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HEAT TRANSFER IN HYDROMAGNETIC ROTATING FLOW OF VISCOUS FLUID THROUGH A NON-HOMOGENEOUS POROUS MEDIUM WITH CONSTANT HEAT SOURCE/SINK

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ABSTRACT

The aim of this paper is to study an unsteady free convection unsteady flow of a viscous incompressible, electrically conducting, rotating liquid in a porous medium past an infinite isothermal vertical plate with constant heat source / sink in the presence of a uniform magnetic field applied perpendicular to the flow region. Expressions for primary and secondary velocities and temperature distribution are obtained by solve the governing equations using perturbation technique. The expressions for the skin-friction coefficient and rate of heat transfer are also derived. The effects on Prandtl Number (Pr), Grashof Number (Gr), Rotation Parameter (E), Heat Sources/ Sink Parameter (a_0), Magnetic Parameter (M) and Permeability (K_0) obtained on the above flow quantities are studied through graphs and tables.

***Key words:** Heat transfer, MHD, Porous media.*

INTRODUCTION:

Free convection problem has attracted the attention of the researchers due to its possible applications in atmospheric and oceanic sciences, nuclear reactors, power transformers etc. Extensive literature on free convection flow has been introduced by Ede [1] and Morton [2]. Seth and Banergee[3] have studied hydromagnetic convective flow in a rotating channels Recently, Singh and Singh[4] have presented an analysis and mass transfer flow of a viscous fluid in rotating system. Pai [5], Schlichtirtg [6] and Bansal [7] presented free convection flow of a viscous incompressible fluid. Free convective flow through a rotating porous medium was considered by Rott and Lowellen[8]. Unsteady free convection flow through a porous medium was discussed by Raptis and Singh [9]. Raptis and Perdikis[10] studied oscillatory flow through a porous medium in the presence of free concoctive flow. Free convection effects on the flow past a porous medium bounded by a vertical infinite surface with constant suction and constant heat flux have been investigated by Sharma[11]. Baghel, Kumar and Sharma[12] considered two dimensional unsteady free convective flow of a viscous incompressible fluid through a rotating porous medium. Two dimensional unsteady free convective flow through a rotating porous medium in the presence of constant heat flux was investigated by Sharma and Pareek[13].

The aim of this investigation is to study MHD free convection unsteady flow of an incompressible, electrically conducting, viscous, rotating liquid in a porous medium past an infinite isothermal vertical plate with constant source / Sink in the presence of a uniform magnetic field applied perpendicular to the flow region. Expressions for primary and secondary velocities and temperature distribution, skin-friction coefficient and rate of heat transfer are obtained and discussed numerically.

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FORMULATION OF THE PROBLEM:

Consider an oscillatory flow of a viscous incompressible electrically conducting, viscous liquid through a porous medium past an infinite, isothermal, vertical porous plate with constant heat source/sink in the presence of a uniform magnetic field.

The Cartesian coordinate system (x, y, z) rotating uniformly with the liquid in a rigid state of rotation with a constant angular velocity (0,0, Ω) about z-axis. The vertical plate is assumed to coincide with the plance z=0 and z-axis is taken normal to the plate pointing towards the flow medium.

In this analysis we made the following assumptions:

1. The constant heat source Q is assumed at z = 0.
2. The suction velocity at the plate is $w = -w_0\left(1+\varepsilon e^{i\omega t}\right)$ where w_0 is a positive constant real number.
3. Permeability of the porous medium is assumed as $k = k_0\left(1+\in e^{i\omega t}\right)$
4. We have neglected viscous dissipation.
5. Electric field is neglected.
6. Joules heating and Hall current effects are neglected.
7. The concentration at the plate z=0 is assumed to vary harmonically with time.

Under the above stated assumption, the governing equations are as follows:

Momentum equation:

$$\frac{\partial u}{\partial t} - 2\Omega v - \omega_0\left(1+\varepsilon e^{i\omega t}\right)\frac{\partial u}{\partial z} = g\beta*\left(T-T_\infty\right) + \upsilon\frac{\partial^2 u}{\partial z^2} - \frac{\upsilon}{k}u - \frac{\sigma_e B_0^2 u}{\rho} \quad (1)$$

$$\frac{\partial v}{\partial t} + 2\Omega u - \omega_0\left(1+\varepsilon e^{i\omega t}\right)\frac{\partial v}{\partial z} = \upsilon\frac{\partial^2 v}{\partial z^2} - \frac{\upsilon}{k}v - \frac{\sigma_e B_0^2 v}{\rho} \quad (2)$$

Energy equation:

$$\frac{\partial T}{\partial t} - \omega_0\left(1+\varepsilon e^{i\omega t}\right)\frac{\partial T}{\partial z} = \frac{K^{-1}}{\rho c_p}\frac{\partial^2 T}{\partial z^2} - Q\left(T-T_\infty\right) \quad (3)$$

The corresponding boundary conditions are:

$$u=0, v=0, T=T_w+\varepsilon\left(T_w-T_\infty\right)e^{i\omega t} \quad \text{at} \quad z=0$$
$$u\to 0, v\to 0, T\to 0 \quad \text{as} \quad z\to\infty \quad (4)$$

5.4 NON DIMENSIONALISATION:

Now we introduce the following non-dimensional variables as follows:

$$u^* = \frac{u}{u_0}, v^* = \frac{v}{u_0}, T^* = \frac{T-T_\infty}{T_\omega - T_\infty}, z^* = \frac{\omega_o z}{V_0},$$

$$t^* = \frac{t\omega_0^2}{V_0}, n^* = \frac{nV_0}{\omega_0{}^2}, k_0^* = \frac{\omega_0^2 k_0}{V_0^2}$$

Using the above variables the equations (1) to (3) after removing the stars, reduces to:

$$\frac{\partial u}{\partial t} - \left(1+\varepsilon c^{i\omega t}\right)\frac{\partial u}{\partial z} - 2Ev = \frac{\partial^2 u}{\partial z^2} + Grt - \frac{1}{Ko\left(1+\varepsilon e^{iwt}\right)}u - M^2 u \quad (5)$$

$$\frac{\partial V}{\partial t} - \left(1+\varepsilon c^{i\omega t}\right)\frac{\partial v}{\partial z} + 2Ev = \frac{\partial^2 v}{\partial z^2} - \frac{1}{K_0\left(He^{iet}\right)}.V - M^2 v \quad (6)$$

$$\Pr\frac{\partial T}{\partial t}-\Pr\left(1+\varepsilon e^{i\omega t}\right)\frac{\partial T}{\partial z}=\frac{\partial^2 T}{\partial z^2}-\alpha_o T \tag{7}$$

where

$$\Pr=\frac{\mu c_p}{K}\left(\text{Prandtl number}\right)$$

$$\text{Gr}=\frac{\text{V}_0 gB\left(T_w-T_\alpha\right)}{U_0^2\omega_0^2}\left(\text{Grashof number}\right)$$

$$\text{E}=\frac{\Omega V_0}{\omega_0^2}\left(\text{Rotation parameter}\right)$$

$$\alpha_0=\frac{\text{V}_0^2}{\text{K}\omega_0^2}\left(\text{Heal source parameter}\right)$$

$$M^2=\frac{\sigma\beta^2\nu}{\rho U_0^2}$$

Now using $q=u+iv$ in the equations (1) – (2) we get:

$$\frac{\partial q}{\partial t}-\left(1+\varepsilon e^{i\omega t}\right)\frac{\partial q}{\partial z}+\frac{1}{K_0\left(1+\varepsilon e^{i\omega t}\right)}q+2iEq=\frac{\partial q^2}{\partial z^2}+GrT-M^2q \tag{8}$$

The boundary conditions become

$$\begin{array}{lll} q=0,\quad T=1+\varepsilon c^{i\omega t} & \text{at} & z=0 \\ q\to 0,\ T\to 0 & \text{as} & Z\to\infty \end{array} \tag{9}$$

5.5 SOLUTION OF THE PROBLEM:

In order to solve the equations, we assume the velocity $q\left(z,t\right)$ and temperature $T\left(z,t\right)$ of the liquid as

$$\begin{aligned} q\left(z,t\right)&=q_0\left(z\right)+\varepsilon q_1\left(z\right)e^{iwt} \\ T\left(z,t\right)&=T_0\left(z\right)+\varepsilon T_1\left(z\right)e^{i\omega t} \end{aligned} \tag{10}$$

Introducing the equation (9) into (7) & (8) we get the following set of equations.

$$\frac{\partial^2 T_0}{\partial z^2}+\Pr\frac{\partial T_0}{\partial z}-\alpha_0 T_0=0 \tag{11}$$

$$\frac{\partial^2 T_1}{\partial z^2}+\Pr\frac{\partial T_1}{\partial z}-\left(i\omega p_r+\alpha_0\right)T_1=-\Pr\frac{\partial T_0}{\partial z} \tag{12}$$

$$\frac{\partial^2 q_0}{\partial z^2}+\frac{\partial q_0}{\partial z}-\left(\frac{1}{k_0}+M^2+2iE\right)q_0=-GrT_0 \tag{13}$$

$$\frac{\partial^2 q_1}{\partial z^2}+\frac{\partial q_1}{\partial z}-\left(\frac{1}{k_0}+M^2-i\left(2E+\omega\right)\right)q_1=-\frac{q_0\left(z\right)}{k_0}-\frac{\partial q_0}{\partial z}-G_r\ T_1 \tag{14}$$

The transformed boundary condition (9) is:

$$\begin{array}{lll} q_0=q_1=0,\quad T_0=1,T_1=1, & \text{at} & z=0 \\ q_0=q_1\to 0,\quad T_0=T_1\to 0 & \text{as} & z\to\alpha \end{array} \tag{15}$$

The solutions (5) to (8) under the transformed boundary conditions (15) using (10) are.

$$q(z,t) = -L_4 e^{-K_5 z} + L_4 e^{-K_1 z} + \varepsilon\left[-\left(p_1 + p_2 + p_3\right)_e^{-k_7 z} + p_1 e^{-k_5 z} + p_2 e^{-k_3 z} + p_3 e^{-k_1 z}\right] e^{i\omega t} \tag{16}$$

$$T(z,t) = e^{-k_1 z} + \varepsilon\left[\left(1 - L_2\right) e^{-k_3 z} + L_2 e^{-k_1 z}\right] \tag{17}$$

The steady parts of the primary and secondary velocity are:

$$q_{0r} = -e^{X_4 z}\left[X_2 \cos Y_4 z - Y_2 \sin Y_4 z\right] + X_2 e^{-K_1 z} \tag{18}$$

$$q_{0i} = -e^{X_4 z}\left[X_2 \sin Y_4 z - Y_2 \cos Y_4 z\right] + Y_2 e^{-K_1 z} \tag{19}$$

The unsteady parts of i.e., time dependent parts of the primary and secondary velocity are:

$$q_{1r} = e^{-X_9 z}\left[X_{10} \cos Y_{10} + Y_{10} \sin Y_9\right] + e^{-X_4 z}\left[X_5 \cos Y_4 z + Y_5 \sin Y_4 z\right] + e^{-X_1 z}\left[X_6 \cos Y_1 z + Y_6 \sin Y_1 z\right] + X_7 e^{-k_1 z} \tag{20}$$

$$q_{1i} = e^{-X_9 z}\left[Y_{10} \cos Y_9 z - X_{10} \sin Y_9 z\right] + e^{-X_4 z}\left[Y_5 \cos Y_4 z - X_5 \sin Y_4 z\right] + e^{-X_1 z}\left[Y_6 \cos Y_1 z - X_6 \sin Y_1 z\right] + Y_7 e^{-k_1 z} \tag{21}$$

Hence, the primary velocity, secondary velocity and temperature field are:

$$u(z,t) = q_0 r + \varepsilon\left[q_1 r \cos \omega t - q_{1i} \sin \omega t\right] \tag{22}$$

$$v(z,t) = q_0 i + \varepsilon\left[q_1 r \sin \omega t + q_{1i} \cos \omega t\right] \tag{23}$$

$$T(z,t) = e^{-k_1 z} + \varepsilon\left[T_{ir} \cos \omega t - T_{1i} \sin \omega t\right] \tag{24}$$

Where $T_{1r} = e^{-X_1 z}\left(\left(1 - X_0\right)\cos Y_1 z - Y_0 \sin Y_1 z\right) + X_0 e^{-k_1 z}$

$$T_{1i} = e^{-k_1 z}\left(-\left(1 - X_0\right)\sin Y_1 z - Y_0 \cos Y_1 z\right) + Y_0 e^{k_1 z}$$

The skin – friction (τ_s) due to primary velocity and skin friction (τ_s) due to secondary velocity at the plate are obtained as follows:

$$\tau_\rho = \left(\frac{\partial u}{\partial z}\right)_{z=0} = S_1 + \left[\varepsilon\, S_2 \cos \omega t + S_3 \sin \omega t\right] \tag{25}$$

$$\tau_s = \left(\frac{\partial v}{\partial z}\right)_{z=0} = S_4 + \varepsilon\left[\, S_2 \sin \omega t - S_3 \cos \omega t\right] \tag{26}$$

The rate of heat transfer in terms of nusselt number (N_u) at the plate is

$$Nu = \left(\frac{\partial T}{\partial z}\right)_{z=0} = -K_1 + \varepsilon\left[S_5 \cos \omega t - S_6 \sin \omega t\right] \tag{27}$$

Tables:

Table No. 1: Values of the skin fiction due to primary and secondary velocity for Gr=10.0, E=1.0

α_0	k_0	**M**	$\tau = \sqrt{\tau_\rho^2 + \tau_s^2}$
1.0	10.0	2	5.42826
1.5	10.0	2	4.96832
2.0	10.0	2	4.42601
2.0	15.0	2	4.41511
2.0	20.0	2	4.41143
1.5	10.0	1.5	4.10361
1.5	10.0	2.5	12.71083

Table No. 2: Values of the skin fiction due to primary and secondary velocity for fixed values α_0 =1.5, k_0 =10, M=2

r	E	$\tau=\sqrt{\tau_\rho^2+\tau_s^2}$
10.0	1	5.42826
10.0	2	10.79800
10.0	3	24.67648
25.0	2	26.99874
20.0	2	21.59848

Table No. 3: Values of Heat Transfer in terms of Nusselt Number N

r	α_0	Nu
0.71	1.5	-2.0370
0.71	1.0	-1.7848
7.00	1.0	-9.8840
11.4	1.0	-15.9556
0.71	0	-0.9807

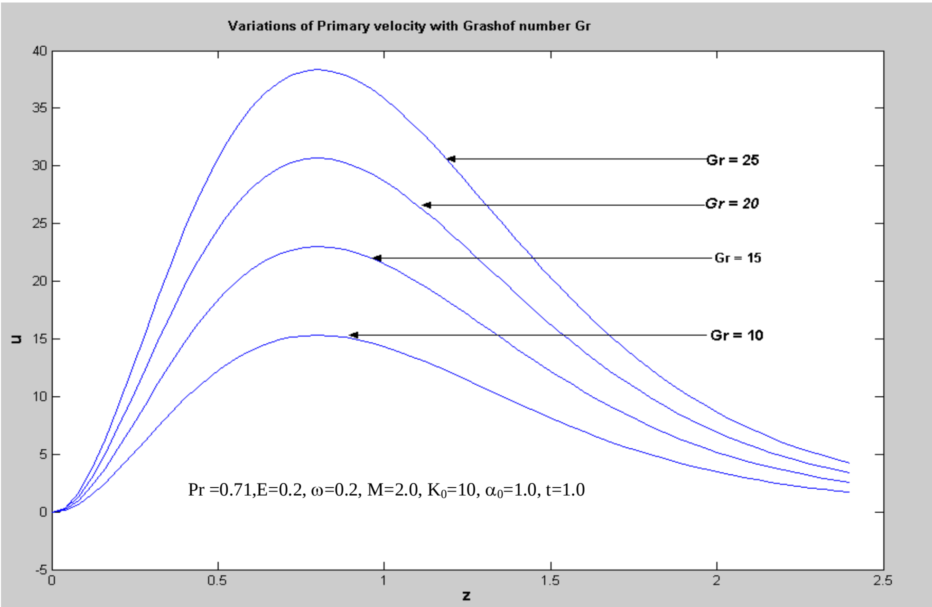


Figure 1

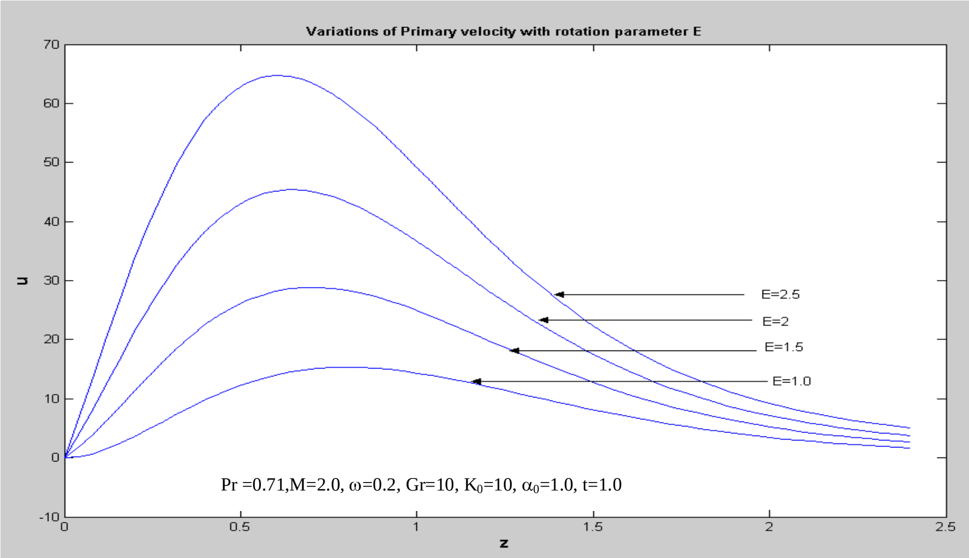


Figure 2

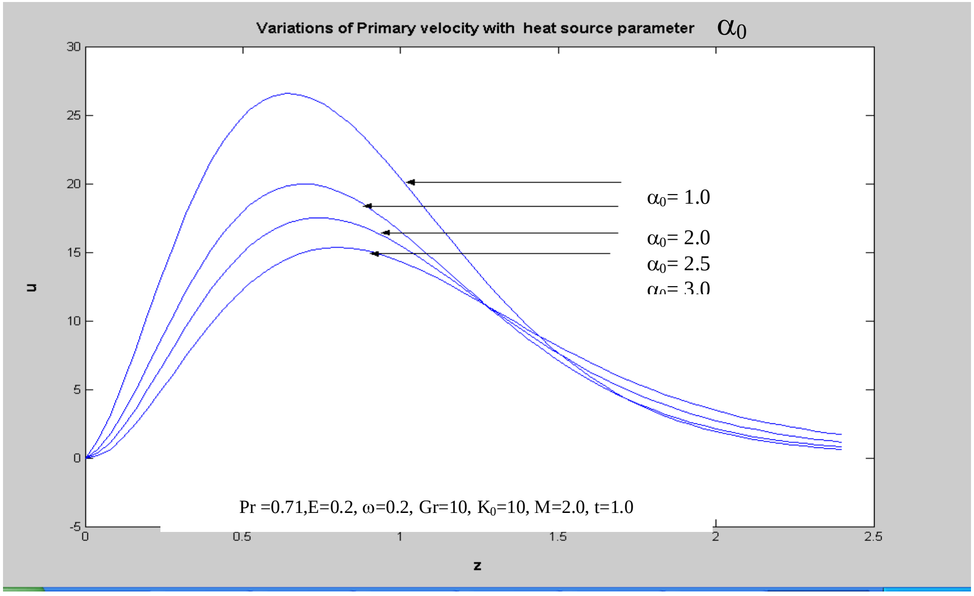


Figure-3

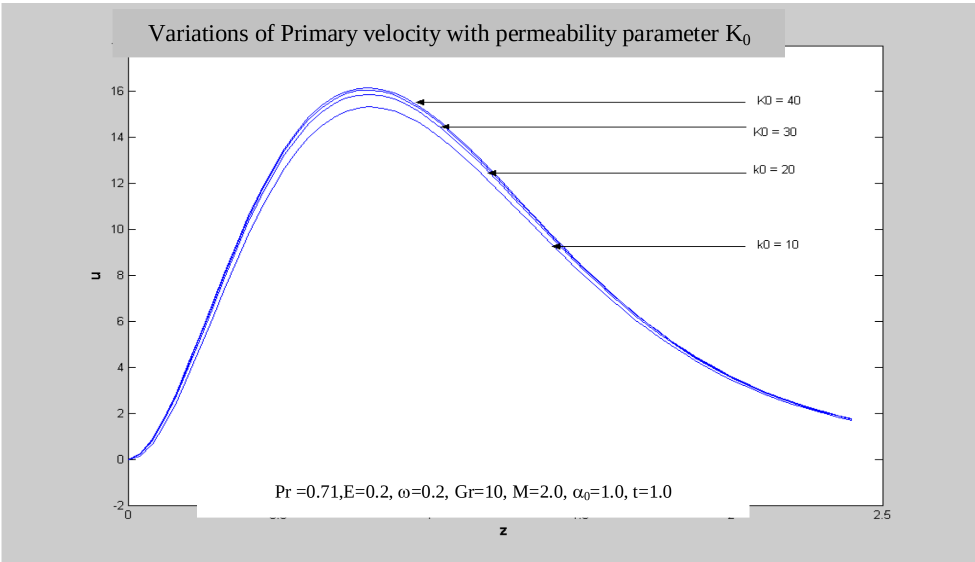
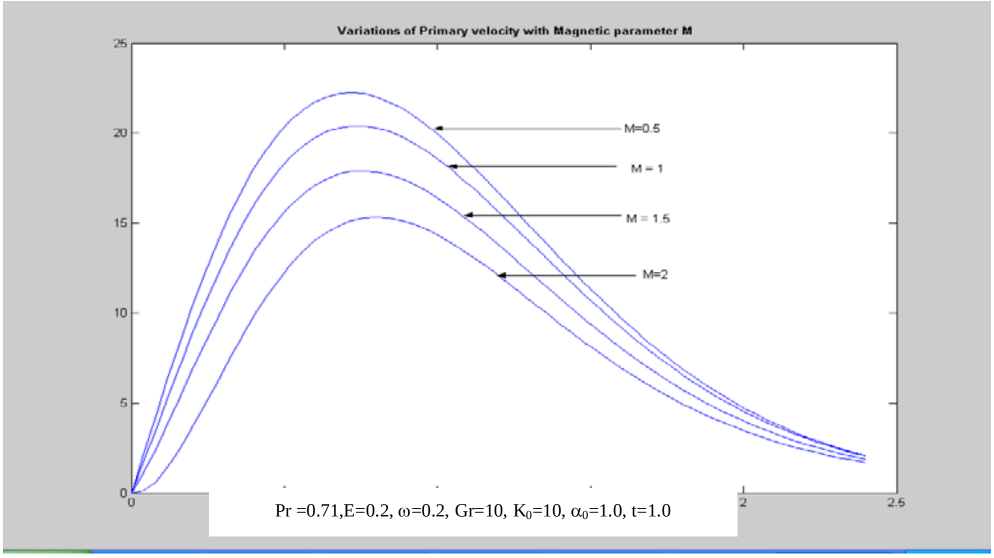


Figure -4



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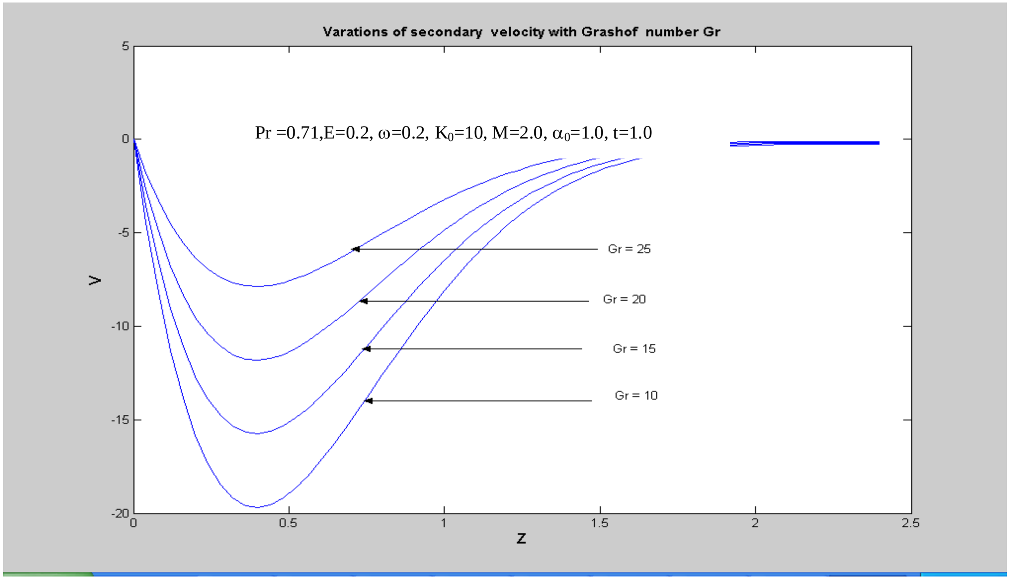

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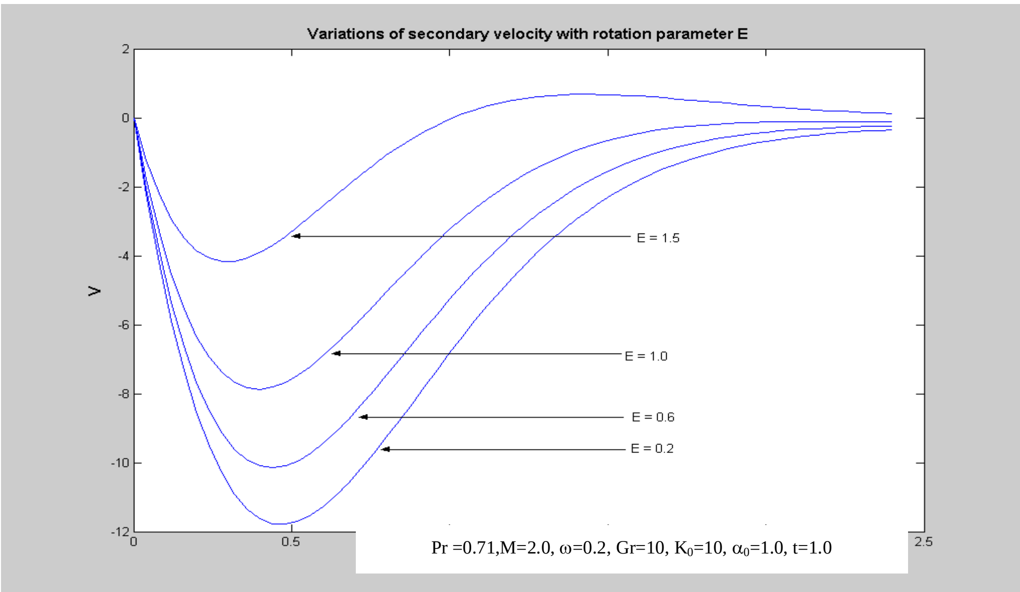
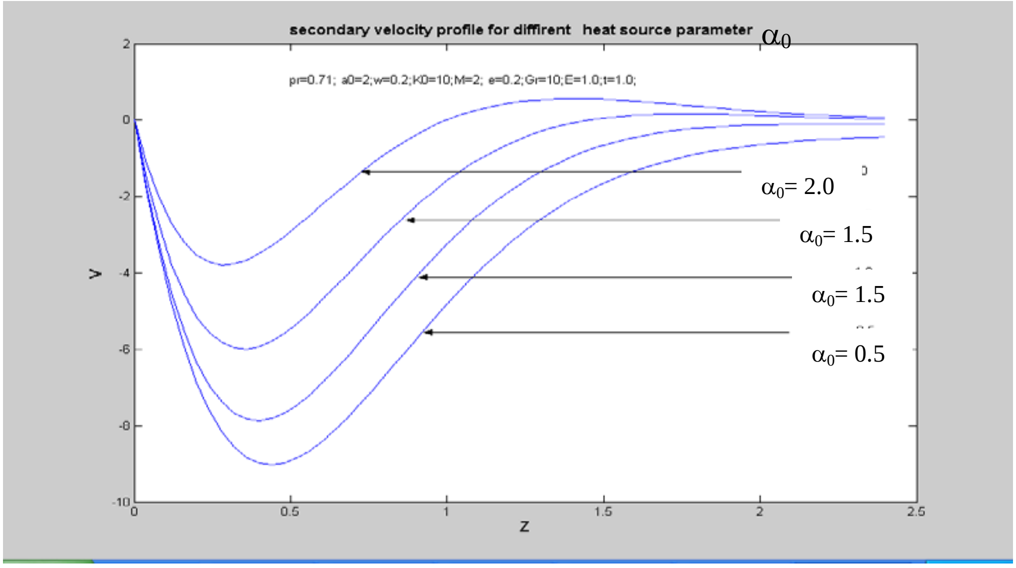


Figure -7



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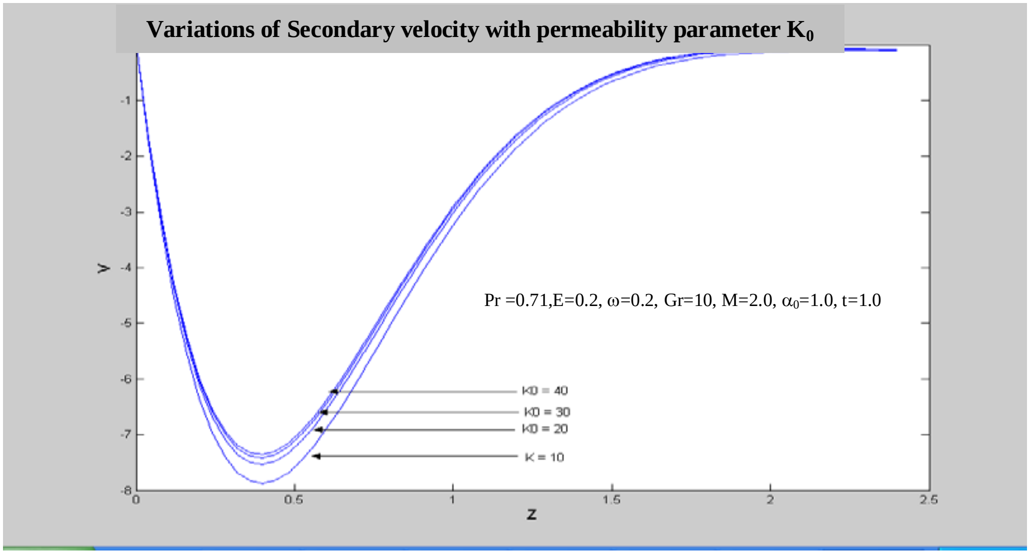


Figure-9

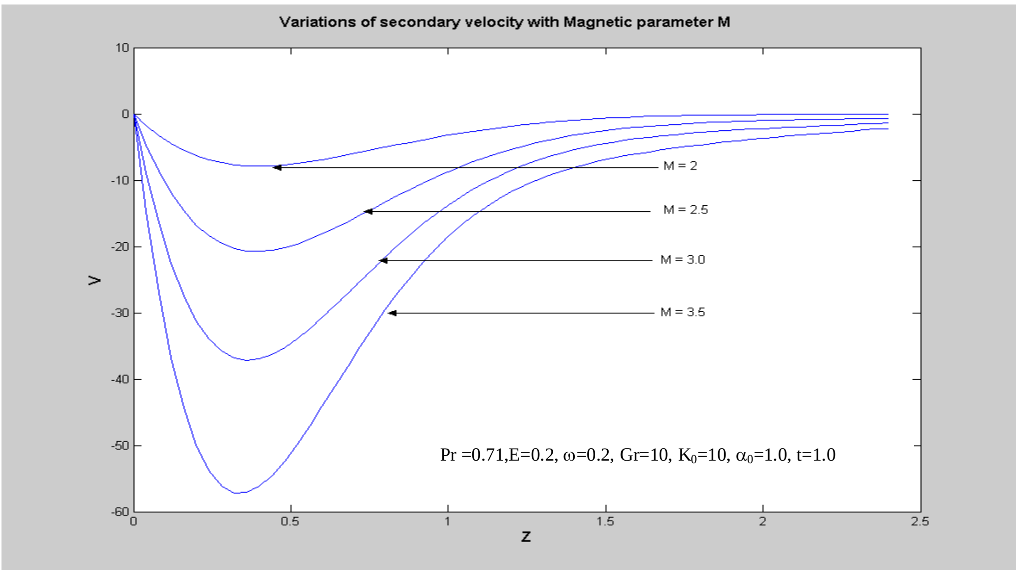


Figure -10

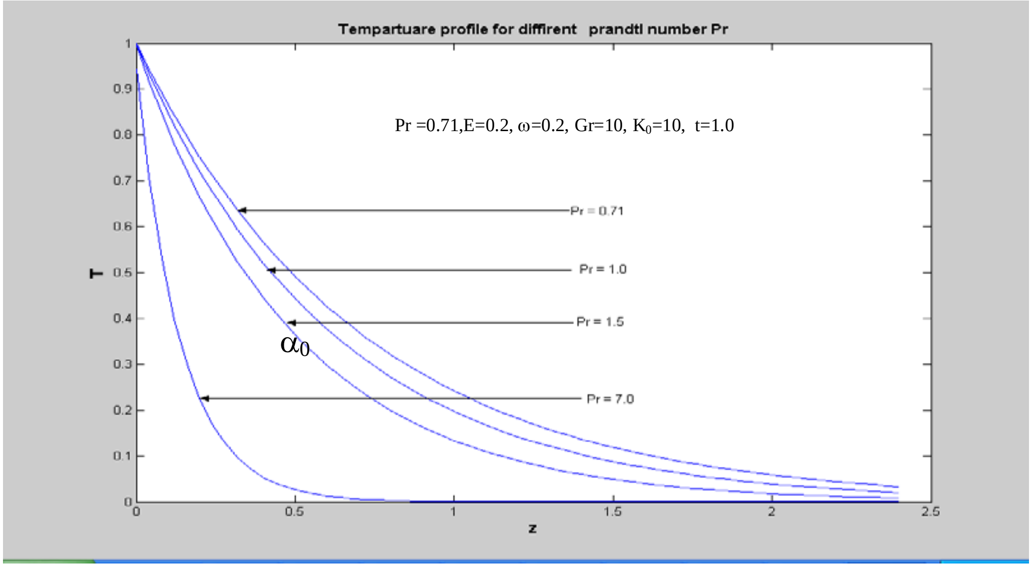
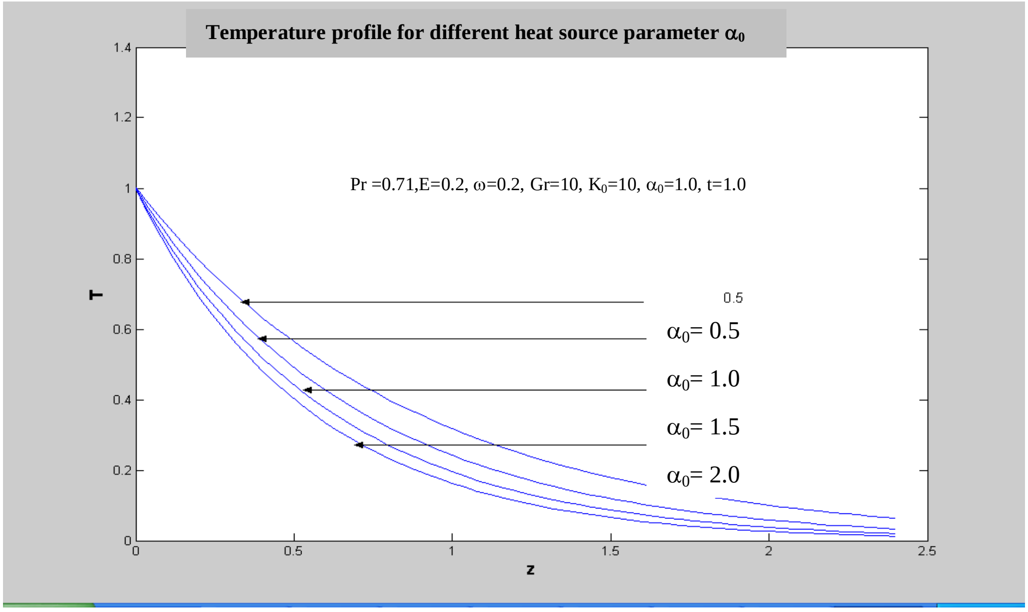


Figure -11



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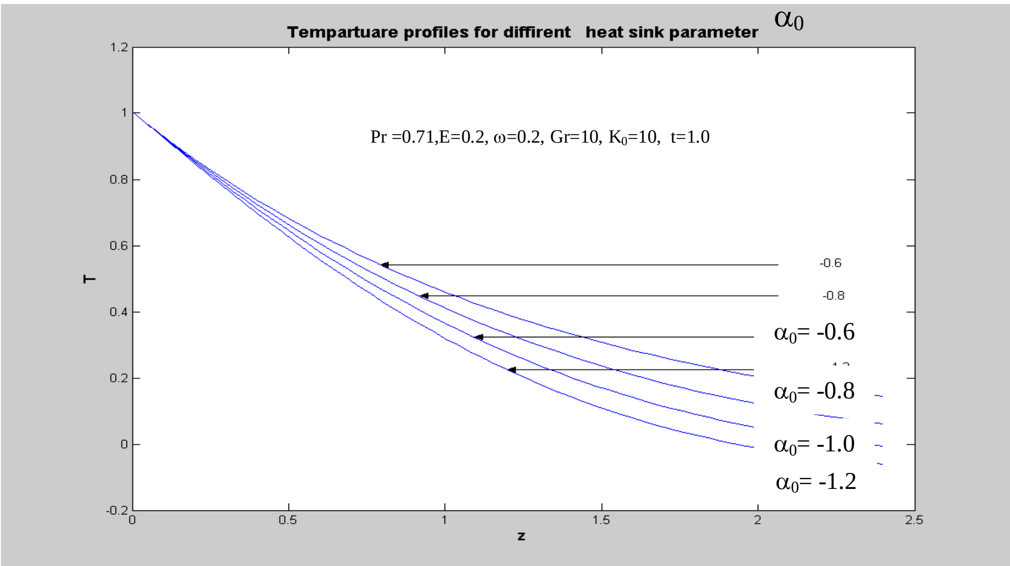

Figure -13

RESULTS AND DISCUSSIONS:

In order to get physical insight to the problem, the numerical calculations carried out for velocity and temperature fields against z for different parameters such as the Grashof number (G_r), the heat source / sink parameter (α_0), The prandtl number P_r, the permeability parameter K_o, The rotation parameter (E) and the magnetic parameter M.

In figure (1), the behaviour of primary velocity is studied for different values of Gr. It is observed that the primary velocity u increases as the Grashoft number G_r increases. From figure (2), it is noticed that, the primary velocity u increases as with increasing the rotation parameter E. From figure (3), we noticed that as the heat source parameter α_0 increased the primary velocity u increases in the region 0<z<1.25 and decreasing for z > 1.75 (nearly), from figure (4), we observed that the primary velocity U increases with increasing the permeability parameter K_o. From figure (5), It is observed that the primary velocity decreasing with increasing the magnetic parameter M. For each M, the primary velocity is gradually increases attains maximum value and then decreasing rapidly. The maximum velocity is slightly shifted to the right (in the +ve direction of z) with increasing M.

In figure (6), behaviour of secondary velocity is studied for different values of G_r. It is seen that the secondary velocity v increases with increasing Gras off number G_r. From figure (7), it is noticed that, the secondary velocity v decreases with increasing the rotation parameter E. In the back flow can be reduced by increasing E. From figure (8), It is observed that the secondary velocity v decreases as the heat source / sink parameter α_o increases. From figure (9), we observed that secondary velocity v decreasing as the permeability parameter K_o. Increasing. From figure (10), the secondary velocity V deceasing with increasing magnetic parameter M. For each M, the secondary velocity decreases attains the minimum value and minimum the increasing gradually. The minimum value is shifted towards the right with increasing M.

From figure (11), the behaviour of temperature is studied for different values of P_r. it is noticed that temperature decreases with increasing prandtl number P_r. From figure (12), it is observed that the temperature decreases as the heat / source parameter α_o increases. From figure (13), It is seen that temperature increases as the heat / sink parameter $|\alpha_o|$ increases.

From table 1 & 2, it is notice that $\boldsymbol{\alpha_0}$ increases as Gr or E or M increases. While it decreases has K_0 or $\boldsymbol{\alpha_0}$ increases.

From table 3 it is observed that the $|Nu|$ increases as Pr or α_0

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