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EFFECT OF RADIATION AND HALL CURRENTS ON UNSTEADY OSCILLATORY CONVECTIVE HEAT TRANSFER FLOW IN A VERTICAL WAVY CHANNEL WITH HEAT SOURCES

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ABSTRACT

In this paper we investigate the convective study of heat and mass transfer flow of a viscous electrically conducting fluid in a vertical wavy channel under the influence of an inclined magnetic fluid with heat generating sources. The walls of the channels are maintained at constant temperature and concentration. The equations governing the flow heat and concentration are solved by employing perturbation technique with a slope δ of the wavy wall. The velocity, temperature and concentration distributions are investigated for a different values of M, m, β, λ, and N. The rate of heat transfer are numerically evaluated for a different variations of the governing parameters.

Keywords: *Heat transfer, Hall effect, Radiation effect, Wavy channel.*

1. INTRODUCTION

It has been established [1] that channels with diverging – converging geometries augment the transportation of heat transfer and momentum. As the fluid flows through a tortuous path viz., the dilated – constricted geometry, there will be more intimate contact between them. The flow takes place both axially (primary) and transversely (secondary) with the secondary velocity being towards the axis in the fluid bulk rather than confining within a thin layer as in straight channels. Hence it is advantageous to go for converging-diverging geometries for improving the design of heat transfer equipment. Vajravelu and Nayfeh [2] have investigated the influence of the wall waviness on friction and pressure drop of the generated coquette flow. Vajravelu and Sastry [3] have analyzed the free convection heat transfer in a viscous, incompressible fluid confined between long vertical wavy walls is the presence of constant heat source. Later Vajravelu and Debnath [4] have extended this study to convective flow is a vertical wavy channel in four different geometrical configurations. This problem has been extended to the case of wavy walls by McMicheal and Deutsch [1], Deshikachar et al [5] Rao et. al., [6] .Alam et. al., [7] have studied unsteady free convective heat and mass transfer flow in a rotating system with Hall currents, viscous dissipation and Joule heating. Taking Hall effects in to account Krishna et. al.,[8] have investigated Hall effects on the unsteady hydromagnetic boundary layer flow. Rao et. al., [6] have analyzed Hall effects on unsteady Hydromagnetic flow. Siva Prasad et. al., [9] have studied Hall effects on unsteady MHD free and forced convection flow in a porous rotating channel.

2. FORMULATION OF THE PROBLEM

We consider the steady flow of an incompressible, viscous, electrically conducting fluid confined in a vertical channel bounded by two wavy walls under the influence of an inclined magnetic field of intensity Ho lying in the plane (y-z).The magnetic field is inclined at an angle α to the axial direction k and hence its components are $(0, H_0 Sin(\alpha), H_0 Cos(\alpha))$ .In view of the waviness of the wall the velocity field has components (u, 0, w) The

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magnetic field in the presence of fluid flow induces the current $(J_x, 0, J_z)$.We choose a rectangular Cartesian co-ordinate system O(x, y, z) with z-axis in the vertical direction and the walls at $x = \pm f\left(\frac{\delta z}{L}\right)$.

When the strength of the magnetic field is very large we include the Hall current so that the generalized Ohm's law is modified to

$$\bar{J} + \omega_e \tau_e \bar{J} x \bar{H} = \sigma(\bar{E} + \mu_e \bar{q} x \bar{H}) \tag{2.1}$$

where q is the velocity vector. H is the magnetic field intensity vector E is the electric field, J is the current density vector, is the cyclotron frequency, τ_e is the electron collision time,σ is the fluid conductivity and μ_e is the magnetic permeability. Neglecting the electron pressure gradient ion-slip and thermo-electric effects and assuming the electric field E=0

$$J_x - m H_0 J_z Sin(\alpha) = -\sigma \mu_e H_0 w Sin(\alpha) \tag{2.2}$$

$$J_z + m H_0 J_x Sin(\alpha) = \sigma \mu_e H_0 u \, Sin(\alpha) \tag{2.3}$$

where m= $\omega_e \tau_e$ is the Hall parameter.

On solving equations (2.2) & (2.3) we obtain

$$J_x = \left(\frac{\sigma \mu_e H_0 Sin(\alpha)}{1 + m^2 H_0^2 Sin^2(\alpha)}\right)\left(m H_0 Sin(\alpha) - w\right) \tag{2.4}$$

$$J_z = \left(\frac{\sigma \mu_e H_0 Sin(\alpha)}{1 + m^2 H_0^2 Sin^2(\alpha)}\right)\left(u + m H_0 w Sin(\alpha)\right) \tag{2.5}$$

where u, w are the velocity components along x and z directions respectively,

The Momentum equations are

$$\left(u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}\right) = -\left(\frac{\partial p}{\partial x}\right) + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) + \mu_e\left(-H_0 J_z Sin(\alpha)\right) \tag{2.6}$$

$$\left(u\frac{\partial W}{\partial x} + w\frac{\partial W}{\partial z}\right) = -\left(\frac{\partial p}{\partial z}\right) + \mu\left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial z^2}\right) + \mu_e\left(H_0 J_x Sin(\alpha)\right) \tag{2.7}$$

Substituting J_x and J_z from equations (2.4) & (2.5) in equations (2.6) & (2.7) we obtain

$$\left(u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}\right) = -\left(\frac{\partial p}{\partial x}\right) + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) - \left(\frac{\sigma \mu_e H_{00}^2 Sin^2(\alpha)}{1 + m^2 H_0^2 Sin^2(\alpha)}\right)\left(u + m H_0 w Sin(\alpha)\right) \tag{2.8}$$

$$\left(u\frac{\partial W}{\partial x} + w\frac{\partial W}{\partial z}\right) = -\left(\frac{\partial p}{\partial z}\right) + \mu\left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial z^2}\right) - \left(\frac{\sigma \mu_e H_0^2 Sin^2(\alpha)}{1 + m^2 H_0^2 Sin^2(\alpha)}\right)\left((w - m H_0 u Sin(\alpha)\right) - \rho g \tag{2.9}$$

The energy equation is

$$\rho C_p\left(u\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial z}\right) = k_f\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) - \left(\frac{\partial (q_r)}{\partial x}\right) \tag{2.10}$$

The equation of state is

$$\rho-\rho_0=-\beta(T-T_o) \tag{2.11}$$

Where T, is the temperature and concentration in the fluid. k_f is the thermal conductivity, Cp is the specific heat constant pressure, β is the coefficient of thermal expansion, Q is the strength of the heat source and q_r is the radiative heat flux. By Roseland approximation (Brew ester (*)) the radiative heat flux is given by

$$q_r=-\left(\frac{4\sigma^{\bullet}}{3\beta_r}\frac{\partial(T'^4)}{\partial y}\right) \tag{2.12}$$

Expanding T'^4 about Te by Taylor expansion and neglecting the higher order terms we get

$$T'^4\cong 4TT_e^3-3T_e^4 \tag{2.13}$$

Where $\sigma^{\bullet}$ is the Stefan-Boltzman constant and β_r is the mean absorption coefficient. Substituting (2.13) & (2.14) in (2.10) we obtain

$$\rho C_p\left(u\frac{\partial T}{\partial x}+w\frac{\partial T}{\partial z}\right)=k_f\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right)+\left(\frac{16\sigma^* T_e^3}{3\beta_R}\frac{\partial^2 T}{\partial x^2}\right) \tag{2.14}$$

The flow is maintained by a constant volume flux for which a characteristic velocity is defined as

$$q=\frac{1}{L}\int_{-Lf}^{Lf} w\,dx \tag{2.15}$$

The boundary conditions are

u= 0, w=0 T=T1, on $x=-f\left(\frac{\delta z}{L}\right)$ (2.16)

u=0, w=0, T=T_2 , on $x=f\left(\frac{\delta z}{L}\right)$ (2.17)

Eliminating the pressure from equations (2.8) & (2.9) and introducing the Stokes Stream function ψ as

$$u=-\left(\frac{\partial\psi}{\partial y}\right),\ w=\left(\frac{\partial\psi}{\partial x}\right) \tag{2.18}$$

The equations (2.8) & (2.9), (2.15) & (2.11) in terms of ψ is

$$\left(\frac{\partial\psi}{\partial z}\frac{\partial(\nabla^2\psi)}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial(\nabla^2\psi)}{\partial z}\right)=\mu\nabla^4\psi+\beta g\frac{\partial(T-T_e)}{\partial x}-\left(\frac{\sigma\mu_e^2 H_0^2 Sin^2(\alpha)}{1+m^2H_0^2\ Sin^2(\alpha)}\right) \tag{2.19}$$

$$\rho C_p\left(\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial z}-\frac{\partial\psi}{\partial z}\frac{\partial T}{\partial x}\right)=k_f\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right)+\left(\frac{16\sigma^* T_e^3}{3\beta_R}\frac{\partial^2 T}{\partial x^2}\right) \tag{2.20}$$

On introducing the following non-dimensional variables

$$(x',z')=\frac{(x,z)}{L},\ \psi'=\frac{\psi}{qL},\ \theta=\frac{T-T_2}{T_1-T_2}$$

The equation of momentum and energy in the non-dimensional form are

$$\nabla^4\psi-M_1^2\nabla^2\psi+\frac{G}{R}\left(\frac{\partial\theta}{\partial x}\right)=R\left(\frac{\partial\psi}{\partial z}\frac{\partial(\nabla^2\psi)}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial(\nabla^2\psi)}{\partial z}\right) \tag{2.21}$$

$$PR\left(\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial z}-\frac{\partial\psi}{\partial z}\frac{\partial\theta}{\partial x}\right)=\nabla^2\theta+\left(\frac{4}{3N_1}\frac{\partial^2\theta}{\partial x^2}\right) \tag{2.22}$$

where

$G=\dfrac{\beta g\,\Delta T_e L^3}{\nu^2}$ (Grashof Number)

$M^2=\dfrac{\sigma\mu_e^2 H_o^2 L^2}{\nu^2}$ (Hartman Number)

$R=\dfrac{qL}{\nu}$ (Reynolds Number)

$P=\dfrac{\mu C_p}{K_f}$ (Prandtl Number)

$N_1=\dfrac{3\beta_R K_f}{4\sigma^{\bullet}T_e^3}$ (Radiation parameter)

The corresponding boundary conditions are

$$\psi(1)-\psi(-1)=1$$

$$\frac{\partial\psi}{\partial z}=0,\frac{\partial\psi}{\partial x}=0,\theta=1,\ \ at\ \ x=-f(\delta z)$$

$$\frac{\partial\psi}{\partial z}=0,\frac{\partial\psi}{\partial x}=0,\theta=0\ \ \ at\ \ x=+f(\delta z)$$

3. NUSSELT NUMBER

The rate of heat transfer (Nusselt Number) on the walls has been calculated using the formula

$$Nu=\frac{1}{f(\theta_m-\theta_w)}\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=\pm1}$$

where

$$\theta_m=0.5\int_{-1}^{1}\theta\,d\eta$$

$$(Nu)_{\eta=+1}=\frac{1}{f\theta_m}(a_{54}+\delta\ b56)$$

$$(Nu)_{\eta=-1}=\frac{1}{f(\theta_m-1)}(b_{54}+\delta\ b_{55})$$

$$\theta_m=b_{57}+\delta b_{58}$$

4. DISCUSSION OF THE NUMERICAL RESULTS

In this analysis we investigate the effect of Hall currents and radiation on connective Heat Transfer flow of viscous electrically conducting fluid in a vertical channel bounded by wavy walls under the action of an inclined magnetic field. The equations governing the flow and Heat transfer are solved by employing a regular perturbation technique with the slope 'δ' as a perturbation parameter. The analysis has been carried out P=0.71 and δ=0.01.

The secondary velocity u which arises due to the wavyness of the boundaries is exhibited in (Figs.1-4). Fig. 1 represents the variation of u with M and m respectively. It is found that higher the Lorentz force (M≤4). Smaller |u| and for further increasing the Lorentz force M≥6 larger |u| in the entire region. Also |u| enhances with |u| increase in the Hall Parameters 'm' with maximum at η= -0.2. (Fig.1).From Fig.2 we observe that higher the constriction of the channel wavy larger |u| in the region. Fig.3 represents the variation of u with inclination λ we found that the secondary velocity in towards the boundary in the Left half and towards an increase in the inclination of magnetic field λ enhances the secondary velocity in the flow region. It is clear from Fig.4, the variation of 'u' with Radiation Parameter N shows that an increase in the results in an enhancement in |u| in the flow region.

The axial velocity 'w' is shown in Figs.5-9 for different values of M, m,β,λ, and N. Figs.5 and 6 represents the variation of w with M and m. The reversal flow which appears in the right region shrinks in its size with increase in M and Hall parameter m. Also |w| experiences a retardation with increase in M and m. They Higher the Lorentz force smaller |w| in the flow region (Figs.5 and 6). The influence of surface geometry on w is shown in Fig.7. We notice that higher the construction of the channel wavy smaller |w| in the flow region. The effect of inclination of the magnetic field on w is shown in Fig.8. An increase in the inclination of the magnetic field depreciates |w| in the entire region. Thus higher the inclination smaller |w| in the flow region. The influence of the radiative Heat Transfer on w is shown in Fig.9. |w| experiences an enhancement with increase in the radiation parameter N.

The Non-dimensional Temperature θ is shown in Figs.10-14 with different Parametric values we follow the convention that the Non-dimensional temperature Positive or Negative according as the actual temperature greater than or lesser than T_2. From Fig.10 shows that an increase in M reduces the actual temperature in the entire flow region (Fig.10). While an increase in the Hall Parameter 'm' enhances the actual temperature in left half and reduces in the right half (Fig.11). Fig.12 represents the variation of θ with β. It is found that higher the constriction of the channel wavy larger the actual temperature of the left half and smaller in the right half. From Fig.13 we notice that an increase in the inclination of magnetic field reduces actual temperature in the left half and enhances it in the right half. An increase in the Radiation Parameter N results in an enhancement of the actual temperature in the left half and reduces in the right half (Fig.14).

The average Nusselt number (Nu) with measures the rate of Heat Transfer at the boundaries $\eta=\pm1$ are evaluated for different values of G, M, m, β,λ are shown in (Tables. 1 and 2).

It is found that the rate of Heat Transfer experiences an enhancement increase in the thermal buoyancy parameter |G|. The variation of Nu with Hartmann number M shows that the Nusselt Number at η=+1 enhances with increase in M. In both heating and cooling cases η=-1. The Nusselt number enhances with $M\leq4$ and depreciates with $M\geq6$. In the heating case and in the cooling case |Nu| enhances with M. An increase in the Hall Parameter 'm' deceases the magnitude of Nu at $|G|=10^3$ and at higher $|G|= 3X10^3$ enhances with m at η=+1 and at η=-1. The rate of Heat Transfer enhances in m in the heating case and reduces in the Cooling case. The variation of Nu with β shows that higher the constriction of the channel wavy larger the state of Heat Transfer at both the wavy (Tables 1 and 2).

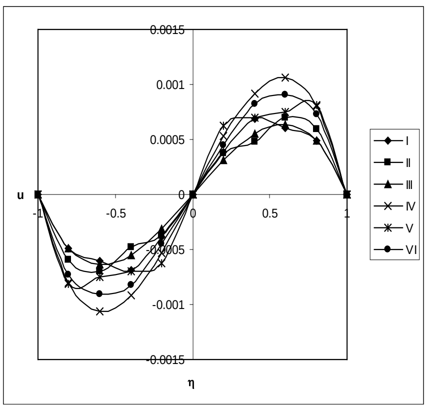

Fig. 1: u with M & m

	I	II	III	IV	V	VI
M	2	4	6	2	2	2
m	0.5	0.5	0.5	0.5	1.5	2.5

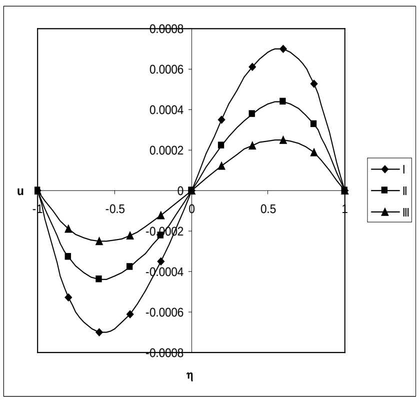

Fig. 2: u with β

	I	II	III
β	-0.5	-0.7	-0.9

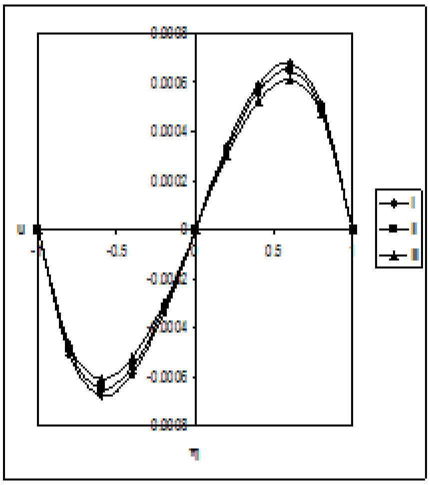

Fig. 3: u with λ

	I	II	III
λ	0.5	0.75	1

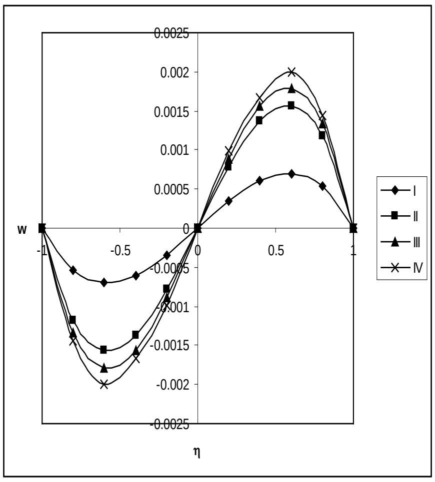

Fig. 4: u with N

	I	II	III
N	1.5	5	10

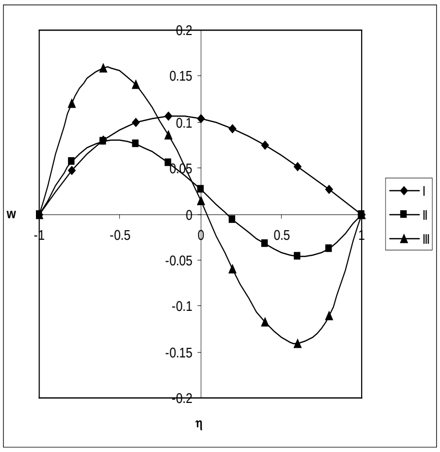

Fig. 5: w with M

	I	II	III
M	2	4	6

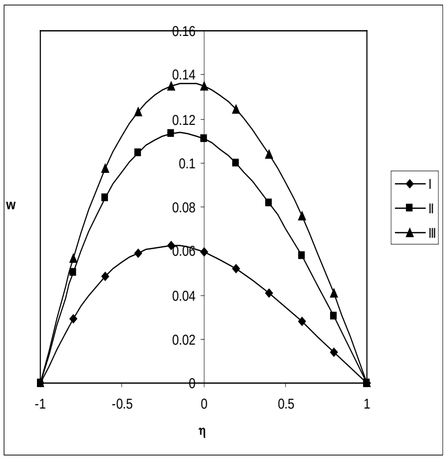

Fig. 6: w with m

	I	II	III
m	0.5	1.5	2.5

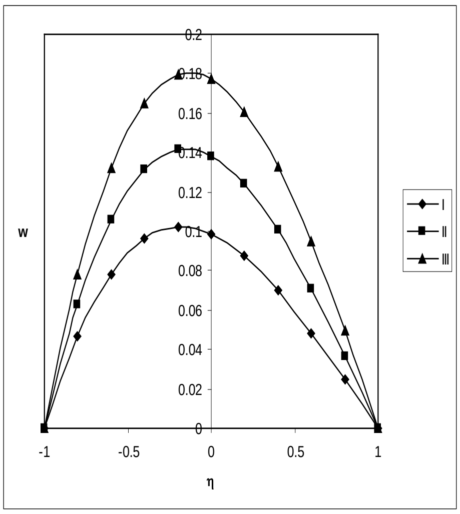

Fig. 7: w with β

	I	II	III
β	-0.5	-0.7	-0.9

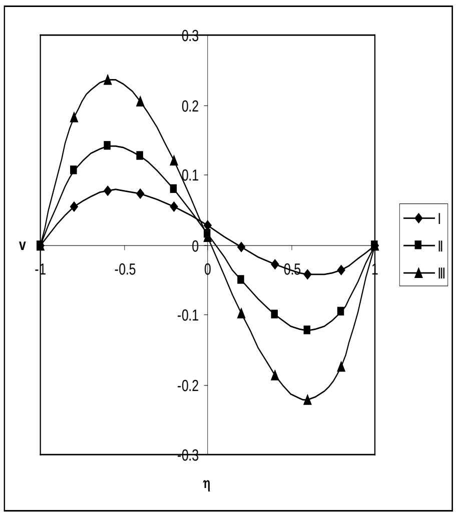

Fig. 8: w with λ

	I	II	III
λ	0.5	0.75	1

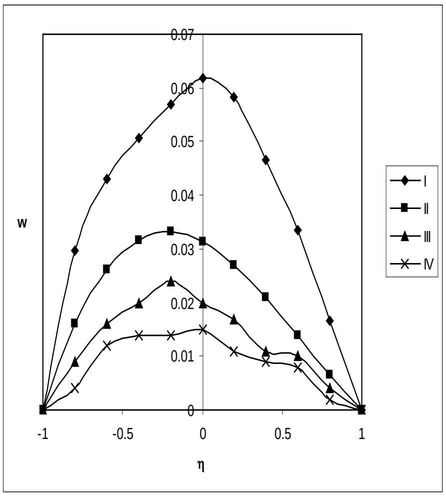

Fig. 9: w with N

	I	II	III
N	1.5	5	10

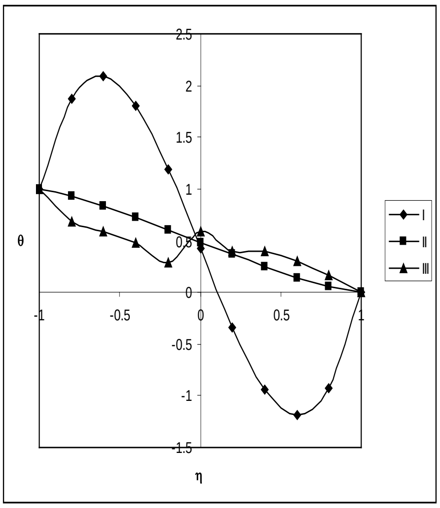

Fig. 10: θ with M

	I	II	III
M	2	4	6

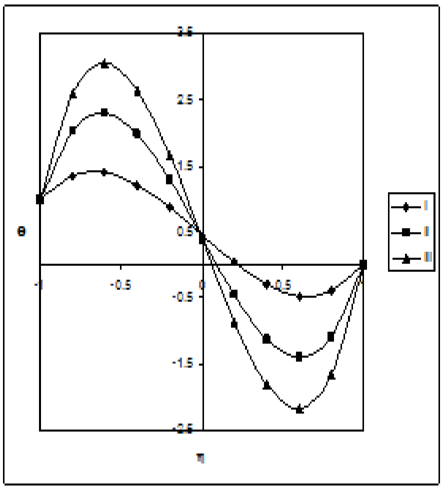

Fig. 11: θ with m

	I	II	III
m	0.5	1.5	2.5

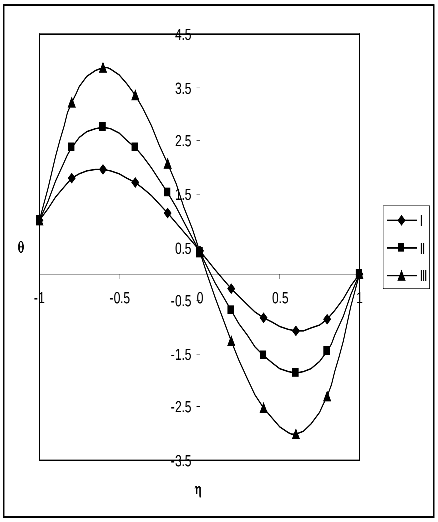

Fig. 12: θ with β

	I	II	III
β	-0.5	-0.7	-0.9

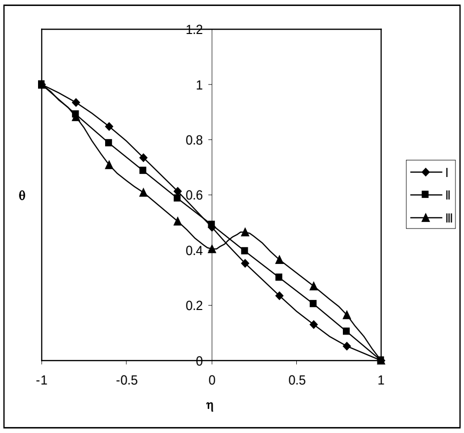

Fig. 13: θ with λ

	I	II	III
λ	0.5	0.75	1

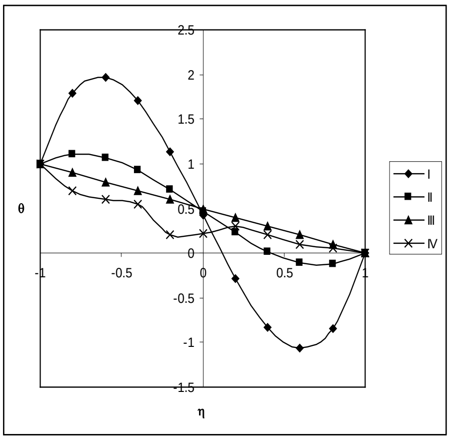

Fig. 14: θ with N

	I	II	III
N	1.5	5	10

Table: 1

Average Nusselt Number (Nu) at η =+1, P=0.71, λ=0.25, $x = \frac{\pi}{4}$

G	I	II	III	IV	V	VI	VII	VIII
10^3	0.18188	-0.69317	0.35664	0.62454	-0.19806	-0.55787	-0.64445	-0.72578
$3x10^3$	-1.84977	-0.68896	0.65772	0.90043	-0.98422	0.60952	-0.68508	-0.69678
-10^3	-0.62884	-0.71548	-2.41294	-5.80161	-0.36364	0.52204	-0.71827	-0.76984
$-3x10^3$	-0.77814	-0.71964	-3.35727	-15.67063	-39.25876	0.78138	-0.82347	-0.99678

Table: 2

Average Nusselt Number (Nu) at η = -1, p=0.71, λ=0.25, $x = \frac{\pi}{4}$

G	I	II	III	IV	V	VI	VII	VIII
10^3	1.28555	3.83430	1.68329	1.59190	5.35666	2.69584	5.12018	7.2056
$3x10^3$	2.32725	4.04440	2.50482	2.91947	-6.05592	2.45190	7.64079	10.76130
-10^3	5.20524	6.1256	7.70786	3.99413	2.66667	0.52633	0.64860	0.84590
$-3x10^3$	-6.39798	7.2096	8.88633	3.66655	2.85088	0.59426	0.7525	0.9595

M	2	4	6	2	2	2	2	2
m	0.5	0.5	0.5	1.5	2.5	0.5	0.5	0.5
β	-0.5	-0.5	-0.5	-0.5	-0.5	-0.3	-0.7	-0.9

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