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A COINCIDENCE POINT THEOREM FOR FOUR SELF MAPPINGS IN DISLOCATED QUASI METRIC SPACES

K. P. R. Sastry[1], S. Kalesha Vali[2], Ch. Srinivasa Rao[3] and M. A. Rahamatulla[4*]

[1]8-28-8/1, Tamil Street, Chinna Waltair, Visakhapatanam-530 017, India

[2]Department of Mathematics, GITAM University, Visakhapatnam- 530 045, India

[3]Department of Mathematics, Mrs. A.V.N. College, Visakhapatnam -530 001, India

[4]Department of Mathematics, Al-Aman College of Engineering, Visakhapatnam – 531 173, India

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ABSTRACT

In this paper we prove a coincidence point theorem in dislocated metric spaces, provide a supporting example and extend the theorem to dislocated quasi metric spaces. We observe that the supporting example of a result of K.P.R. Rao and P. Ranga Swamy ([7]) , on the existence of a coincidence point for four self maps on a dislocated metric space is not valid . We also make a modification of their result.

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1. INTRODUCTION

In 2005, F.M. Zayeda, G.H. Hassan and M.A. Ahmed [10] defined dislocated quasi metric spaces and dislocated metric spaces. C.T. Aage and J.N. Salunke [1] and A. Isufati [4] proved fixed point theorems for a single self map and a pair of self mappings in dislocated metric spaces, K.P.R. Rao and P. Ranga Swamy ([7]) proved a common coincidence point theorem for four self maps in a dislocated metric space. In this paper we prove a coincidence point theorem for four self maps on a dislocated quasi metric space and observe that (Theorem 2.1, [7]) is a special case of our result.

First we recall some Definitions from [10]

Definition 1.1: Let X be a non empty set and let $d : X \times X \longrightarrow [0, \infty)$ be a function . The following conditions on d are refered subsequently

$d(x,x) = 0 \;\forall\, x \in X$. (1.1.1)

$d(x,y) = d(y,x) = 0 \Rightarrow x = y \;\forall\; x,y \in X.$ (1.1.2)

$d(x,y) = d(y,x) \;\forall\, x,y \in X.$ (1.1.3)

$d(x,y) \le d(x,z) + d(z,y) \;\forall\, x,y,z \in X.$ (1.1.4)

$d(x,y) \le \max\{d(x,z),\; d(z,y)\} \;\forall\; x,y,z \in X.$ (1.1.5)

(i) If d satisfies (1.1.2) and (1.1.4) then d is called a dislocated quasi metric (or) dq - metric and (X, d) is called a dq - metric space.

(ii) If d satisfies (1.1.2), (1.1.3) and (1.1.4) then d is called a dislocated metric and (X, d) is called a dislocated metric space.

(iii) If d satisfies (1.1.1), (1.1.2) and (1.1.4) then d is called a quasi metric and (X, d) is called a quasi metric space.

(iv) If d satisfies (1.1.1), (1.1.2), (1.1.3) and (1.1.4) then d is called a metric and (X, d) is called a metric space.

(v) If d satisfies (1.1.1), (1.1.2), (1.1.3) and (1.1.5) then d is called an Ultra metric and (X, d) is called an Ultra metric space.

Corresponding author: *M. A. Rahamatulla[4*]*
[4]Department of Mathematics, Al-Aman College of Engineering, Visakhapatnam – 531 173, India

We observe that every ultra metric is a metric.

Definition 1.2 ([10]): A sequence $\{x_n\}$ in a dq – metric space (X, d) is called a Cauchy sequence if for any given $\epsilon > 0$ there exists $n_0 \in N$ such that for all $m, n \geq n_0$, $d(x_m, x_n) < \epsilon$.

Definition 1.3 ([10]): A sequence $\{x_n\}$ in a dq – metric space is said to be dislocated quasi convergent to x. if $\lim_{n\to\infty} d(x_n, x) = \lim_{n\to\infty} d(x, x_n) = 0$.

In this case x is called a dq – limit of $\{x_n\}$ and we write $x_n \to x$.

Lemma 1.4 ([10]): Dq – limit in a dq – metric space is unique.

Definition 1.5 ([10]): A dq – metric space (X, d) is called complete if every Cauchy sequence in it is dq – convergent .

Definition 1.6 ([10]): Let (X, d_1) and (Y, d_2) be dq – metric spaces and let $f: X \to Y$ be a function. Then f is said to be continuous at $x_0 \in X$, if the sequence $f\{x_n\}$ is d_2q - convergent to $f(x_0) \in Y$ whenever the sequence $\{x_n\}$ in X is d_1q - convergent to x_0 .

Definition 1.7 ([10]): Let (X, d) be a dq – metric space. A map T: $X \to X$ is called a contraction if there exists $0 \leq \lambda < 1$ such that

$$d(Tx, Ty) \leq \lambda\, d(x, y)\ \forall\, x, y \in X.$$

Note: In a metric space, a contraction map is continuous. However, in a dq – metric space a contraction map need not be continuous (Rutten [9], Example [3.6]). **Zayeda.et.al [10] proved the dq - metric version of Banach contraction principle.**

Theorem 1.8 ([10]): Let (X, d) be a dq – metric space and let $T: X \to X$ be a continuous contraction mapping. Then T has unique fixed point.

K. P. R. Rao and P. Ranga Swamy [7] proved the following coincidence theorem for four self maps on a dislocated metric space.

Theorem 1.9: ([7], Theorem 2.1) Let (X, d) be a complete dislocated metric space and let F, G, S and T: $X \to X$ be continuous mappings satisfying

$S(X) \subseteq G(X)$ and $T(X) \subseteq F(X)$ (1.9.1)

$SF = FS$ and $TG = GT$ and (1.9.2)

$d(Sx, Ty) \leq \varphi(\max\{d(Fx, Gy), d(Fx, Sx), d(Gy, Ty), \frac{d(Fx,Sx)\,d(Gy,Ty)}{d(Fx,Gy)}\})$ (1.9.3)

for all $x, y \in X$, where $\varphi: R^+ \longrightarrow R^+$ is monotonically non – decreasing and $\sum_{n=1}^{\infty} \varphi^n(t) < \infty$ for all $t > 0$.

Then (i) F and S (or) G and T have coincidence point (or) (ii) the pairs (F, S) and (G ,T) have a common coincidence point.

The following Example is given in [7] in support of Theorem 1.9 ([7], Example 2.2)

Example 1.10: Let X = [0, 1] and $d(x, y) = max\{x, y\}$. Then (X, d) is a dislocated metric space. Define $Sx = 0$, $Tx = \frac{x}{6}, Fx = x, Gx = \frac{x}{3}$. Clearly S, T, F and G are continuous and (1.9.1) and (1.9.2) are satisfied.

Also $d(Sx, Ty) \leq \varphi(d(Fx, Gy))$ for all $x, y \in X$, where $\varphi(t) = \frac{t}{2}$, clearly 0 is a common coincidence point of (F, S) and (G, T). However the above example does not support Theorem 1.9, since at $x = y = 0$ (1.9.3) is not satisfied (In fact it has no meaning). Consequently condition (1.9.3) should be assumed to hold whenever $d(Fx, Gy) \neq 0$. Example 1.10 cannot be considered as an Example in support of Theorem 1.9,

since $d(Fx, Gy) = 0$ for $x = y = 0$.

2. MAIN RESULTS

In this section, we prove a coincidence point theorem for four self maps on a dislocated metric space and provide a supporting example. Extend this to dislocated quasi metric spaces. We also obtain the result of [7], with modification, as a corollary.

Theorem 2.1: Let (X, d) be a complete dislocated metric space and let F, G, S and T: $X \to X$ be continuous mappings satisfying

$S(X) \subseteq G(X)$ and $T(X) \subseteq F(X)$ (2.1.1)

$SF = FS$ and $TG = GT$ and (2.1.2)

$d(Sx, Ty) \leq \varphi \max\{d(Fx, Gy), d(Fx, Sx), d(Gy, Ty)\})$ (2.1.3)

for all $x, y \in X$, where $\varphi : R^+ \longrightarrow R^+$ is monotonically non –decreasing and $\sum_{n=1}^{\infty} \varphi^n(t) < \infty$ for all $t > 0$. Then the pairs (F, S) and (G,T) have a common coincidence point.

Proof: It is clear that $\varphi^n(t) \to 0$ as $n \to \infty$ and $\varphi(t) < t \; \forall \; t > 0$. Suppose $x_0 \in X$. Define the sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$y_{2n} = Sx_{2n} = Gx_{2n+1}$$

$$y_{2n+1} = Tx_{2n+1} = Fx_{2n+2}, \quad n = 0,1,2, \ldots.$$

Now

$d(y_{2n}, y_{2n+1}) = d(Sx_{2n}, Tx_{2n+1})$ (by (2.1.3))

$$\leq \varphi(\max\{d(Fx_{2n}, Gx_{2n+1}), d(Fx_{2n}, Sx_{2n}), d(Gx_{2n+1}, Tx_{2n+1})\})$$

$$= \varphi(max\{d(y_{2n-1}, y_{2n}), d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1})\})$$

$$= \varphi(max\{d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1})\}) \quad (2.1.4)$$

Now from (2.1.4)

$$d(y_{2n-1}, y_{2n}) \leq d(y_{2n}, y_{2n+1})$$

$$\Rightarrow d(y_{2n}, y_{2n+1}) \leq \varphi(d(y_{2n}, y_{2n+1}))$$

$$\Rightarrow d(y_{2n}, y_{2n+1}) = 0$$

$$\Rightarrow d(y_{2n}, y_{2n+1}) \leq 0 \leq \varphi(d(y_{2n-1}, y_{2n})) \quad (2.1.5)$$

Now suppose

$d(y_{2n-1}, y_{2n}) > d(y_{2n}, y_{2n+1})$. Then from (2.1.4)

follows that $d(y_{2n}, y_{2n+1}) \leq \varphi(d(y_{2n-1}, y_{2n}))$ (2.1.6)

From (2.1.5) and (2.1.6) follows that

$d(y_{2n}, y_{2n+1}) \leq \varphi(d(y_{2n-1}, y_{2n}))$ for $n = 1,2,3 \ldots$

Similarly we can show that

$d(y_{2n+1}, y_{2n+2}) = d(Tx_{2n+1}, Sx_{2n+2})$ (by (1.1.3))

$$= d(Sx_{2n+2}, Tx_{2n+1})$$

$$\leq \varphi(max\{d(Fx_{2n+2}, Gx_{2n+1}), d(Fx_{2n+2}, Sx_{2n+2}), d(Gx_{2n+1}, Tx_{2n+1})\})$$

$$= \varphi(max\{d(y_{2n+1}, y_{2n}), d(y_{2n+1}, y_{2n+2}), d(y_{2n}, y_{2n+1})\})$$

$$= \varphi(max\{d(y_{2n+1}, y_{2n+2}), d(y_{2n}, y_{2n+1})\})$$

$$\leq \varphi(d(y_{2n}, y_{2n+1}))$$

Hence $d(y_n, y_{n+1}) \leq \varphi(d(y_{n-1}, y_n))$ for $n = 1,2,3\ldots$

$$\begin{aligned} \therefore\ d(y_n, y_{n+1}) &\leq \varphi(d(y_{n-1}, y_n)) \\ &\leq \\ &\vdots \\ &\leq \varphi^n(d(y_0, y_1)) \end{aligned}$$
$$\therefore\ d(y_n, y_{n+1}) \leq \varphi^n(d(y_0, y_1))$$

Since $\varphi^n(t) \to 0 \ as\ n \to \infty$, we may suppose with out loss of generality that $d(y_0, y_1) < 1$

$$\begin{aligned} \therefore\ d(y_n, y_{n+k}) &\leq d(y_n, y_{n+1}) + \cdots + d(y_{n+k-1}, y_{n+k}) \\ &= \varphi^n(d(y_0, y_1)) + \cdots + \varphi^{n+k-1}(d(y_0, y_1)) \\ &= \frac{\varphi^n(d(y_0, y_1))}{1-\varphi(d(y_0, y_1))} \to 0 \quad as \quad n \to \infty \end{aligned}$$

$\therefore$ $\{y_n\}$ is a Cauchy sequence. Hence there exists $u \in X$ such that

$\{y_n\}$ converges to u. Since $FS = SF$ and S and F are continuous,

we have $Su = \lim_{n\to\infty} SFx_{2n} = \lim_{n\to\infty} FSx_{2n} = Fu$,

since $TG = GT$ and T and G are continuous ,

we have $Tu = \lim_{n\to\infty} TGx_{2n+1} = \lim_{n\to\infty} GTx_{2n+1} = Gu$

Thus u is a common coincidence point of the pairs (F, S) and (G, T) ,

consequently u is a common coincidence point of F, S, G and T.

The following Example supports our theorem

Example 2.2: Let $X = [0,1]$, $d(x,y) = \max(x, y)$. Then (X,d) is a complete dislocated metric space. Define $Sx = 0$, $Tx = \frac{x}{2}$, $Fx = Gx = x$. Take $\varphi(t) = \frac{t}{2}$. Then clearly (2.1.1), (2.1.2) and (2.1.3) are satisfied and 0 is a common coincidence point of S, F, G and T.

The following is a modified version of Theorem 1.9

Theorem 2.3: Let (X, d) be a complete dislocated metric space and let F, G, S and $T : X \to X$ be continuous mappings satisfying

$$S(X) \subseteq G(X) \text{ and } T(X) \subseteq F(X) \tag{2.3.1}$$

$$SF = FS \ and\ TG = GT \ and \tag{2.3.2}$$

$$d(Tx, Sy) \leq \varphi(max\{d(Gx,Fy), d(Fy,Sy), d(Gx,Tx), \frac{d(Fy,Sy)d(Gx,Tx)}{d(Gx,Fy)}\}) \tag{2.3.3}$$

for all $x, y \in X$, whenever $d(Gx,Fy) \neq 0$, where $\varphi : R^+ \longrightarrow R^+$ is monotonically non–decreasing and $\sum_{n=1}^{\infty} \varphi^n(t) < \infty$ for all $t > 0$.

Then F, S, G and T have a common coincidence point .

Proof: It is clear that $\varphi^n(t) \to 0 \ as\ n \to \infty \ and\ \varphi(t) < t \ \forall\ t > 0$. $Suppose\ x_0 \in X$.

Define the sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$y_{2n} = Sx_{2n} = Gx_{2n+1}$$

$$y_{2n+1} = Tx_{2n+1} = Fx_{2n+2},\ n = 0,1,2,\ldots.$$

If $y_{2n} = y_{2n+1}$ for some n then $Gx_{2n+1} = Tx_{2n+1}$. Hence x_{2n+1} is a coincidence point of G and T.

If $y_{2n+1} = y_{2n+2}$ for some n then $Fx_{2n+2} = Sx_{2n+2}$. Hence x_{2n+2} is a coincidence point of F and S. Assume that $y_n \neq y_{n+1}$ for all n. Then as in the proof of Theorem 2.1, we can show that $\{ y_n \}$ is a Cauchy sequence .

Hence there exists $u \in X$ such that $\{ y_n \}$ converges to u. Since $FS = SF$ and S and F are continuous ,

we have $Su = \lim_{n\to\infty} SFx_{2n} = \lim_{n\to\infty} FSx_{2n} = Fu$,

since $TG = GT$ and T and G are continuous , we have $Tu = \lim_{n\to\infty} TGx_{2n+1} = \lim_{n\to\infty} GTx_{2n+1} = Gu.$

Thus u is a common coincidence point of the pairs (F , S) and (G , T) , consequently u is a common coincidence point of F , S ,G and T.

The following Example shows that Theorem 2.3 may not hold, even in metric spaces, if (2.3.2) is dropped.

Example 2.4: Let $X = [0,1]$ with the usual metric d. Define $Sx = \frac{x}{3}$, $Tx = 0$, $Fx = x$, $Gx = 1 - x \ \ \forall x \in X$. Then (X, d) is a complete metric space, (2.3.1) is clearly satisfied (2.3.3) is satisfied with $\varphi(t) = \frac{t}{2}$. But (2.3.2) is not satisfied since $TG \neq GT$. Here the pair (*F, S*) has a coincidence point, namely 0. The pair (*G, T*) has a coincidence point namely 1. But *S, T, F* and *G* do not have a common coincidence point.

Now we extend Theorem 2.1 to dislocated quasi metric spaces as follows

Theorem 2.5: Let (X , d) be a complete dislocated quasi metric space, let F, G , S and $T : X \to X$ be continuous mappings satisfying

$$S (X) \subseteq G (X) \text{ and } T (X) \subseteq F (X) \tag{2.5.1}$$

$$SF = FS \text{ and } TG = GT \quad and \tag{2.5.2}$$

$$d (Sx , Ty) \leq \varphi(max\{d(Fx, Gy), d(Fx, Sx), d(Gy, Ty)\}) \tag{2.5.3}$$

$$d (Tx , Sy) \leq \varphi(max\{d(Gx, Fy), d(Fy, Sy), d(Gx, Tx)\}) \tag{2.5.4}$$

for all $x, y \in X$, where $\varphi : R^+ \longrightarrow R^+$ is monotonically non – decreasing and $\sum_{n=1}^{\infty} \varphi^n \ (t) < \infty$ for all $t > 0$.

Then the pairs (F, S) and (G,T) have a common coincidence point which is also a common coincidence point of F,G,S and T

Proof: It is clear that $\varphi^n \ (t) \to 0 \ \ as \ \ n \to \ \infty \ and \ \varphi(t) < t \ \forall \ t > 0 \ . Suppose \ x_0 \in X$.

Define the sequences $\{x_n\} \ and \ \{y_n \}$ in X such that

$$y_{2n} = Sx_{2n} = Gx_{2n+1}$$

$$y_{2n+1} = Tx_{2n+1} = Fx_{2n+2} \ , \ n = 0,1,2,\ldots.$$

Now

$$d(y_{2n} , y_{2n+1}) = d(Sx_{2n} , Tx_{2n+1}) \qquad \text{(by (2.5.3))}$$

$$\leq \varphi(max\{d (Fx_{2n} , Gx_{2n+1}) , d(Fx_{2n} , Sx_{2n}), \ d(Gx_{2n+1} , Tx_{2n+1})\})$$

$$= \varphi(max\{d(y_{2n-1} , y_{2n}) , d(y_{2n-1} , y_{2n}) , d(y_{2n} , y_{2n+1})\})$$

$$= \varphi(max\{d(y_{2n-1} , y_{2n}) , d(y_{2n} , y_{2n+1})\}) \tag{2.5.5}$$

Now from (2.5.5)

$$d(y_{2n-1} , y_{2n}) \leq d(y_{2n} , y_{2n+1})$$

$$\Rightarrow d(y_{2n} , y_{2n+1}) \leq \varphi(d(y_{2n} , y_{2n+1}))$$

$$\Rightarrow d(y_{2n} , y_{2n+1}) = 0$$

$$\Rightarrow \; d(y_{2n}, y_{2n+1}) \leq 0 \leq \varphi(d(y_{2n-1}, y_{2n})) \tag{2.5.6}$$

Now suppose

$d(y_{2n-1}, y_{2n}) > d(y_{2n}, y_{2n+1})$. Then from (2.5.5)

follows that

$$d(y_{2n}, y_{2n+1}) \leq \varphi(d(y_{2n-1}, y_{2n})) \tag{2.5.7}$$

From (2.5.6) and (2.5.7) shows that

$d(y_{2n}, y_{2n+1}) \leq \varphi(d(y_{2n-1}, y_{2n}))$ for $n = 1,2,3\ldots$

Similarly using (2.5.4), we can show that

$d(y_{2n+1}, y_{2n+2}) \leq \varphi(d(y_{2n}, y_{2n+1}))$

Now

$$\begin{aligned} d(y_{2n}, y_{2n+1}) &= d(Tx_{2n+1}, Sx_{2n}) \text{ (by (2.5.4))} \\ &\leq \varphi(max\{d(Gx_{2n+1}, Fx_{2n}), d(Fx_{2n}, Sx_{2n}), d(Gx_{2n+1}, Tx_{2n+1})\}) \\ &= \varphi(max\{d(y_{2n}, y_{2n-1}), d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1})\}) \\ &= \varphi(max\{d(y_{2n}, y_{2n-1}), d(y_{2n}, y_{2n+1})\}) \end{aligned}$$

$\therefore$ If $d(y_{2n}, y_{2n+1}) \leq d(y_{2n}, y_{2n+1}) < d(y_{2n}, y_{2n+1})$

since $d(y_{2n}, y_{2n+1}) > 0$, which is contradiction

$\therefore \; d(y_{2n}, y_{2n+1}) \leq d(y_{2n-1}, y_{2n})$

$\therefore \; d(y_{2n}, y_{2n+1}) \leq \varphi(d(y_{2n-1}, y_{2n}))$ this is true for $n = 1,2,3$ ...

$$\begin{aligned} \therefore \; d(y_n, y_{n+1}) &\leq \varphi(d(y_{n-1}, y_n)) \\ &\leq \\ &\vdots \\ &\leq \varphi^n(d(y_0, y_1)) \end{aligned}$$

Here again, we may suppose without loss of generality that $\varphi(d(y_0, y_1)) < 1$

$$\begin{aligned} \therefore \; d(y_n, y_{n+k}) &\leq d(y_n, y_{n+1}) + \cdots + d(y_{n+k-1}, y_{n+k}) \\ &= \varphi^n(d(y_0, y_1)) + \cdots + \varphi^{n+k-1}(d(y_0, y_1)) \\ &= \frac{\varphi^n(d(y_0, y_1))}{1-\varphi(d(y_0, y_1))} \to 0 \quad as \quad n \to \infty \end{aligned}$$

$\therefore \{y_n\}$ is a Cauchy sequence .

Since X is a complete dislocated quasi metric space there exists $u \in X$ such that $\{y_n\}$ converges to u . Since $FS = SF$ and S and F are continuous,

we have $Su = \lim_{n\to\infty} SFx_{2n} = \lim_{n\to\infty} FSx_{2n} = Fu$,

since $TG = GT$ and T and G are continuous ,

we have $Tu = \lim_{n\to\infty} TGx_{2n+1} = \lim_{n\to\infty} GTx_{2n+1} = Gu$

Thus the pairs (F , S) and (G , T) have common coincidence point which is also a common coincidence point of F , S ,G and T .

The following is a common fixed point theorem, with the control function containing rational terms.

Theorem 2.6: Let (X, d) be a complete dislocated quasi metric space.

Let F, G, S and T: $X \to X$ be continuous mappings satisfying

$S(X) \subseteq G(X)$ and $T(X) \subseteq F(X)$ (2.6.1)

$SF = FS \, and \, TG = GT \, and$ (2.6.2)

$d(Sx, Ty) \le \varphi(max\{d(Fx, Gy), d(Fx, Sx), d(Gy, Ty), \frac{d(Fx, Sx)d(Gy, Ty)}{d(Fx, Gy)}\})$ for all $x, y \in X \, and$

$d(Fx, Gy) \neq 0 \, and$ (2.6.3)

$d(Tx, Sy) \le \varphi(\max\{d(Gx, Fy), d(Fy, Sy), d(Gx, Tx), \frac{d(Fy,Sy)d(Gx,Tx)}{d(Gx,Fy)}\})$ (2.6.4)

for all $x, y \in X \, and \, d(Gx, Fy) \neq 0$, where $\varphi : R^+ \longrightarrow R^+$ is monotonically non – decreasing and

$\sum_{n=1}^{\infty} \varphi^n(t) < \infty$ for all $t > 0$.

Then the pairs *(F, S)* and (G, T) have a common coincidence point.

Proof: It is clear that

$\varphi^n(t) \to 0 \; as \; n \to \infty \, and \, \varphi(t) < t \, \forall \, t > 0$. $Suppose \, x_0 \in X$.

Define the sequences $\{x_n\} \, and \, \{y_n\}$ in X such that

$$y_{2n} = Sx_{2n} = Gx_{2n+1}$$

$$y_{2n+1} = Tx_{2n+1} = Fx_{2n+2}, \; n = 0,1,2,\ldots.$$

Now

$d(y_{2n}, y_{2n+1}) = d(Sx_{2n}, Tx_{2n+1})$ (by (2.6.3))

$$\le \varphi(max\{d(Fx_{2n}, Gx_{2n+1}), d(Fx_{2n}, Sx_{2n}), d(Gx_{2n+1}, Tx_{2n+1}), \frac{d(Fx_{2n}, Sx_{2n})\, d(Gx_{2n+1}, Tx_{2n+1})}{d(Fx_{2n}, Gx_{2n+1})}\})$$

$$= \varphi(max\{d(y_{2n-1}, y_{2n}), d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1}), \frac{d(y_{2n-1}, y_{2n})\, d(y_{2n}, y_{2n+1})}{d(y_{2n-1}, y_{2n})}\})$$

$$= \varphi(max\{d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1})\}) \quad (2.6.5)$$

Now from (2.6.5)

$$d(y_{2n-1}, y_{2n}) \le d(y_{2n}, y_{2n+1})$$

$$\Rightarrow d(y_{2n}, y_{2n+1}) \le \varphi(d(y_{2n}, y_{2n+1}))$$

$$\Rightarrow d(y_{2n}, y_{2n+1}) = 0$$

$$\Rightarrow d(y_{2n}, y_{2n+1}) \le 0 \le \varphi(d(y_{2n-1}, y_{2n})) \quad (2.6.6)$$

Now suppose

$d(y_{2n-1}, y_{2n}) > d(y_{2n}, y_{2n+1})$. Then from (2.6.5)

follows that

$$d(y_{2n}, y_{2n+1}) \le \varphi(d(y_{2n-1}, y_{2n})) \quad (2.6.7)$$

From (2.6.6) and (2.6.7) follows that

$d(y_{2n}, y_{2n+1}) \le \varphi(d(y_{2n-1}, y_{2n}))$ for $n = 1,2,3 \ldots$

Similarly we can show that

$$d(y_{2n+1}, y_{2n+2}) \leq \varphi(d(y_{2n}, y_{2n+1})) \quad \text{for} \quad n = 1,2,3 \ldots$$

Now

$$d(y_{2n}, y_{2n+1}) = d(Tx_{2n+1}, Sx_{2n}) \qquad \text{(by (2.6.4))}$$

$$\leq \varphi(max\{d(Gx_{2n+1}, Fx_{2n}), d(Fx_{2n}, Sx_{2n}), d(Gx_{2n+1}, Tx_{2n+1}), \frac{d(Fx_{2n}, Sx_{2n})\, d(Gx_{2n+1}, Tx_{2n+1})}{d(Gx_{2n+1}, Fx_{2n})}\})$$

$$= \varphi(max\{d(y_{2n}, y_{2n-1}), (y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1}), \frac{d(y_{2n-1}, y_{2n})\, d(y_{2n}, y_{2n+1})}{d(y_{2n-1}, y_{2n}),}\})$$

$$= \varphi(max\{d(y_{2n}, y_{2n-1}), d(y_{2n}, y_{2n+1})\})$$

$\therefore$ If $d(y_{2n}, y_{2n+1}) \leq d(y_{2n}, y_{2n+1}) < d(y_{2n}, y_{2n+1})$

since $d(y_{2n}, y_{2n+1}) > 0,$ which is contradiction

$$\therefore d(y_{2n}, y_{2n+1}) \leq d(y_{2n-1}, y_{2n})$$

$\therefore d(y_{2n}, y_{2n+1}) \leq \varphi(d(y_{2n-1}, y_{2n}))$ this is true for $n = 1,2,3 \ldots$

Similarly

$$\begin{aligned}\therefore d(y_n, y_{n+1}) &\leq \varphi(d(y_{n-1}, y_n)) \\ &\leq \\ &\vdots \\ &\leq \varphi^n(d(y_0, y_1))\end{aligned}$$

Since $\varphi^n(t) \to 0$ as $n \to \infty$, we may suppose with out loss of generality that $d(y_0, y_1) < 1$

$$\begin{aligned}\therefore d(y_n, y_{n+k}) &\leq d(y_n, y_{n+1}) + \cdots + d(y_{n+k-1}, y_{n+k}) \\ &= \varphi^n(d(y_0, y_1)) + \cdots + \varphi^{n+k-1}(d(y_0, y_1)) \\ &= \frac{\varphi^n(d(y_0, y_1))}{1-\varphi(d(y_0, y_1))} \to 0 \quad as \quad n \to \infty\end{aligned}$$

$\therefore \{y_n\}$ is a Cauchy sequence.

Since X is a complete dislocated quasi metric space there exists $u \in X$ such that $\{y_n\}$ converges to u.

Since $FS = SF$ and S and F are continuous, we have $Su = \lim_{n\to\infty} SFx_{2n} = \lim_{n\to\infty} FSx_{2n} = Fu$,

since $TG = GT$ and T and G are continuous ,

we have $Tu = \lim_{n\to\infty} TGx_{2n+1} = \lim_{n\to\infty} GTx_{2n+1} = Gu$

Thus the pairs (F, S) and (G, T) have common coincidence point which is also a common coincidence point of F, S, G and T.

Note: In theorem 2.6, if we assume the space to be a complete dislocated metric space, (2.6.4) can be drooped and consequently we get a modified version of theorem 1.9 as a corollary.

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