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SUM CORDIAL LABELING FOR SOME GRAPHS

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ABSTRACT

In this paper I investigate a new labeling called the Sum Cordial labeling and some Sum Cordial graphs. A binary vertex labeling of a graph G with induced edge labeling $f^* : E(G) \rightarrow \{0,1\}$ defined by $f^*(uv) = (f(u) + f(v))\ (\mathrm{mod}2)$ is called a sum cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. Any graph which satisfies the sum cordial labeling is called a Sum Cordial graph. Here I prove some graphs like path P_n, cycle C_n, star $K_{1,n}$, bistar $B_{n,n}$ and the graphs obtained by joining apex vertices of k copies of double stars to a new vertex are Sum cordial graphs.

Keywords: *Sum cordial labeling, Sum Cordial graphs,*

INTRODUCTION

All graphs in this paper are finite and undirected. For all other terminology and notations I follow Harrary [3]. Let G (V, E) be a graph where the symbols V (G) and E (G) denotes the vertex set and the edge set. The cardinality of the vertex set is called the order of the graph G and it is denoted by p. The cardinality of the edge set is called the size of the graph G and it is denoted by q. Hence the graph is denoted by G(p, q).If the vertices or edges or both of the graph are assigned values subject to certain conditions it is known as graph labeling. A dynamic survey on graph labeling is regularly updated by Gallian [4] and it is published by Electronic Journal of Combinatorics. Some basic concepts are taken from [5],[6]and [14]. This labeling can also be investigated for the graphs in the papers [7], [8], [9], [10] and [11] Labeled graphs have variety of applications in graph theory, particularly for missile guidance code, design good radar type codes and convolution codes with optimal autocorrelation properties. Labeled graph plays vital role in the study of X-ray crystallography, communication network and to determine optimal circuit layouts. A detailed study on variety of applications on graph labeling is carried out in Bloom and Golomb [1]

Definition-1: Consider k copies of stars $K_{1,n,n}^{(1)}, K_{1,n,n}^{(2)}, K_{1,n,n}^{(3)}, \ldots, K_{1,n,n}^{(k)}$ then
G = $< K_{1,n,n}^{(1)} : K_{1,n,n}^{(2)} : K_{1,n}^{(3)} : \ldots : K_{1,n,n}^{(k)} >$ is the graph obtained by joining apex vertices of each $K_{1,n,n}^{(p-1)}$ and $K_{1,n,n}^{(p)}$ to a new vertex x_{p-1} where $2 \leq p \leq k$. G has k(2n+2)- 1 vertices and k(2n+2)- 2 edges.

Definition-2: A mapping f: V (G) → {0, 1} is called binary vertex labeling of G and f (v) is called the label of vertex v of G under f.

The induced edge labeling $f^* : E(G) \rightarrow \{0,1\}$ is given by $f^*(uv) = |f(u) - f(v)|$ for every uv ∈ E(G) . Let $v_f(0)$, $v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f and let $e_f(0)$, $e_f(1)$ be the number of edges of G having labels 0 and 1 respectively under f^*

Definition-3: A binary vertex labeling of a graph G is called cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$
A graph G is cordial if it admits cordial labeling.

The concept of cordial labeling was introduced by Cahit[2] in which he investigated several results .The product cordial labeling was investigated by Sundaram [12] .Also some new graphs are investigated as product cordial graphs by Vaidya [13]

Definition-4: A binary vertex labeling of a graph G with induced edge labeling $f^* : E(G) \rightarrow \{0,1\}$ defined by $f^*(uv) = (f(u) + f(v))\ (\mathrm{mod}2)$ is called sum cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$
A graph G is sum cordial if it admits sum cordial labeling

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2. MAIN RESULTS

Theorem 2.1: The path P_n admits sum cordial labeling.

Proof: Let G = $P_n = v_1\ v_2 \dots v_n$ be a path of length n with n vertices and n-1edges

Define f: V (G) → {0, 1}, we consider four cases

Case-1: When n = 4k
P_n: $v_1\ v_2 \dots v_{4k}$ is a path of length 4k with 4k vertices and 4k-1edges
Define
$f(v_{4i-3}) = f(v_{4i-2}) = 1$, for $1 \leq i \leq k$ and
$f(v_{4i-1}) = f(v_{4i}) = 0$,

Case-2: When n = 4k -1
P_n: $v_1\ v_2 \dots v_{4k-1}$ is a path of length 4k- 1with 4k -1 vertices and 4k-2edges
Define
$f(v_{4i-3}) = f(v_{4i-2}) = 1$
$f(v_{4i-1}) = 0$ for $1 \leq i \leq k$ and $f(v_{4i}) = 0$ for $1 \leq i \leq k-1$

Case-3: When n = 4k +1
P_n: $v_1\ v_2 \dots v_{4k+1}$ is a path of length 4k+ 1with 4k +1 vertices and 4k edges
Define
$f(v_{4i-3}) = 1$ for $1 \leq i \leq k+1$
$f(v_{4i-2}) = 1$, $f(v_{4i-1}) = f(v_{4i}) = 0$ for $1 \leq i \leq k$

Case-4: When n = 4k +2
P_n: $v_1\ v_2 \dots v_{4k+2}$ is a path of length 4k+ 2with 4k +2 vertices and 4k - 1 edges
Define
$f(v_1) = f(v_2) = f(v_5) = 1$
$f(v_3) = f(v_4) = f(v_6) = 0$
$f(v_{4i-1}) = f(v_{4i}) = 1$
and $f(v_{4i+1}) = f(v_{4i+2}) = 0$, for $2 \leq i \leq k$

In all the above cases, $| v_f(0) - v_f(1) | \leq 1$ and $| e_f(0) - e_f(1) | \leq 1$

Hence the path P_n admits sum cordial labeling.

Example 2.2: The paths P_7 and P_{10} are sum cordial graphs

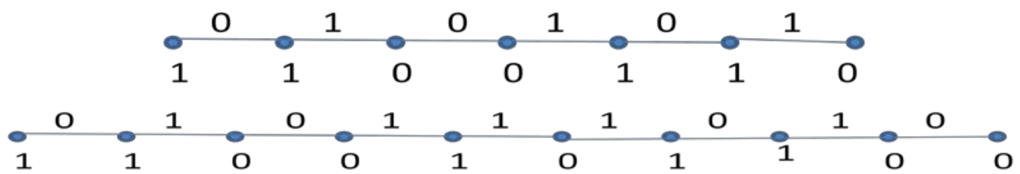

Fig-1: Sum cordial labeling of the paths P_7 and P_{10}

Theorem2.3: The Star $K_{1,n}$ admits sum cordial labeling.

Proof: Let $K_{1,n}$ be a star with n+1vertices and n edges.

Define f: V (G) → {0, 1}, we consider two cases

Case- 1: n is even
Define
$f(u_i) = 1$, $1 \leq i \leq \frac{n}{2}$
$= 0$, otherwise
and $f(c) = 0$

Case-2: n is odd
Define'

$f(u_i) = 1, \quad 1 \le i \le \lceil \frac{n}{2} \rceil$
$\quad = 0$, otherwise
and $f(c) = 0$

In both the cases

$| v_f(0) - v_f(1) | \le 1$ and $| e_f(0) - e_f(1) | \le 1$

Hence the star $K_{1,n}$ admits sum cordial labeling.

Example 2.4: The stars $K_{1,6}$ and $K_{1,7}$ are sum cordial graphs

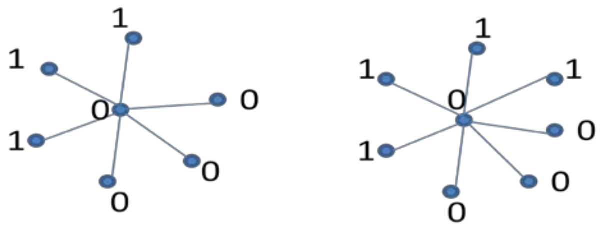

Fig-2: Sum cordial labeling of the stars $K_{1,6}$ and $K_{1,7}$

Theorem 2.5: The bistar $B_{n,\, n}$ admits sum cordial labeling.

Proof: Let $u, u_1, u_2, \ldots, u_n, v, v_1, v_2, \ldots, v_n$, be the vertices of the bistar $B_{n,n}$,

Let $G = B_{n,n}$, . Then $|V(G)| = 2n+2$ and $|E(G)| = 2n+1$

Define $f: V(G) \to \{0, 1\}$

Case-i: n is even
Define
$f(u) = 0$
$f(v) = 1$
and $f(u_i) = f(v_i) = 0$ for $1 \le i \le \frac{n}{2}$
$\quad = 1$ for $\frac{n}{2} + 1 \le i \le n$

Case-ii: n is odd
Define
$f(u) = 0$
$f(v) = 1$
and $f(u_i) = 0$ for $1 \le i \le \lceil \frac{n}{2} \rceil$
$\quad = 1$ for $\lceil \frac{n}{2} \rceil + 1 \le i \le n$'

and $f(v_i) = 0$ for $1 \le i \le \lfloor \frac{n}{2} \rfloor$
$\quad = 1$ for $\lfloor \frac{n}{2} \rfloor + 1 \le i \le n$

In both the cases $| v_f(0) - v_f(1) | \le 1$ and $| e_f(0) - e_f(1) | \le 1$

Hence any bistar $B_{n,n}$, admits sum cordial labeling

Example 2.6: The bistars $B_{3,3}$ and $B_{6,6}$ are sum cordial graphs

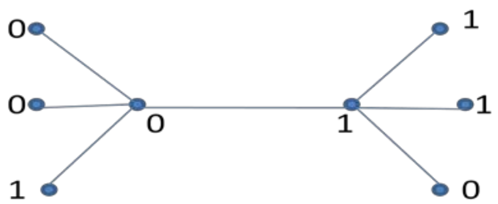

Fig-3: Sum cordial labeling of the bistars $B_{3,3}$ and $B_{6,6}$

Theorem 2.7: The double star $K_{1,n,n}$ is a sum cordial graph

Proof: Let u, u_1 , u_2 , … , u_n , v_1 , v_2 , … , v_n be the vertices of the double star $K_{1,\ n,n,}$

Let G = $K_{1,n,n,}$ Then |V(G) | = 2n+1 and |E(G) | = 2n
Define f: V (G) → {0, 1} by
f(u) = 1
f (u_i) = 1 , $1\leq i\leq n$ and
f(v_i) = 0 , $1\leq i\leq n$

Then| v_f (0) - v_f (1) | ≤ 1 and | e_f (0) - e_f (1) | ≤ 1

Therefore f is a sum cordial labeling.

Hence any double star $K_{1,n,n}$ admits sum cordial labeling.

Example 2.8: The double stars $K_{1,4,4}$ and $K_{17,7}$ are sum cordial graphs

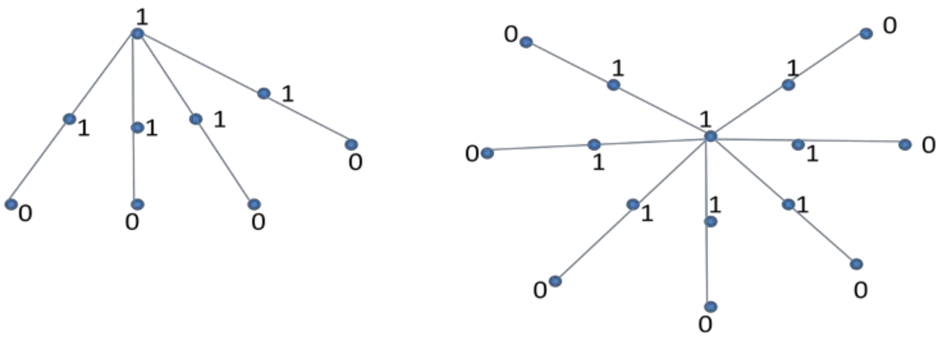

Fig-4: Sum cordial labeling of the double stars $K_{1,4,4}$ and $K_{17,7}$

Theorem2.9: Graph G = $< K^{(1)}_{1,n,n} : K^{(2)}_{1,n,n} : K^{(3)}_{1,n,n} : \ldots : K^{(k)}_{1,n,n} >$ is sum cordial.

Proof: Let $u^{(1)}_1, u^{(1)}_2, u^{(1)}_3, \ldots, u^{(1)}_n$ $v^{(1)}_1, v^{(1)}_2, v^{(1)}_3, \ldots, v^{(1)}_n$, be the vertices of $K^{(1)}_{1,n,n}$, $u^{(2)}_1, u^{(2)}_2, u^{(2)}_3, \ldots, u^{(2)}_n$ $v^{(2)}_1, v^{(2)}_2, v^{(2)}_3, \ldots, v^{(2)}_n$ be the vertices of $K^{(2)}_{1,n,n}$ and $u^{(k)}_1, u^{(k)}_2, u^{(k)}_3, \ldots, u^{(k)}_n$ $v^{(k)}_1, v^{(k)}_2, v^{(k)}_3, \ldots, v^{(k)}_n$ be the vertices of $K^{(k)}_{1,n,n}$ .Let c_1, $c_{2,}$ … , c_k be the apex vertices of $K^{(1)}_{1,n,n}$, $K^{(2)}_{1,n,n}$, … , $K^{(k)}_{1,n,n}$ respectively and c_{p-1} and c_p are adjacent to a new vertex x_{p-1}.$2 \leq p \leq k$. Let G =$< K^{(1)}_{1,n,n} : K^{(2)}_{1,n,n} : K^{(3)}_{1,n,n} : \ldots : K^{(k)}_{1,n,n} >$ .Here |V(G) | = 2k(n+1) -1 and |E(G)| = 2k(n+1) -2

Define sum cordial labeling f: V (G) → {0, 1} as follows.
For $1\leq i\leq n$
f($u^{(p)}_i$) = 1 , if p is odd and $1\leq p\leq k$
f($v^{(q)}_i$) = 1, if q is even $1\leq q\leq n$
f($v^{(p)}_i$) = 0 , if p is odd and $1\leq p\leq k$
f($u^{(q)}_i$) = 0 , if q is even and $1\leq p\leq k$
f(c_i) = 1; if i is odd , $1\leq i\leq k$
f(c_i) = 0; if i is even , $1\leq i\leq k$

$f(x_{p-1}) = 1$; if p is even and $2 \le p \le k$

$f(x_{p-1}) = 0$; if p is odd and $2 \le p \le k$

Also $| v_f(0) - v_f(1) | \le 1$ and $| e_f(0) - e_f(1) | \le 1$

Hence $G = < K_{1,n,n}^{(1)} : K_{1,n,n}^{(2)} : K_{1,n,n}^{(3)} : \ldots : K_{1,n,n}^{(k)} >$ admits sum cordial labeling .

Example 2.10: G = $< K_{1,7,7}^{(1)}, K_{1,7,7,}^{(2)} K_{1,7,7,}^{(3)} >$ is a sum cordial.

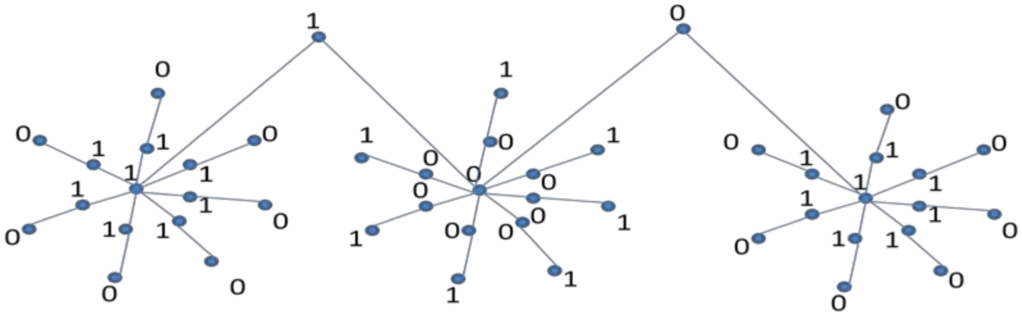

Fig-5: Sum cordial labeling of the graph **G =** $< K_{1,7,7}^{(1)}, K_{1,7,7,}^{(2)} K_{1,7,7,}^{(3)} >$

Example 2.10: G = $< K_{1,6,6}^{(1)}, K_{1,6,6}^{(2)} >$ is a sum cordial.

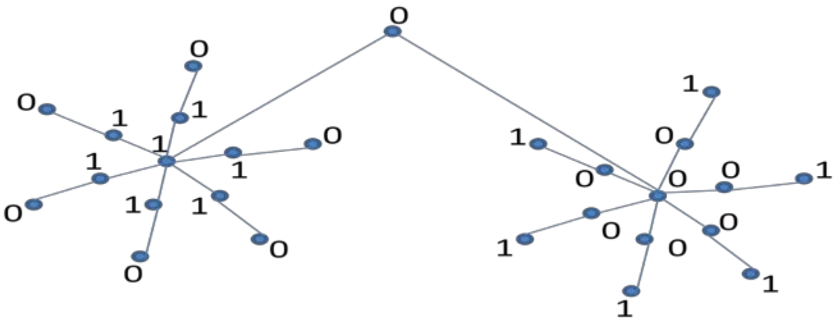

Fig-6: Sum cordial labeling of the graph **G =** $< K_{1,6,6}^{(1)}, K_{1,6,6}^{(2)} >$

CONCLUSION

In this paper some graphs like path, star, bistar, doublestar and the graphs obtained by joining apex vertices of k copies of double stars to a new vertex are investigated for the sum cordial labeling. This labeling can be verified for some other graphs.

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