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SOME COMMON FIXED POINT THEOREMS IN FUZZY METRIC SPACE

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ABSTRACT

This paper presents some common fixed point theorems for occasionally weakly compatible mappings in fuzzy metric space under various conditions.

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Keywords: *Occasionally weakly compatible mappings, fuzzy metric space.*

INTRODUCTION

Fuzzy set was defined by Zadeh [24].Kramosil and Michalek [12] introduced fuzzy metric space, George and Veeramani [4] modified the notion of fuzzy metric spaces with the help of continuous t-norms. Many researchers have obtained common fixed point theorems for mappings satisfying different types of commutativity conditions.

Vasuki[23] proved fixed point theorems for R-weakly commutating mappings. Pant [16, 17, 18] introduced the new concept reciprocally continuous mappings and established some common fixed point theorems.

Balasubramaniam et al. [2] have shown that Rhoades [20] open problem on the existence of contractive definition which generates a fixed point but does not force the mappings to be continuous at the fixed point, posses an affirmative answer.

Pant and Jha [18] obtained some analogous results proved by Balasubramaniam et al.

Recent literature on fixed point in fuzzy metric space can be viewed in [7, 14, 22].This paper presents some common fixed point theorems for more general commutative condition i.e, occasionally weakly compatible mappings in fuzzy metric space. Before giving the results, some preliminary definitions are given below.

1. PRELIMINARIES

Definition 1.1 [21]: A fuzzy set A in X is a function with domain X and values in [0, 1]

Definition 1.2[18]: A binary operation $*$:[0,1]x[0,1]→[0,1] is a continuous t-norm if $*$ is satisfying conditions:
(1) $*$ is an commutative and associative
(2) * is continuous
(3) $a * 1 = a$ for all $a \epsilon [0,1]$
(4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ and $a, b, c, d \epsilon [0,1]$

Example 1.3[4]: Example for continuous t-norms are
(1) $a * b = ab$ for all $a, b \epsilon [0,1]$
(2) $a * b = min\{a, b\}$ for all $a, b \epsilon [0,1]$

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Definition 1.4[4]: A 3-tuple $(X, M, *)$ is said to be a fuzzy metric space if X is an arbitrary set,$*$ is a continuous t-norm and M is a fuzzy set on X^2x$[0,\infty]$ satisfying the following conditions for all $x, y, z\epsilon X$ such that t is in [0,∞]
(FM-1) $M(x, y, t) > 0$;
(FM-2) $M(x, y, t) = 1$ if and only if $x = y$
(FM-3) $M(x, y, t) = M(y, x, t)$;
(FM-4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;
(FM-5) $M(x, y, *): (0, \infty) \rightarrow (0,1]$ is continuous

Then M is called a fuzzy metric on X.

Then $M(x, y, t)$ denotes the degree of nearness between x and y with respect to t

Example 1.5 [4]: Let (X, d) be a metric space. Denote $a * b = ab$ for all $a, b\epsilon[0,1]$ and let M_d be fuzzy sets on X^2x(0,∞) defined as follows:

$$M_d(x, y, t) = \frac{t}{t+d(x,y)}$$

Then $(X, M_d, *)$ is a fuzzy metric space.

We call this fuzzy metric induced by a metric d the standard intuitionistic fuzzy metric

Definition 1.6 [4]: Let $(X, M, *)$ be a fuzzy metric space.Then
(1) a sequence $\{x_n\}$ in X is said to converge to x in X if for each $\varepsilon > 0$ and each $t > 0$, there exists $n_0\epsilon N$ such that $M(x_n, x, t) > 1 - \varepsilon$ for all $n \geq n_0$
(2) a sequence $\{x_n\}$ in X is said to be Cauchy if for each $\varepsilon > 0$ and each t>0 there exists $n_0\epsilon N$ such that $M(x_n, x_m, t) > 1 - \varepsilon$ for all $n, m \geq n_0$
(3) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 1.7[20]: A pair of self mappings (f, g) of a fuzzy metric space $(X, M, *)$ is said to be
(1) weakly commuting if $M(fgx, gfx, t) \geq M(fx, gx, t)$ for all $x\epsilon X$ and t> 0
(2) R-weakly commuting if there exists some R> 0 such that $M(fgx, gfx, t) \geq M(fx, gx, {}^t/_R)$ for all $x\epsilon X$ and $t > 0$

Definition 1.8[11]: Two self-mappings f and g of a fuzzy metric space $(X, M, *)$ are called compatible if $\lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$ for some $x\epsilon X$

Definition 1.9 [5]: Two self-maps f and g of a fuzzy metric $(X, M, *)$ are called reciprocally continuous on X if $\lim_{n\to\infty} fgx_n$=fx and $\lim_{n\to\infty} gfx_n$=gx whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$ for some $x\epsilon X$

Definition 1.10: Let X be a set, f , g self-maps of X . A point $x\epsilon X$ is called a coincidence point f and g if and only if $fx = gx$.

We shall call $w = fx = gx$ a point of coincidence of f and g.

Definition 1.11[11]: A pair maps S and T is called weakly compatible pair if they commute at coincidence points

Definition 1.12: Two self-maps f and g of a set X are occasionally weakly compatible if and only if there is point $x\epsilon X$ which is a coincidence point of f and g at which f and g are commute

Remark 1.13: A. Al-Thagafi and Naseer Shahzad [1] have shown that occasionally weakly is weakly compatible but converse is not true

Example 1.14 [1]: Let R be usual metric space.

Define $S, T: R \rightarrow R$ by $Sx = 3x$ and $Tx = x^2$ for all $x\epsilon R$.

Then $Sx = Tx$ for $x = 0,3$ but $ST0 = TS0$ and $3 \neq TS$. S and T are occasionally weakly compatible self-maps but not weakly compatible.

Lemma 1.15 [10]: Let X be a set, f, g be occasionally weakly compatible self-maps of X.

If f and g have unique point of coincidence, $w = fx = gx$, then w is the unique common fixed point of f and g.

2. MAIN RESULT

Theorem 2.1: Let $(X, M,*)$ be complete fuzzy metric space and let A, B and $\{S_i\}_{i=1}^{n}$ and $\{T_j\}_{j=1}^{m}$ be self-mappings of X.

Let pairs $\{A, S_i\}_{i=1}^{n}$ and $\{B, T_j\}_{j=1}^{m}$ be occasionally weakly compatible at x and y respectively.

If there exists $q\epsilon(0,1)$ such that

$$M(Ax, By, qt) \geq min\left\{\begin{array}{c}\{M(S_1x, T_jy, t)\}_{j=1}^{m}, \{M(S_2x, T_jy, t)\}_{j=1}^{m}, \dots, \{M(S_nx, T_jy, t)\}_{j=1}^{m}, \{M(S_ix, Ax, t)\}_{i=1}^{n}, \\ \{M(By, T_jy, t)\}_{j=1}^{m}, \frac{\sum_{j=1}^{m}\alpha_j M(Ax, T_jy, t) + \sum_{i=1}^{n}\beta_i M(By, S_ix, t)}{\sum_{j=1}^{m}\alpha_j + \sum_{i=1}^{n}\beta_i}\end{array}\right\} \quad (1)$$

for all $x, y\epsilon X$,for all $t > 0$ and α_j's ≥ 0 and $\beta_i's \geq 0$ with atleast one of α_j's or one of β_i's is non-zero, then there exists a unique point $w\epsilon X$ such that $Aw = S_iw = w$ for all i and a unique point $z\epsilon X$ such that $Bz = T_jz = z$ for all j.

More over $z = w$ so that there is a unique common fixed point of $A, B, \{S_i\}_{i=1}^{n} and \{T_j\}_{j=1}^{m}$

Proof 2.1: Let the pairs $\{A, S_i\}_{i=1}^{n}$ *and* $\{B, T_j\}_{j=1}^{m}$ be occasionally weakly compatible at the points $x, y\epsilon X$ respectively.

Then $Ax = S_ix \ and\ By = T_jy\ for\ all\ i\ and\ j$

Claim:$Ax = By$

If not,

$$M(Ax, By, t) \geq min\left\{\begin{array}{c}\{M(S_1x, T_jy, t)\}_{j=1}^{m}, \{M(S_2x, T_jy, t)\}_{j=1}^{m}, \dots, \{M(S_nx, T_jy, t)\}_{j=1}^{m}, \{M(S_ix, Ax, t)\}_{i=1}^{n}, \\ \{M(B_y, T_jy, t)\}_{j=1}^{m}, \frac{\sum_{j=1}^{m}\alpha_j M(Ax, T_jy, t) + \sum_{i=1}^{n}\beta_i M(By, S_ix, t)}{\sum_{j=1}^{m}\alpha_j + \sum_{i=1}^{n}\beta_i}\end{array}\right\}$$

$$= min\left\{M(Ax, By, t), M(Ax, Ax, t), M(By, By, t), \frac{\sum_{j=1}^{m}\alpha_j M(Ax, By, t) + \sum_{i=1}^{n}\beta_i M(By, Ax, t)}{\sum_{j=1}^{m}\alpha_j + \sum_{i=1}^{n}\beta_i}\right\}$$

$$= min\left\{M(Ax, By, t), 1,1, M(Ax, By, t)\left(\frac{\sum_{j=1}^{m}\alpha_j + \sum_{i=1}^{n}\beta_i}{\sum_{j=1}^{m}\alpha_j + \sum_{i=1}^{n}\beta_i}\right)\right\}$$

$$= min\{M(Ax, By, t), 1,1, M(Ax, By, t)\}$$

$$= M(Ax, By, t)$$

a contradiction.

Therefore $Ax = By$

i.e,$Ax = S_ix = By = T_jy$

Suppose that there is a another point z such that $Az = S_iz$ then by (1) we have $Az = S_iz = By = T_jy\ so\ Az = Ax\ and\ w = Ax = S_ix$ is the unique point of coincidence of $A\ and\ S_i$

By lemma1.15, w is the only common fixed point of $A\ and\ S_i$

Similarly there is a unique point $z\epsilon X$ such that $z = Bz = T_jz$

Uniqueness: Assume that $w \neq z$

We have

$$M(w,z,qt) = M(Aw,Bz,t)$$

$$\geq min\left\{\begin{array}{c}\{M(S_1w,T_jz,t)\}_{j=1}^{m}, \{M(S_2w,T_jz,t)\}_{j=1}^{m}, \dots, \{M(S_nw,T_jz,t)\}_{j=1}^{m} \\ \{M(S_iw,Aw,t)\}_{i=1}^{n}, \{M(Bz,T_jz,t)\}_{j=1}^{m}, \\ \frac{\sum_{j=1}^{m}\alpha_j M(Aw,T_jz,t)+\sum_{i=1}^{n}\beta_i M(Bz,S_iw,t)}{\sum_{j=1}^{m}\alpha_j+\sum_{i=1}^{n}\beta_i}\end{array}\right\}$$

$$= min\left\{M(w,z,t), M(w,w,t), M(z,z,t), \frac{\sum_{j=1}^{m}\alpha_j M(w,z,t)+\sum_{i=1}^{n}\beta_i M(z,w,t)}{\sum_{j=1}^{m}\alpha_j+\sum_{i=1}^{n}\beta_i}\right\}$$

$$= min\left\{M(w,z,t), 1,1, M(w,z,t)\left(\frac{\sum_{j=1}^{m}\alpha_j+\sum_{i=1}^{n}\beta_i}{\sum_{j=1}^{m}\alpha_j+\sum_{i=1}^{n}\beta_i}\right)\right\}$$

$$= min\{M(w,z,t), 1,1, M(w,z,t)\}$$

$$= M(w,z,t)$$

a contradiction.

Therefore we have $z = w$ by lemma1.15, z is a common fixed point of A, B, $\{S_i\}_{i=1}^{n} and \ \{T_j\}_{j=1}^{m}$.The uniqueness of the fixed point holds from (1).

Corollary 2.2 [25]: Let $(X, M,*)$ be a complete fuzzy metric space and let $A, B, S \ and \ T$ be self-mappings of X. Let the pairs $\{A, S\} \ and \ \{B, T\}$ be occasionally weakly compatible.

If there exists $q\epsilon(0,1)$ such that

$$M(Ax,By,t) \geq min\left\{M(Sx,Ty,t), M(Sx,Ax,t), M(By,Ty,t), \left(M(Ax,Ty,t) + M(By,Sx,t)\big/_2\right)\right\} \quad (2)$$

for all$x, y\epsilon X$ and for all $t > 0$ then there exists a unique point $w\epsilon X$ such that$Aw = Sw = w$ and a unique point $z\epsilon X$ such that $Bz = Tz = z$.

Moreover $z = w$,so that there is a unique common fixed point of $A, B, S \ and \ T$.

Proof 2.2: Take n=m=1 and $\alpha_1 = \beta_1 = 1$ in the above theorem 2.1, the corollary follows.

Theorem 2.3: Let $(X, M,*)$ be complete fuzzy metric space and let A, B and $\{S_i\}_{i=1}^{n}$ and $\{T_j\}_{j=1}^{m}$ be self-mappings of X.

Let pairs $\{A, S_i\}_{i=1}^{n}$ and $\{B, T_j\}_{j=1}^{m}$ be occasionally weakly compatible at x and y respectively.

If there exists $q\epsilon(0,1)$ such that

$$M(Ax,By,qt) \geq \Psi\left(min\left\{\begin{array}{c}\{M(S_1x,T_jy,t)\}_{j=1}^{m}, \{M(S_2x,T_jy,t)\}_{j=1}^{m}, \dots, \{M(S_nx,T_jy,t)\}_{j=1}^{m}, \{M(S_ix,Ax,t)\}_{i=1}^{n}, \\ \{M(B_y,T_jy,t)\}_{j=1}^{m}, \frac{\sum_{j=1}^{m}\alpha_j M(Ax,T_jy,t)+\sum_{i=1}^{n}\beta_i M(By,S_ix,t)}{\sum_{j=1}^{m}\alpha_j+\sum_{i=1}^{n}\beta_i}\end{array}\right\}\right) \quad (3)$$

for all $x, y\epsilon X$,for all $t > 0$ and α_j's ≥ 0 and $\beta_i^{'}s \geq 0$ with atleast one of $\alpha_j 's$ or one of $\beta_i 's$ is non-zero, and $\Psi: [0,1] \to [0,1]$ such that $\Psi(t) > t$ for all $0 < t < 1$ then there exists a unique common fixed point of $A, B, \{S_i\}_{i=1}^{n} and \{T_j\}_{j=1}^{m}$

Proof 2.3: Let the pairs $\{A, S_i\}_{i=1}^{n} \ and \ \{B, T_j\}_{j=1}^{m}$ be occasionally weakly compatible at the points $x, y\epsilon X$ respectively.

Then $Ax = S_ix \ and \ By = T_jy \ for \ all \ i \ and \ j$

Claim:$Ax = By$

If not,

$$M(Ax,By,t) \geq \Psi\left(min \begin{Bmatrix} \{M(S_1x,T_jy,t)\}_{j=1}^m, \{M(S_2x,T_jy,t)\}_{j=1}^m, \dots, \{M(S_nx,T_jy,t)\}_{j=1}^m, \{M(S_ix,Ax,t)\}_{i=1}^n, \\ \{M(B_y,T_jy,t)\}_{j=1}^m, \frac{\sum_{j=1}^m \alpha_j M(Ax,T_jy,t)+\sum_{i=1}^n \beta_i M(By,S_ix,t)}{\sum_{j=1}^m \alpha_j+\sum_{i=1}^n \beta_i} \end{Bmatrix} \right)$$

$$= \Psi\left(min\left\{M(Ax,By,t), M(Ax,Ax,t), M(By,By,t), \frac{\sum_{j=1}^m \alpha_j M(Ax,By,t)+\sum_{i=1}^n \beta_i M(By,Ax,t)}{\sum_{j=1}^m \alpha_j+\sum_{i=1}^n \beta_i}\right\}\right)$$

$$= \Psi\left(min\left\{M(Ax,By,t), 1,1, M(Ax,By,t)\left(\frac{\sum_{j=1}^m \alpha_j+\sum_{i=1}^n \beta_i}{\sum_{j=1}^m \alpha_j+\sum_{i=1}^n \beta_i}\right)\right\}\right)$$

$$= \Psi(min\{M(Ax,By,t),1,1,M(Ax,By,t)\})$$

$$= \Psi(M(Ax,By,t)) > M(Ax,By,t)$$

a contradiction.

Therefore $Ax = By$

i.e,$Ax = S_ix = By = T_jy$

Suppose that there is a another point z such that $Az = S_iz$ then by (3) we have

$Az = S_iz = By = T_jy$ *so* $Az = Ax$ *and* $w = Ax = S_ix$ is the unique point of coincidence of A *and* S_i

By lemma1.15, w is the only common fixed point of A *and* S_i

Similarly there is a unique point $z \epsilon X$ such that $z = Bz = T_jz$

Uniqueness: Assume that $w \neq z$

We have

$M(w,z,qt) = M(Aw,Bz,t)$

$$\geq \Psi\left(min \begin{Bmatrix} \{M(S_1w,T_jz,t)\}_{j=1}^m, \{M(S_2w,T_jz,t)\}_{j=1}^m, \dots, \{M(S_nw,T_jz,t)\}_{j=1}^m \\ \{M(S_iw,Aw,t)\}_{i=1}^n, \{M(Bz,T_jz,t)\}_{j=1}^m, \\ \frac{\sum_{j=1}^m \alpha_j M(Aw,T_jz,t)+\sum_{i=1}^n \beta_i M(Bz,S_iw,t)}{\sum_{j=1}^m \alpha_j+\sum_{i=1}^n \beta_i} \end{Bmatrix} \right)$$

$$= \Psi\left(min\left\{M(w,z,t), M(w,w,t), M(z,z,t), \frac{\sum_{j=1}^m \alpha_j M(w,z,t)+\sum_{i=1}^n \beta_i M(z,w,t)}{\sum_{j=1}^m \alpha_j+\sum_{i=1}^n \beta_i}\right\}\right)$$

$$= \Psi\left(min\left\{M(w,z,t), 1,1, M(w,z,t)\left(\frac{\sum_{j=1}^m \alpha_j+\sum_{i=1}^n \beta_i}{\sum_{j=1}^m \alpha_j+\sum_{i=1}^n \beta_i}\right)\right\}\right)$$

$$= \Psi(min\{M(w,z,t),1,1,M(w,z,t)\})$$

$$= \Psi(M(w,z,t))$$

$$> M(w,z,t)$$

a contradiction.

Therefore we have $z = w$ by lemma1.15, z is a common fixed point of A, B, $\{S_i\}_{i=1}^n$ *and* $\{T_j\}_{j=1}^m$.The uniqueness of the fixed point holds from (3).

Corollary 2.4 [25]: Let $(X, M,*)$ be a complete fuzzy metric space and let A, B, S *and* T be self-mappings of X.Let the pairs $\{A,S\}$ *and* $\{B,T\}$ be occasionally weakly compatible.

If there exists $q\epsilon(0,1)$ such that

$$M(Ax,By,t) \geq \Psi\left(min\left\{M(Sx,Ty,t), M(Sx,Ax,t), M(By,Ty,t), \left(M(Ax,Ty,t)+M(By,Sx,t)/_2\right)\right\}\right) \tag{4}$$

for all $x,y\epsilon X$ and $\Psi:[0,1]\rightarrow[0,1]$ such that $0<t<1$, then there exists a unique common fixed point of A,B,S and T.

Proof 2.4: Take n=m=1 and $\alpha_1 = \beta_1 = 1$ in the above theorem 2.1, the corollary follows.

Theorem 2.5: Let $(X,M,*)$ be complete fuzzy metric space and let A,B and $\{S_i\}_{i=1}^n$ and $\{T_j\}_{j=1}^m$ be self-mappings of X.

Let pairs $\{A,S_i\}_{i=1}^n$ and $\{B,T_j\}_{j=1}^m$ be occasionally weakly compatible at x and y respectively.

If there exists $q\epsilon(0,1)$ such that

$$M(Ax,By,qt) \geq \Psi\begin{Bmatrix}\{M(S_1x,T_jy,t)\}_{j=1}^m, \{M(S_2x,T_jy,t)\}_{j=1}^m, \dots, \{M(S_nx,T_jy,t)\}_{j=1}^m, \{M(S_ix,Ax,t)\}_{i=1}^n, \\ \{M(By,T_jy,t)\}_{j=1}^m, \frac{\sum_{j=1}^m \alpha_j M(Ax,T_jy,t)+\sum_{i=1}^n \beta_i M(By,S_ix,t)}{\sum_{j=1}^m \alpha_j+\sum_{i=1}^n \beta_i}, \{M(By,S_ix,t)\}_{i=1}^n\end{Bmatrix} \tag{5}$$

for all $x,y\epsilon X$,for all $t>0$ and α_j's ≥ 0 and $\beta_i^{'}s \geq 0$ with atleast one of α_j's or one of β_i's is non-zero, and $\Psi:[0,1]^{nm+2n+m+1}\rightarrow[0,1]$ such that $\Psi(t,t,\dots,t_{(nm\ times)},1,1,\dots,1_{(n\ times)},1,1,\dots,1_{(m\ times)},t,t,\dots,t_{(n\ times)},t) > t$ for all $0<t<1$, then there exists a unique common fixed point of $A,B,\{S_i\}_{i=1}^n and \{T_j\}_{j=1}^m$.

Proof 2.5: Let the pairs $\{A,S_i\}_{i=1}^n$ and $\{B,T_j\}_{j=1}^m$ be occasionally weakly compatible at the points $x,y\epsilon X$ respectively.

Then $Ax = S_ix$ and $By = T_jy$ for all i and j

Claim:$Ax = By$

If not,

$$M(Ax,By,qt) \geq \Psi\begin{Bmatrix}\{M(S_1x,T_jy,t)\}_{j=1}^m, \{M(S_2x,T_jy,t)\}_{j=1}^m, \dots, \{M(S_nx,T_jy,t)\}_{j=1}^m, \{M(S_ix,Ax,t)\}_{i=1}^n, \\ \{M(By,T_jy,t)\}_{j=1}^m, \dfrac{\sum_{j=1}^m \alpha_j M(Ax,T_jy,t)+\sum_{i=1}^n \beta_i M(By,S_ix,t)}{\sum_{j=1}^m \alpha_j+\sum_{i=1}^n \beta_i}, \{M(By,S_ix,t)\}_{i=1}^n\end{Bmatrix}$$

$$= \Psi\begin{Bmatrix}M(Ax,By,t),\dots,M(Ax,By,t)_{(nm\ times)}, M(Ax,Ax,t),\dots,M(Ax,Ax,t)_{(n\ times)}, \\ M(By,By,t),\dots,M(By,By,t)_{(m\ times)}, \frac{\sum_{j=1}^m \alpha_j M(Ax,By,t)+\sum_{i=1}^n \beta_i M(By,Ax,t)}{\sum_{j=1}^m \alpha_j+\sum_{i=1}^n \beta_i}, \\ M(By,Ax,t),\dots M(By,Ax,t)_{(n\ times)}\end{Bmatrix}$$

$$= \Psi\begin{Bmatrix}M(Ax,By,t),\dots,M(Ax,By,t)_{(nm\ times)}, 1,1,\dots,1_{(n+mtimes)}, M(Ax,By,t),\dots, \\ M(Ax,By,t)_{(n+1\ times)}\end{Bmatrix}$$

$$> M(Ax,By,t)$$

a contradiction.

Therefore $Ax = By$

i.e,$Ax = S_ix = By = T_jy$

Suppose that there is a another point z such that $Az = S_iz$ then by (5) we have $Az = S_iz = By = T_jy$ so $Az = Ax$ and $w = Ax = S_ix$ is the unique point of coincidence of A and S_i

By lemma1.15, w is the only common fixed point of A and S_i

Similarly there is a unique point $z\epsilon X$ such that $z = Bz = T_j z$

Uniqueness: Assume that $w \neq z$

We have

$$M(w,z,qt) = M(Aw,Bz,t)$$

$$\geq \Psi \left\{ \begin{array}{c} \{M(S_1 w, T_j z, t)\}_{j=1}^{m}, \{M(S_2 w, T_j z, t)\}_{j=1}^{m}, \ldots, \{M(S_n w, T_j z, t)\}_{j=1}^{m} \\ \{M(S_i w, Aw, t)\}_{i=1}^{n}, \{M(Bz, T_j z, t)\}_{j=1}^{m}, \\ \frac{\sum_{j=1}^{m} \alpha_j M(Aw, T_j z, t) + \sum_{i=1}^{n} \beta_i M(Bz, S_i w, t)}{\sum_{j=1}^{m} \alpha_j + \sum_{i=1}^{n} \beta_i}, \{M(Bz, S_i w, t\}_{i=1}^{n} \end{array} \right\}$$

$$= \Psi \left\{ \begin{array}{c} M(w,z,t), \ldots, M(w,z,t)_{(nm\ times)} M(w,w,t), \ldots, M(w,w,t)_{(n\ times)}, \\ M(z,z,t), \ldots, M(z,z,t)_{(m\ times)}, \frac{\sum_{j=1}^{m} \alpha_j M(w,z,t) + \sum_{i=1}^{n} \beta_i M(z,w,t)}{\sum_{j=1}^{m} \alpha_j + \sum_{i=1}^{n} \beta_i}, \\ M(w,z,t), \ldots, M(w,z,t)_{(n\ times)} \end{array} \right\}$$

$$= \Psi \left\{ \begin{array}{c} M(w,z,t), \ldots, M(w,z,t)_{(nm\ times)}, 1,1,\ldots,1_{(n+m\ times)}, M(w,z,t), \ldots, \\ M(w,z,t)_{(n+1\ times)} \end{array} \right\}$$

$$> M(w,z,t)$$

a contradiction.

Therefore we have $z = w$ by lemma1.15, z is a common fixed point of A, B, $\{S_i\}_{i=1}^{n} and$ $\{T_j\}_{j=1}^{m}$.The uniqueness of the fixed point holds from (5).

Corollary2.6 [25]: Let $(X, M, *)$ be a complete fuzzy metric space and let $A, B, S\ and\ T$ be self-mappings of X.Let the pairs $\{A, S\}\ and\ \{B, T\}$ be occasionally weakly compatible.

If there exists $q\epsilon(0,1)$ such that

$$M(Ax,By,t) \geq \Psi \left(\begin{array}{c} M(Sx,Ty,t), M(Sx,Ax,t), M(By,Ty,t), \frac{(M(Ax,Ty,t)+M(By,Sx,t)}{2}, \\ M(By,Sx,t) \end{array} \right) \quad (6)$$

for all$x, y\epsilon X$ and $\Psi: [0,1]^5 \to [0,1]$ such that$\Psi(t,1,1,t,t) > t$ for all $0 < t < 1$,then there exists a unique common fixed point of $A, B, S\ and\ T$.

Proof 2.6: Take n=m=1 and $\alpha_1 = \beta_1 = 1$ in the above theorem 2.5, the corollary follows.

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