $E(S)$ and $Cov(T,S)$.

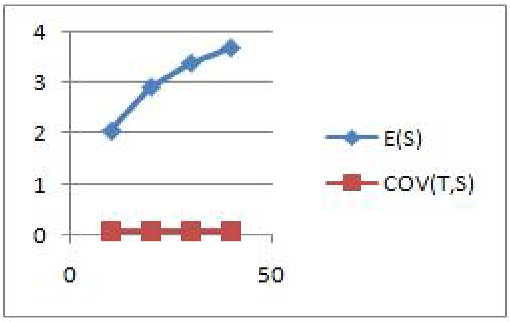
$E(S)$

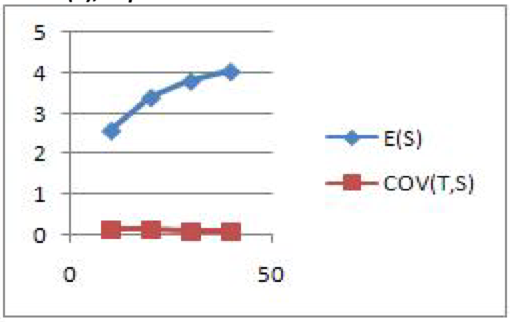
λ \ μ	5	10	15	20
10	3.3898	2.5641	2.0619	1.7241
20	4.0404	3.3898	2.9197	2.5641
30	4.3165	3.7975	3.3898	3.0612
40	4.4693	4.0404	3.6866	3.3898

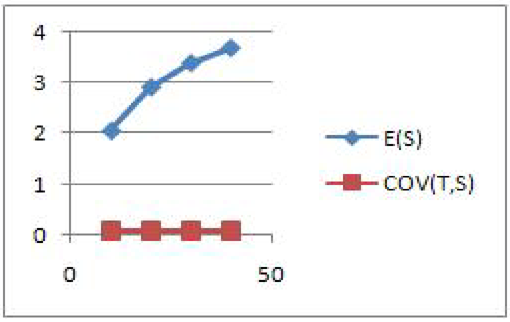
$Cov(T,S)$

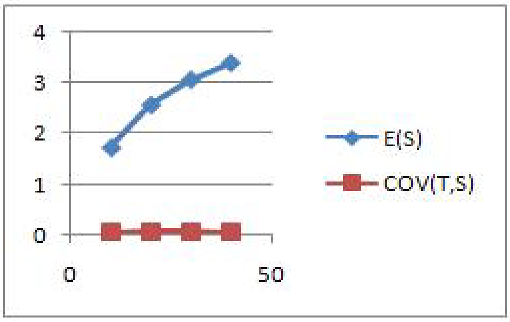
λ \ μ	5	10	15	20
10	0.2295	0.1315	0.0850	0.0595
20	0.1633	0.1149	0.0853	0.0657
30	0.1242	0.0961	0.0766	0.0624
40	0.0999	0.0816	0.0680	0.0575

The effect of variation of 'λ' and 'μ' on $E(S)$ and $Cov(T,S)$









The increase in the values of 'μ' decreases expected recruitment time and the increase in the values of 'λ' increases expected recruitment time also when 'λ' and 'μ' increase $Cov(T,S)$ decreases.

Finally we conclude that, when'a' and 'μ' increase $E(S)$ decreases and when 'a' and 'λ' increase $E(S)$ decreases. When 'a' and 'μ' increase $Cov(T,S)$ decreases and when'a' and 'μ'increase $Cov(T,S)$ decreases.

CONCLUSION

From the above results it is found that,

- When the parameter 'μ' alone increases both the expected time for recruitment and the co variances between the busy period and recruitment time decreases.
- When the parameter 'λ' alone increases both the expected time for recruitment increases and the co variances between the busy period and recruitment time decreases.
- When both the parameters 'λ' and 'μ' increase the expected time for recruitment remains same and the co variances between the busy period and recruitment time decreases.
- when 'a' increases both the expected time for recruitment and the covariances between the busy period and recruitment time decrease.
- When the parameter 'μ'and 'a' increase both the expected time for recruitment and the co variances between the busy period and recruitment time decrease.
- When the parameter 'λ' and 'a' increase both the expected time for recruitment and the co variances between the busy period and recruitment time decrease.

In all the cases the co variances between the busy period and recruitment time decreases.

ACKNOWLEDGEMENTS

The authors thank Queen Mary's College Chennai, Sacred Heart College Tirupattur, and VelTech Dr. RR and Dr.SR, Technical University for providing necessary facilities for research.

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Source of support: Queen Mary's College Chennai, Sacred Heart College Tirupattur,
Conflict of interest: None Declared