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HOMOMORPHISM IN INTUITIONISTIC L-FUZZY SUBHEMIRINGS OF A HEMIRING

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of an intuitionistic L-fuzzy subhemiring of a hemiring under homomorphism and anti-homomorphism.

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INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring (*R ; + ; .*). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. An algebra (R ; +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups satisfying a. (b+c) = a. b+a. c and (b+c) .a = b. a+c. a for all a, b and c in R. A semiring R is said to be additively commutative if a+b = b+a for all a, b in R. A semiring R may have an identity 1, defined by 1. a = a = a. 1 and a zero 0, defined by 0+a = a = a+0 and a.0 = 0 = 0.a for all a in R. A semiring R is said to be a hemiring if it is an additively commutative with zero. After the introdution of fuzzy sets by L.A.Zadeh[17], several researchers explored on the generalization of the notion of fuzzy set. The concept of intuitionistic L-fuzzy subset was introduced by K.T.Atanassov[6,7], as a generalization of the notion of fuzzy set. The notion of homomorphism and anti-homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by N.Palaniappan & K.Arjunan[11]. Some properties of intuitionistic fuzzy subgroups was introduced by Palaniappan. N & K.Arjunan[13]. In this paper, we introduce the some Theorems in intuitionistic L-fuzzy subhemiring of a hemiring under homomorphism and anti-homomorphism.

1. PRELIMINARIES:

1.1 Definition: Let X be a non-empty set and L = (L,≤) be a lattice with least element 0 and greatest element 1. A **L-fuzzy subset** A of X is a function A: X → L.

1.2 Definition: Let (R, + , .) be a hemiring. A L-fuzzy subset A of R is said to be a L-fuzzy subhemiring (LFSHR) of R if it satisfies the following conditions:
(i) $\mu_A(x+y) \geq \mu_A(x) \wedge \mu_A(y)$,
(ii) $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$, for all x and y in R.

1.3 Definition: Let (R , + , .) be a hemiring. A L-fuzzy subset A of R is said to be an anti L-fuzzy subhemiring (ALFSHR) of R if it satisfies the following conditions:
(i) $\mu_A(x+y) \leq \mu_A(x) \vee \mu_A(y)$,
(ii) $\mu_A(xy) \leq \mu_A(x) \vee \mu_A(y)$, for all x and y in R.

1.4 Definition: Let (L, ≤) be a complete lattice with an involutive order reversing operation N : L → L. A **intuitionistic L-fuzzy subset** (ILFS) A in X is defined as an object of the form $A = \{ < x, \mu_A(x), \nu_A(x) > / x \text{ in } X \}$, where $\mu_A : X \to L$ and $\nu_A : X \to L$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$ satisfying $\mu_A(x) \leq N(\nu_A(x))$.

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1.5 Definition: Let (R, +, .) be a hemiring. An intuitionistic L-fuzzy subset A of R is said to be an intuitionistic L-fuzzy subhemiring (ILFSHR) of R if it satisfies the following conditions:
(i) $\mu_A(x + y) \geq \mu_A(x) \wedge \mu_A(y)$,
(ii) $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$,
(iii) $\nu_A(x + y) \leq \nu_A(x) \vee \nu_A(y)$,
(iv) $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$, for all x and y in R.

2. PROPERTIES OF INTUITIONISTIC L-FUZZY SUBHEMIRINGS

2.1 Theorem: Let (R, +, .) and (R^l, +, .) be any two hemirings. The homomorphic image of an intuitionistic L-fuzzy subhemiring of R is an intuitionistic L-fuzzy subhemiring of R^l.

Proof: Let (R, +, .) and (R^l, +, .) be any two hemirings. Let f: $R \rightarrow R^l$ be a homomorphism. Then, f (x+y) = f(x) + f(y) and f(xy) = f(x) f(y), for all x and y in R. Let V = f(A), where A is an intuitionistic L-fuzzy subhemiring of R. We have to prove that V is an intuitionistic L-fuzzy subhemiring of R^l. Now, for f(x), f(y) in R^l, $\mu_v(f(x) + f(y)) = \mu_v(f(x+y)) \geq \mu_A(x + y) \geq \mu_A(x) \wedge \mu_A(y)$, which implies that $\mu_v(f(x) + f(y)) \geq \mu_v(f(x)) \wedge \mu_v(f(y))$. Again, $\mu_v(f(x)f(y)) = \mu_v(f(xy)) \geq \mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$, which implies that $\mu_v(f(x)f(y)) \geq \mu_v(f(x)) \wedge \mu_v(f(y))$. Now, for f(x), f(y) in R^l, $\nu_v(f(x) + f(y)) = \nu_v(f(x+y)) \leq \nu_A(x+ y) \leq \nu_A(x) \vee \nu_A(y)$, which implies that $\nu_v(f(x) + f(y)) \leq \nu_v(f(x)) \vee \nu_v(f(y))$. Again, $\nu_v(f(x)f(y)) = \nu_v(f(xy)) \leq \nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$, which implies that $\nu_v(f(x)f(y)) \leq \nu_v(f(x)) \vee \nu_v(f(y))$.

Hence V is an intuitionistic L-fuzzy subhemiring of R^l.

2.2 Theorem: Let (R, +, .) and (R^l, +, .) be any two hemirings. The homomorphic preimage of an intuitionistic L-fuzzy subhemiring of R^l is a intuitionistic L-fuzzy subhemiring of R.

Proof: Let (R, +, .) and (R^l, +, .) be any two hemirings. Let f: $R \rightarrow R^l$ be a homomorphism. Then, f(x+y) = f(x) + f(y) and f(xy) = f(x) f(y), for all x and y in R. Let V = f(A), where V is an intuitionistic L-fuzzy subhemiring of R^l. We have to prove that A is an intuitionistic L-fuzzy subhemiring of R. Let x and y in R. Then, $\mu_A(x + y) = \mu_v(f(x + y)) = \mu_v(f(x) + f(y)) \geq \mu_v(f(x)) \wedge \mu_v(f(y)) = \mu_A(x) \wedge \mu_A(y)$, which implies that $\mu_A(x+y) \geq \mu_A(x) \wedge \mu_A(y)$. Again, $\mu_A(xy)= \mu_v(f(xy)) = \mu_v(f(x)f(y)) \geq \mu_v(f(x)) \wedge \mu_v(f(y)) = \mu_A(x) \wedge \mu_A(y)$, which implies that $\mu_A(xy) \geq \mu_A(x) \wedge\mu_A(y)$.

Let x and y in R. Then, $\nu_A(x+ y) = \nu_v(f(x + y))= \nu_v(f(x)+f(y)) \leq \nu_v(f(x)) \vee \nu_v(f(y))= \nu_A(x) \vee \nu_A(y)$, which implies that $\nu_A(x+ y) \leq \nu_A(x) \vee \nu_A(y)$. Again, $\nu_A(xy) = \nu_v(f(xy)) = \nu_v(f(x)f(y)) \leq \nu_v(f(x)) \vee \nu_v(f(y)) = \nu_A(x) \vee \nu_A(y)$, which implies that $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$. Hence A is an intuitionistic L-fuzzy subhemiring of R.

2.3 Theorem: Let (R, +, .) and (R^l, +, .) be any two hemirings. The anti-homomorphic image of an intuitionistic L-fuzzy subhemiring of R is an intuitionistic L-fuzzy subhemiring of R^l.

Proof: Let (R, +, .) and (R^l, +, .) be any two hemirings. Let f: $R \rightarrow R^l$ be an anti-homomorphism. Then, f (x+y) = f (y) + f (x) and f(xy) = f(y) f(x), for all x and y in R. Let V = f(A), where A is an intuitionistic L-fuzzy subhemiring of R. We have to prove that V is an intuitionistic L-fuzzy subhemiring of R^l. Now, for f(x), f(y) in R^l, $\mu_v(f(x) + f(y)) = \mu_v(f(y + x)) \geq \mu_A(y + x) \geq \mu_A(y) \wedge \mu_A(x) = \mu_A(x) \wedge \mu_A(y)$, which implies that $\mu_v(f(x) + f(y)) \geq \mu_v(f(x)) \wedge \mu_v(f(y))$. Again, $\mu_v(f(x)f(y)) = \mu_v(f(yx)) \geq \mu_A(yx) \geq \mu_A(y)\wedge\mu_A(x)= \mu_A(x)\wedge \mu_A(y)$, which implies that $\mu_v(f(x)f(y)) \geq \mu_v(f(x)) \wedge \mu_v(f(y))$. Now, for f(x), f(y) in R^l, $\nu_v(f(x) + f(y)) = \nu_v(f(y + x)) \leq \nu_A(y + x) \leq \nu_A(y) \vee \nu_A(x) = \nu_A(x) \vee \nu_A(y)$, which implies that $\nu_v(f(x)+f(y)) \leq \nu_v(f(x)) \vee \nu_v(f(y))$. Again, $\nu_v(f(x)f(y)) = \nu_v(f(yx)) \leq \nu_A(yx) \leq \nu_A(y) \vee \nu_A(x) = \nu_A(x) \vee \nu_A(y)$, which implies that $\nu_v(f(x)f(y)) \leq \nu_v(f(x)) \vee \nu_v(f(y))$. Hence V is an intuitionistic L-fuzzy subhemiring of R^l.

2.4 Theorem: Let (R, +, .) and (R^l, +, .) be any two hemirings. The anti-homomorphic preimage of an intuitionistic L-fuzzy subhemiring of R^l is an intuitionistic L-fuzzy subhemiring of R.

Proof: Let (R, +, .) and (R^l, +, .) be any two hemirings. Let f: $R \rightarrow R^l$ be an anti-homomorphism. Then, f (x+y) = f (y) + f (x) and f(xy) = f(y) f(x), for all x and y in R. Let V = f(A), where V is an intuitionistic L-fuzzy subhemiring of R^l. We have to prove that A is an intuitionistic L-fuzzy subhemiring of R. Let x and y in R. Then, $\mu_A(x + y) = \mu_v(f(x + y))= \mu_v(f(y) + f(x))\geq \mu_v(f(y)) \wedge \mu_v(f(x)) = \mu_v(f(x)) \wedge \mu_v(f(y))= \mu_A(x) \wedge \mu_A(y)$, which implies that $\mu_A(x + y) \geq \mu_A(x) \wedge \mu_A(y)$. Again, $\mu_A(xy) = \mu_v(f(xy))= \mu_v(f(y)f(x)) \geq \mu_v(f(y)) \wedge\mu_v(f(x))= \mu_v(f(x)) \wedge \mu_v(f(y)) = \mu_A(x)\wedge\mu_A(y)$, which implies that $\mu_A(xy) \geq \mu_A(x)\wedge\mu_A(y)$. Then, $\nu_A(x+y)=\nu_v(f(x+y)) = \nu_v(f(y) + f(x))\leq \nu_v(f(y)) \vee \nu_v(f(x))= \nu_v(f(x)) \vee \nu_v(f(y)) = \nu_A(x) \vee \nu_A(y)$, which implies that $\nu_A(x + y) \leq \nu_A(x) \vee \nu_A(y)$.

Again, $\nu_A(xy) = \nu_v(f(xy)) = \nu_v(f(y)f(x)) \leq \nu_v(f(y)) \vee \nu_v(f(x)) = \nu_v(f(x)) \vee \nu_v(f(y)) = \nu_A(x) \vee \nu_A(y)$, which implies that $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$. Hence A is an intuitionistic L-fuzzy subhemiring of R.

2.5 Theorem: Let A be an intuitionistic L-fuzzy subhemiring of a hemiring H and f is an isomorphism from a hemiring R onto H. Then A◦f is an intuitionistic L-fuzzy subhemiring of R.

Proof: Let x and y in R and A be an intuitionistic L-fuzzy subhemiring of a hemiring H. Then we have, $(\mu_A \circ f)(x+y) = \mu_A(f(x+y)) = \mu_A(f(x)+f(y)) \geq \mu_A(f(x)) \wedge \mu_A(f(y)) = (\mu_A \circ f)(x) \wedge (\mu_A \circ f)(y)$, which implies that $(\mu_A \circ f)(x+y) \geq (\mu_A \circ f)(x) \wedge (\mu_A \circ f)(y)$. And, $(\mu_A \circ f)(xy) = \mu_A(f(xy)) = \mu_A(f(x)f(y)) \geq \mu_A(f(x)) \wedge \mu_A(f(y)) = (\mu_A \circ f)(x) \wedge (\mu_A \circ f)(y)$, which implies that $(\mu_A \circ f)(xy) \geq (\mu_A \circ f)(x) \wedge (\mu_A \circ f)(y)$. Then we have, $(\nu_A \circ f)(x+y) = \nu_A(f(x+y)) = \nu_A(f(x)+f(y)) \leq \nu_A(f(x)) \vee \nu_A(f(y)) = (\nu_A \circ f)(x) \vee (\nu_A \circ f)(y)$, which implies that $(\nu_A \circ f)(x+y) \leq (\nu_A \circ f)(x) \vee (\nu_A \circ f)(y)$. And $(\nu_A \circ f)(xy) = \nu_A(f(xy)) = \nu_A(f(x)f(y)) \leq \nu_A(f(x)) \vee \nu_A(f(y)) = (\nu_A \circ f)(x) \vee (\nu_A \circ f)(y)$, which implies that $(\nu_A \circ f)(xy) \leq (\nu_A \circ f)(x) \vee (\nu_A \circ f)(y)$. Therefore (A◦f) is an intuitionistic L-fuzzy subhemiring of a hemiring R.

2.6 Theorem: Let A be an intuitionistic L-fuzzy subhemiring of a hemiring H and f is an anti-isomorphism from a hemiring R onto H. Then A◦f is an intuitionistic L-fuzzy subhemiring of R.

Proof: Let x and y in R and A be an intuitionistic L-fuzzy subhemiring of a hemiring H. Then we have, $(\mu_A \circ f)(x+y) = \mu_A(f(x+y)) = \mu_A(f(y)+f(x)) \geq \mu_A(f(x)) \wedge \mu_A(f(y)) = (\mu_A \circ f)(x) \wedge (\mu_A \circ f)(y)$, which implies that $(\mu_A \circ f)(x+y) \geq (\mu_A \circ f)(x) \wedge (\mu_A \circ f)(y)$. And, $(\mu_A \circ f)(xy) = \mu_A(f(xy)) = \mu_A(f(y)f(x)) \geq \mu_A(f(x)) \wedge \mu_A(f(y)) = (\mu_A \circ f)(x) \wedge (\mu_A \circ f)(y)$, which implies that $(\mu_A \circ f)(xy) \geq (\mu_A \circ f)(x) \wedge (\mu_A \circ f)(y)$. Then we have, $(\nu_A \circ f)(x+y) = \nu_A(f(x+y)) = \nu_A(f(y)+f(x)) \leq \nu_A(f(x)) \vee \nu_A(f(y)) = (\nu_A \circ f)(x) \vee (\nu_A \circ f)(y)$, which implies that $(\nu_A \circ f)(x+y) \leq (\nu_A \circ f)(x) \vee (\nu_A \circ f)(y)$. And, $(\nu_A \circ f)(xy) = \nu_A(f(xy)) = \nu_A(f(y)f(x)) \leq \nu_A(f(x)) \vee \nu_A(f(y)) = (\nu_A \circ f)(x) \vee (\nu_A \circ f)(y)$, which implies that $(\nu_A \circ f)(xy) \leq (\nu_A \circ f)(x) \vee (\nu_A \circ f)(y)$.

Therefore A◦f is an intuitionistic L-fuzzy subhemiring of the hemiring R.

REFERENCES

1. Akram. M and K.H.Dar, On Anti Fuzzy Left h- ideals in Hemirings, International Mathematical Forum, 2(46); 2295 – 2304, 2007.
2. Akram.M and Dar.K.H, On fuzzy d-algebras, Punjab university journal of mathematics, 37(2005), 61-76.
3. Anitha. N and Arjunan. K, Homomorphism in Intuitionistic fuzzy subhemirings of a hemiring, International J.of.Math. Sci.& Engg. Appls.(IJMSEA), Vol.4 (V); 165 – 172, 2010.
4. Anthony.J.M. and H Sherwood, Fuzzy groups Redefined, Journal of mathematical analysis and applications, 69; 124 -130, 1979.
5. Asok Kumer Ray, On product of fuzzy subgroups, fuzzy sets and sysrems, 105; 181-183, 1999.
6. Atanassov.K., Intuitionistic fuzzy sets, fuzzy sets and systems, 20(1), 87-96 (1986).
7. Atanassov.K., Stoeva.S., Intuitionistic L-fuzzy sets, Cybernetics and systems research 2 (Elsevier Sci. Publ., Amsterdam, 1984), 539-540.
8. Biswas. R, Fuzzy subgroups and Anti-fuzzy subgroups, Fuzzy sets and systems, 35; 121-124, 1990.
9. Davvaz.B and Wieslaw.A.Dudek, Fuzzy n-ary groups as a generalization of rosenfeld fuzzy groups, ARXIV-0710.3884VI(MATH.RA)20 OCT 2007,1-16.
10. Naganathan. S, Arjunan.K and Palaniappan. N, Level subsets of intuitionistic L-fuzzy subgroups of a group, International journal of computational and applied mathematics, Volume 4, 177–184, (2009).
11. Palaniappan. N & K. Arjunan, The homomorphism, anti homomorphism of a fuzzy and an anti-fuzzy ideals of a ring, Varahmihir Journal of Mathematical Sciences, 6(1); 181-006, 2008.
12. Palaniappan. N & K. Arjunan, Operation on fuzzy and anti fuzzy ideals , Antartica J. Math ., 4(1); 59-64, 2007.
13. Palaniappan. N & K.Arjunan, Some properties of intuitionistic fuzzy subgroups, Acta Ciencia Indica, Vol.XXXIII (2); 321-328, 2007.
14. Rajesh Kumar, Fuzzy Algebra, University of Delhi Publication Division, Volume 1, 1993.
15. Selvak kumaraen.N & Arjunan.K, A study on anti L-fuzzy subhemirings of a hemiring, IOSR Journal of Engg., Vol. 2, Iss. 6, 1420-1423, 2012.
16. Vasantha kandasamy.W.B, Smarandache fuzzy algebra, American research press , Rehoboth, 2003.
17. Xueling MA . Jianming ZHAN, On Fuzzy h - Ideals of hemirings, Journal of Systems science & Complexity, 20; 470 – 478, 2007.
18. Zadeh . L . A, Fuzzy sets, Information and control, 8; 338-353, 1965.

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