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EFFECT OF ECONOMIC CONSTRAINT ON TIME COST TRADE-OFF PROBLEM IN PROJECT SCHEDULING

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ABSTRACT

The time cost trade-off problem is a type of project scheduling problem which studies how to transform project activities so as to achieve the trade-off between the project cost and the completion time. In the real projects, the trade-off between project cost and project completion time and the economic constraint with respect to renewable and non-renewable resources are considerable aspects of decision makers. In this paper, we present the impact of economic constraint on time cost trade-off problems with renewable and non-renewable resources by constructing new linear programming models. The mathematical linear programming models are illustrated using a project network.

Keywords: *Time cost trade-off, renewable and non-renewable resources, economic constraint, AoA network; AoN network.*

1. INTRODUCTION

An acyclic and directed project network that depicts precedence relationships between activities and includes unique start and finish nodes are termed as activity on arc (AoA). Where as in the activity on node (AoN) network each activity is represented by a node and each arc is the symbolic representation of precedence relationships between two activities. In conventional networks many times dummy activities have to be included to represents the precedence relationship between the activities to maintain the logic, this results in increase number of activities. The project is modeled by an AoA, where its arcs represent project activities and its nodes define specific events. AoN networks can be converted into AoA networks, i.e., activities on nodes are can be converted to activities on arcs where arcs suggest the flow of work or the progress of the project, while activities on nodes make the network appear static. The users of arc diagram argue that the identification of the activities in numeric form, that is by the tail and head event numbers (i,j) makes it more suitable for computer programming. The objective of this paper is to construct the complete and efficient time, cost profile with respect to economic constraint of a project. Few mathematical models have been developed to minimize duration and cost by considering renewable, non-renewable resources, which relates the shortest project duration with respect to the budgetary constraint. The project has to be supported financially and all previous approaches presented earlier solved the time cost trade-off problems by assuming that the project budget as the fixed amount i.e., the project would never deviate from the original estimate which is non-renewable. Sometimes the assumptions are unrealistic as the project is generally funded by one or several external financing sources which includes financial institutions, investors and government, as well as via various financial alternatives such as loans, bonds etc. And these financial alternatives are connected with various levels of market, business and political risk. Therefore the project might be forced to stop either temporarily or even permanently, in either of the cases the project cannot be completed in acceptable time frame. Therefore it is important to compute and incorporate the financial(economic) constraint into the time cost trade – off problems.

2. PROBLEM FORMULATION

A project scheduling network is a set of precedence-constrained relations that has to be scheduled so as to minimize a given objective. In project scheduling network, the jobs additionally compete for limited resources. Due to its universality, the latter problem has a variety of applications in manufacturing, production planning, project management etc. It is one of the most difficult problems in operations research, and has therefore become an accepted platform for the latest optimization techniques, which includes virtually all local search paradigms. An activity is completed in number of ways, each of which corresponds to time and cost requirements. Since the acceleration of an activity requires additional resources and hence contributes to a higher cost. The main objective is to find the sets of decisions that lead to desirable project duration and cost. This process is called as the time cost trade-off analysis. In the time cost trade-off problems the decision maker has to decide to crash the activities to meet a required deadline or

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wants to evaluate whether crashing the activities are worth with the additional cost as presented by murthy [1]. The two objectives of time cost trade-off problems are to minimize cost and to minimize duration, this solution is termed as Pareto optimal i.e., there is no other shorter duration under a given budget or no lower cost under the required duration as presented by De et al. [2]. The Pareto optimal is also termed as Minimum time cost curve, which is negatively sloped to the origin of time cost coordinate axis. Time cost trade-off problems [3-6] have been developed from 1950's parallel with the development of critical path method. The objective is to construct the complete and efficient time, cost profile with respect to financial constraint of a project. Novel mathematical models have been developed to minimize duration and cost by considering renewable, non-renewable resources, which relates the shortest project duration with respect to the budgetary constraint. New linear programming (LP) models of AOA network to minimize the project duration subject to the financial constraint are presented.

Notations and Mathematical Models

C_c = crash cost of an activity
N_c = normal cost of an activity
N_T = normal duration of an activity
C_T = crash duration of an activity
E_i = event time of a node i
T_{req} = project delivery time
s = start event of an activity
f = end event of an activity
t_{ij} = the project duration of an activity i,j
$C_{ij}*$ = total cost of an activity i, j
C_i*= cost of an activity i
t_i = duration of an activity i
Es_i = earliest start time of an activity i
Ef_j = earliest finish time of an activity i
r_{ij} = the amount of renewable resource r
P_r = price of the renewable resource r
$C_{ij}**$ = cost of the activity (i,j) = $t_{ij} \times r_{ij} \times P_r$ + the cost of non renewable resources

The linear programming model of AoA network to minimize the project duration subject to the financial constraint

(i) Minimize E_j

subject to the constraints

$$\sum_{\forall(i,j)} C_{ij}* = \sum_{\forall(i,j)} B_{ij} + \sum_{\forall(i,j)} A_{ij}t_{ij} \leq B_{\max} \quad \text{where } B_{\max} \text{ is maximum available budget}$$

$$E_i + t_{ij} - E_j \leq 0 \forall(i,j)$$

$$C_T \leq t_{ij} \leq N_T, 0\forall(i,j),$$

$$E_s = 0, \; E_j \leq T_{req}$$

$$E_j, t_{ij} \geq 0$$

where A_{ij}, B_{ij} and $C_{ij}*$ are cost slope, intercept on the y-axis (fig :) i.e., cost axis and cost of an activity respectively, are given by

$$A_{ij} = \frac{C_c - N_c}{N_T - C_T}$$

$$B_{ij} = C_c - A_{ij}C_T$$

$$C_{ij}* = B_{ij} + A_{ij}t_{ij}$$

If the study interchanges the objective function and constraints will be of the form

i) Maximize E_j

subject to the constraints

$$\sum_{\forall(i,j)} C_{ij}* = \sum_{\forall(i,j)} B_{ij} + \sum_{\forall(i,j)} A_{ij}t_{ij} \leq B_{\max} \quad \text{where } B_{\max} \text{ is maximum available budget}$$

$$E_i + t_{ij} - E_j \le 0 \quad \forall(i,j)$$
$$C_T \le t_{ij} \le N_T, 0\forall(i,j)$$
$$E_s = 0,\ E_j \ge T_{req}$$
$$E_j, t_{ij} \ge 0$$

ii) Minimize $\sum_{\forall(i,j)} C_{ij}* = \sum_{\forall(i,j)} B_{ij} + \sum_{\forall(i,j)} A_{ij}t_{ij}$

subject to the constraints

$$E_i + t_{ij} - E_j \le 0\forall(i,j)$$
$$C_T \le t_{ij} \le N_T, 0\forall(i,j)$$
$$E_s = 0,\ E_j \le T_{req}$$
$$E_j, t_{ij} \ge 0$$

iii) Maximize $\sum_{\forall(i,j)} C_{ij}* = \sum_{\forall(i,j)} B_{ij} + \sum_{\forall(i,j)} A_{ij}t_{ij}$ subject to the constraints

$$E_i + t_{ij} - E_j \le 0 \quad \forall(i,j)$$
$$C_T \le t_{ij} \le N_T, 0\forall(i,j),$$
$$E_s = 0,\ E_j \ge T_{req}$$
$$E_j, t_{ij} \ge 0$$

The linear programming model of AoN network to minimize the project duration subject to the budgetary constraint

v) Minimize E_j subject to

$$\sum_{\forall i} C_i* = \sum_{\forall i} B_i + \sum_{\forall i} A_i t_i \le B_{\max}$$

where $B_{\max}$ is maximum available budget

$$E_i + t_i - E_j \le 0\forall i$$
$$C_T \le t_i \le N_T, 0\forall i,$$
$$E_s = 0,\ E_f \le T_{req}$$
$$E_i, t_i \ge 0$$

where A_i, B_i and C_i* are cost slope, intercept on the y-axis i.e., cost axis and cost of an activity respectively and $B_{\max}$ denotes the level of budget and is expressed as the random variable in lieu of the fixed estimate and the constraint is called the financial constraint.

vi) Maximize E_j subject to

$$\sum_{\forall i} C_i* = \sum_{\forall i} B_i + \sum_{\forall i} A_i t_i \le B_{\max}$$

where $B_{\max}$ is maximum available budget

$$E_i + t_i - E_j \le 0\forall(i,j)$$
$$C_T \le t_i \le N_T, 0\forall i$$
$$E_s = 0,\ E_f \ge T_{req}$$
$$E_i, t_i \ge 0$$

vi) Minimize

$$\sum_{\forall i} C_i* = \sum_{\forall i} B_i + \sum_{\forall i} A_i t_i$$

Subject to

$$E_i + t_i - E_j \le 0\forall(i,j)$$

$$C_T \le t_i \le N_T, 0 \forall i$$
$$E_s = 0, \; E_f \le T_{req}$$
$$E_i, t_i \ge 0.$$

vii) Maximize

$$\sum_{\forall i} C_i * = \sum_{\forall i} B_i + \sum_{\forall i} A_i t_i$$

Subject to

$$E_i + t_i - E_j \le 0 \forall (i, j)$$
$$C_T \le t_i \le N_T, 0 \forall i$$
$$E_s = 0, \; E_f \ge T_{req}$$
$$E_i, t_i \ge 0$$

The linear programming model for AoA network using renewable and non renewable resources to minimize the project duration subject to the budgetary constraint

i) Minimize E_j subject to

$$\sum_{\forall (i,j)} C_{ij} ** = r_{ij} P_r \sum_{\forall (i,j)} t_{ij} + \sum_{\forall (i,j)} \alpha_{ij} \le B_{\max}$$
$$E_i + t_{ij} - E_j \le 0 \forall (i, j)$$
$$C_T \le t_{ij} \le N_T, 0 \forall (i, j)$$
$$E_s = 0, \; E_j \le T_{req}$$
$$E_j, t_{ij} \ge 0$$

where $B_{\max}$ denotes the level of budget and is expressed as the random variable in lieu of the fixed estimate and the constraint is called the financial constraint.

ii) Maximize E_j subject to

$$\sum_{\forall (i,j)} C_{ij} ** = r_{ij} P_r \sum_{\forall (i,j)} t_{ij} + \sum_{\forall (i,j)} \alpha_{ij} \le B_{\max}$$
$$E_i + t_{ij} - E_j \le 0 \forall (i, j)$$
$$C_T \le t_{ij} \le N_T, 0 \forall (i, j)$$
$$E_s = 0, \; E_j \ge T_{req}$$
$$E_j, t_{ij} \ge 0$$

iii) Minimize

$$\sum_{\forall (i,j)} C_{ij} ** = r_{ij} P_r \sum_{\forall (i,j)} t_{ij} + \sum_{\forall (i,j)} \alpha_{ij}$$

Subject to

$$E_i + t_{ij} - E_j \le 0 \forall (i, j)$$
$$t_{ij} \le N_T, \; t_{ij} \ge C_T$$
$$E_s = 0, \; E_j \le T_{req}$$
$$E_j, t_{ij} \ge 0$$

where

C_{ij}** is the cost calculated using renewable and non renewable resources and is given by
C_{ij}** = cost of the activity (i,j) = $t_{ij} \times r_{ij} \times P_r$ + the cost of non renewable resources
α_{ij} is the cost of non-renewable resources

vi) Maximize $\sum_{\forall(i,j)} C_{ij}{**} = r_{ij} P_r \sum_{\forall(i,j)} t_{ij} + \sum_{\forall(i,j)} \alpha_{ij}$ subject to

$$E_i + t_{ij} - E_j \leq 0 \forall (i, j)$$

$$t_{ij} \leq N_T, \quad t_{ij} \geq C_T$$

$$E_s = 0, \quad E_j \geq T_{req}$$

$E_j, t_{ij} \geq 0$ where $C_{ij}{**}$ = cost of the activity (i,j) = $t_{ij} \times r_{ij} \times P_r$ + the cost of non renewable resources

3. SOLUTION PROCEDURE

For an AoA network, a LP model which includes objective function and constraints is formulated with respect to financial (economic) constraint without considering the renewable and non-renewable resources. And for the same AoA network, a LP model is formulated with financial constraint by considering renewable and non-renewable resources. Finally, these two models are illustrated by an example. Both the models are compared.

Numerical Example:

The data of Housing construction by SSR Constructions is given in Table-I and the project network is represented in Fig. 1.

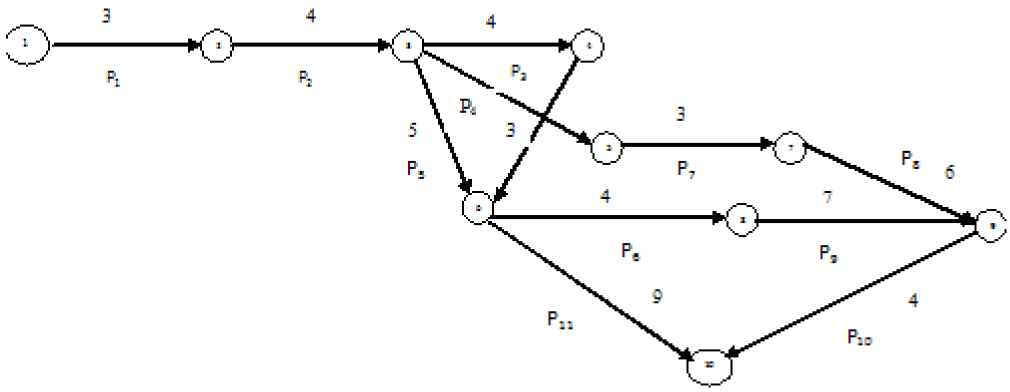

Fig. 1: Project Network of House Construction project

Table I: Housing Project description

Activity	Activity Name	Immediate Predecessor	Normal Time	Crash Time	Normal Cost	Crash Cost	Max Reduction in time	Cost Slope (A_{ij})
1-2	P_1	---	3	2	5000	7000	1	2000
2-3	P_2	P_1	4	2	4000	5000	2	500
3-4	P_3	P_2	4	4	7000	7000	0	---
3-5	P_4	P_2	3	1	3000	5000	2	1000
3-6	P_5	P_2	5	2	6000	10500	3	1500
6-8	P_6	P_5, P_3	4	3	8000	10000	1	2000
5-7	P_7	P_4	3	1	4000	5500	2	750
7-9	P_8	P_7	6	4	6000	9000	2	1500
8-9	P_9	P_6	7	4	5000	8000	3	1000
9-10	P_{10}	P_8, P_9	4	2	6000	7500	2	750
6-10	P_{11}	P_3, P_5	9	7	3000	4000	2	500

Cost intercepts for the project are calculated using the equation $B_{ij} = C_c - A_{ij} C_T$

For example for activities 1-2 and 2-3

B_{12} = 7000 – 2000 × 2 = 3000

B_{23} = 5000 – 500 × 2 = 4000 etc.

Now substituting these values in the model (i), we get

Minimize $E_{P11} = E_{610}$

Subject to constraints

First constraint

$$\sum_{\forall(i,j)} C_{ij} * = \sum_{\forall(i,j)} B_{ij} + \sum_{\forall(i,j)} A_{ij} t_{ij} \le B_{\max}$$
$$= C_{P1} + C_{P2} + C_{P3} + C_{P4} + C_{P5} + C_{P6} + C_{P7} + C_{P8} + C_{P9} + C_{P10} + C_{P11}$$
$$= \left(B_{P1} + B_{P2} + B_{P3} + B_{P4} + B_{P5} + B_{P6} + B_{P7} + B_{P8} + B_{P9} + B_{P10} + B_{P11}\right) +$$
$$\left(2000t_{P1} + 500t_{P2} + 0t_{P3} + 1000t_{P4} + 1500t_{P5} + 2000t_{P6} + 750t_{P7} + 1500t_{P8} + 1000t_{P9} + 750t_{P10} + 500t_{P11}\right)$$
$$\le B_{\max}$$
$$= (47750) + \left(2000t_{12} + 500\,t_{23} + 0t_{34} + 1000\,t_{35} + 1500\,t_{36} + 2000\,t_{68} + 750\,t_{57} + 1500\,t_{79} + 1000\,t_{89} + 750\,t_{910} + 500\,t_{610}\right)$$
$$\le 78{,}500$$
$$= \left(2000\,t_{12} + 500t_{23} + 0\,t_{34} + 1000\,t_{35} + 1500\,t_{36} + 2000\,t_{68} + 750\,t_{57} + 1500\,t_{79} + 1000\,t_{89} + 750t_{910} + 500t_{610}\right)$$
$$\le 30{,}750$$

Second set of constraint are given as follows: $E_i + t_{ij} - E_j \le 0 \forall (i, j)$

For an activity 1-2 and 2-3 are

$$E_1 + t_{12} - E_2 \le 0 \qquad E_2 + t_{23} - E_3 \le 0$$
$$i.e., E_1 + t_{12} \le E_2 \qquad i.e., E_2 + t_{23} \le E_3$$

Third set of constraints are as follows, $C_T \le t_{ij} \le N_T, 0 \forall (i, j)$, Boundary condition

$2 \le t_{12} \le 3$, $2 \le t_{23} \le 4$, $t_{34} = 4$

$1 \le t_{35} \le 3$, $2 \le t_{36} \le 5$, $3 \le t_{68} \le 4$

$1 \le t_{57} \le 3$, $4 \le t_{79} \le 6$, $4 \le t_{89} \le 7$

$2 \le t_{910} \le 4$, $7 \le t_{610} \le 9$

$E_4 \le E_6$, for the dummy activity 4 – 6

Fourth and fifth constraints are

$E_s = 0$, $E_j \le T_{req}$, $E_1 = 0$, $E_{10} \le 17$

Finally the non-negative constraint is

$$E_j, t_{ij} \ge 0$$
$$E_2, E_3, E_4, E_5, E_6, E_7, E_8, E_9, E_{10} \ge 0$$
$$t_{12}, t_{23}, t_{34}, t_{35}, t_{36}, t_{68}, t_{57}, t_{79}, t_{89}, t_{910}, t_{610} \ge 0$$

The LP model is solved by TORA software and the proposed model can help to quantify the importance of budget and can help us to evaluate the financial risks in project management. Therefore with respect to the maximum available budget of ₹. 78,500, the minimum length of the project is 17 weeks. Now consider the same example to illustrate the LP models with respect to renewable and non- renewable resources

Minimize $E_j = E_{10}$

subject to

$$\sum_{\forall(i,j)} C_{ij}^{**} = r_{ij} P_r \sum_{\forall(i,j)} t_{ij} + \sum_{\forall(i,j)} \alpha_{ij} \le B_{\max}$$

$$\begin{aligned} C_{P1}+C_{P2}+C_{P3}+C_{P4}+C_{P5}+C_{P6}+C_{P7}+C_{P8}+C_{P9}+C_{P10}+C_{P11} \\ = 4\times 6\left(t_{P1}+t_{P2}+t_{P3}+t_{P4}+t_{P5}+t_{P6}+t_{P7}+t_{P8}+t_{P9}+t_{P10}+t_{P11}\right) \\ +\left(\alpha_{P1}+\alpha_{P2}+\alpha_{P3}+\alpha_{P4}+\alpha_{P5}+\alpha_{P6}+\alpha_{P7}+\alpha_{P8}+\alpha_{P9}+\alpha_{P10}+\alpha_{P11}\right) \\ \le B_{\max} \end{aligned}$$

$$\left(t_{12}+t_{23}+t_{34}+t_{35}+t_{36}+t_{68}+t_{57}+t_{79}+t_{89}+t_{910}+t_{610}\right) \le 896$$

Rest of the constraints are consider as per the above model.

The LP model is solved using TORA software and the proposed model can help to quantify the importance of budgets by using renewable and non-renewable resources. Similarly the rest of the models can be explained as above illustration.

CONCLUSION

In this paper, we have developed new Linear programming models with impact of financial constraint with renewable and non-renewable resources and without considering the renewable and non-renewable resources. Few mathematical models have been developed to minimize duration and cost by considering renewable, non-renewable resources, which relates the shortest project duration with respect to the budgetary constraint. We adopt the enumeration arithmetic to fit the cost of the project within the maximum available budget.

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