# Available online through www.ijma.info ISSN 2229-5046 

# SOME FIXED POINT THEOREMS IN TWO FUZZY METRIC SPACES 

T. Veerapandi<br>Associate Professor of Mathematics, P. M. T College, Melaneelithanallur-627953, India<br>T. Thiripura Sundari*<br>Department of Mathematics, Sri K. G. S Arts College, Srivaikuntam, India

J. Paulraj Joseph<br>Associate Professor of Mathematics, Manonmaniam Sundaranar University, Tirunelveli, India

(Received on: 19-07-12; Revised \& Accepted on: 11-08-12)


#### Abstract

In this paper we prove some fixed point theorems for generalized contraction mappings in two complete fuzzy metric spaces.


Key words and Phrases: fixed point, common fixed point and complete fuzzy metric space.
AMS Mathematics Subject Classification: 47H10, 54H25.

## 1. INTRODUCTION.

Fuzzy set was defined by Zadeh [9] in 1965, has lead to a rich growth of fuzzy mathematics. Kramosil and Michalek [7] introduced fuzzy metric space, George and Veeramani [5] modified the notion of fuzzy metric spaces with the help of continuous t-norms. Many authors namely Deng [2], Erceg [3] used the concept of fuzzy mathematics in different ways. Recently there are several authors proved many kinds of fixed point theorems in fuzzy metric spaces viz. [4] In this paper we prove some fixed point theorems in two complete fuzzy metric spaces for contractive type mappings and non-expansive mappings by generalizing the results of Veerapandi et al [8] on fuzzy metric space. Now we begin with some known definitions and preliminary concepts.

Definition1.2: A fuzzy set A in X is a function with domain X and values in $[0,1]$.

Definition1.3 [10]: A binary operation $*:[0,1] \times[0,1] \rightarrow[0,1]$ is a continuous $t$-norm if it satisfies the following conditions:
(i) * is commutative and associative,
(ii) ${ }^{*}$ is continuous,
(iii) $\mathrm{a} * 1=\mathrm{a}$ for all $\mathrm{a} \in[0,1]$,
(iv) $\mathrm{a} * \mathrm{~b} \leq \mathrm{c} * \mathrm{~d}$ whenever $\mathrm{a} \leq \mathrm{c}$ and $\mathrm{b} \leq \mathrm{d}$, for each $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d} \in[0,1]$.

Examples of t -norm are $\mathrm{a} * \mathrm{~b}=\mathrm{ab}$ and $\mathrm{a} * \mathrm{~b}=\min (\mathrm{a}, \mathrm{b})$.
Definition1.4 [7]: A 3-tuple ( $\mathrm{X}, \mathrm{M}$, *) is said to be a fuzzy metric space if X is an arbitrary set, * is a continuous $t$-norm and $M$ is a fuzzy set on $X^{2} \times[0, \infty)$ satisfying the following conditions, for all $x, y, z \in X$ and $s, t>0$,
(f-1) $\quad M(x, y, t)>0$,
(f-2) $\quad M(x, y, t)=1$ if and only if $x=y$,
(f-3) $\quad M(x, y, t)=M(y, x, t)$,
(f-4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t+s)$,
(f-5) $\mathrm{M}(\mathrm{x}, \mathrm{y},):.[0, \infty) \rightarrow[0,1]$ is left continuous $\forall \mathrm{x}, \mathrm{y}, \mathrm{z} \in \mathrm{X}$ and $\mathrm{t}, \mathrm{s}>0$.
Then M is called a fuzzy metric on X and $\mathrm{M}(\mathrm{x}, \mathrm{y}, \mathrm{t})$ denotes the degree of nearness between x and y with respect to t .
Example1.5 [5]: (Induced fuzzy metric): Let ( $\mathrm{X}, \mathrm{d}$ ) be a metric space, define $\mathrm{a} * \mathrm{~b}=\min \{\mathrm{a}, \mathrm{b}\}$ for all $\mathrm{a}, \mathrm{b} \in[0,1]$ and $\mathrm{M}_{\mathrm{d}}$ be fuzzy set on $\mathrm{X}^{2} \times[0, \infty)$ defined as,

Corresponding author: T. Thiripura Sundari*,
Department of Mathematics, Sri K. G. S Arts College, Srivaikuntam, India

## T. Veerapandi, T. Thiripura Sundari* and J. Paulraj Joseph/ SOME FIXED POINT THEOREMS IN TWO FUZZY METRIC SPACES/

 IJMA- 3(8), August-2012.$$
\mathrm{M}_{\mathrm{d}}(\mathrm{x}, \mathrm{y}, \mathrm{t})=\frac{t}{t+d(x, y)}
$$

for all $\mathrm{x}, \mathrm{y} \in \mathrm{X}$ and $\mathrm{t}>0$. Then $\left(\mathrm{X}, \mathrm{M}_{\mathrm{d}}\right.$, *) is a fuzzy metric space. We call this fuzzy metric $\mathrm{M}_{\mathrm{d}}$ as the standard intuitionistic fuzzy metric.

Definition 1.6[6]: Let (X, M, *) be a fuzzy metric space. Then
(i) A sequence $\left\{\mathrm{x}_{\mathrm{n}}\right\}$ in X is said to be convergent to a point $\mathrm{x} \in \mathrm{X}$, if
$\lim _{n \rightarrow \infty} \mathrm{M}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}, \mathrm{t}\right)=1$ for all $\mathrm{t}>0$.
(ii) A sequence $\left\{x_{n}\right\}$ in $X$ is said to be a Cauchy sequence if

$$
\lim _{n \rightarrow \infty} \mathrm{M}\left(\mathrm{x}_{\mathrm{n}+\mathrm{p}}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right)=1 \text { for all } \mathrm{t}>0 \text { and } \mathrm{p}>0
$$

(iii) A fuzzy metric space is said to be complete if every Cauchy sequence is convergent to a point in it.

Remark 1.7: Since * is continuous, it follows from ( $\mathrm{f}-4$ ), that the limit of the sequence in fuzzy metric space is uniquely determined.

Let ( $\mathrm{X}, \mathrm{M}$, *) be a fuzzy metric space with the following conditions.
(f-6) $\lim _{t \rightarrow \infty} \mathrm{M}(\mathrm{x}, \mathrm{y}, \mathrm{t})=1$ for all $\mathrm{x}, \mathrm{y} \in \mathrm{X}$
Lemma 1.8[6]: For all $x, y \in X, M(x, y,$.$) is non-decreasing.$
Lemma 1.9[1]: Let $\left\{\mathrm{x}_{\mathrm{n}}\right\}$ be a sequence in a fuzzy metric space ( $\mathrm{X}, \mathrm{M}, *$ *) with the condition ( $\mathrm{f}-6$ ). If there exists a number $\mathrm{q} \in(0,1)$ such that

$$
\mathrm{M}\left(\mathrm{x}_{\mathrm{n}+2}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{qt}\right) \geq \mathrm{M}\left(\mathrm{x}_{\mathrm{n}+1}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right)
$$

for all $\mathrm{t}>0$ and $\mathrm{n}=1,2, \ldots \ldots$, then $\left\{\mathrm{x}_{\mathrm{n}}\right\}$ is a cauchy sequence in X .
Lemma 1.10: If for all $x, y \in X, t>0$ and for a number $q \in(0,1)$,

$$
\mathrm{M}(\mathrm{x}, \mathrm{y}, \mathrm{qt}) \geq \mathrm{M}(\mathrm{x}, \mathrm{y}, \mathrm{t}) \text {, then } \mathrm{x}=\mathrm{y} .
$$

## 2. MAIN RESULTS

Theorem2.1. Let ( $\mathrm{X}, \mathrm{M}_{1}$, *) and ( $\mathrm{Y}, \mathrm{M}_{2}$, * ) be two complete fuzzy metric spaces. If T is a mapping from X into Y and $S$ is a mapping from $Y$ into $X$, satisfying the following conditions:

$$
\begin{align*}
& M_{2}(T x, T S y, q t) \geq \min \left\{M_{1}(x, S y, t), M_{2}(y, T x, t) * M_{2}(y, T S y, t)\right\}  \tag{1}\\
& M_{1}(S y, S T x, q t) \geq \min \left\{M_{1}(x, S y, t) * M_{1}(x, S T x, t), M_{2}(y, T x, t)\right\} \tag{2}
\end{align*}
$$

for all x in X and y in Y where $\mathrm{q}<1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y . Further $\mathrm{Tz}=\mathrm{w}$ and $\mathrm{Sw}=\mathrm{z}$.

Proof: Let $x_{0}$ be an arbitrary point in $X$. Define a sequence $\left\{x_{n}\right\}$ in $X$ and $\left\{y_{n}\right\}$ in $Y$, as follows:

$$
\mathrm{x}_{\mathrm{n}}=(\mathrm{ST})^{\mathrm{n}} \mathrm{x}_{0}, \mathrm{y}_{\mathrm{n}}=\mathrm{T}\left(\mathrm{x}_{\mathrm{n}-1}\right) \text { for } \mathrm{n}=1,2, \ldots
$$

We have

$$
\begin{aligned}
& \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{qt}\right)=\mathrm{M}_{1}\left((\mathrm{ST})^{\mathrm{n}} \mathrm{x}_{0},(\mathrm{ST})^{\mathrm{n}+1} \mathrm{x}_{0}, \mathrm{qt}\right) \\
& =M_{1}\left(S(T(S T))^{n-1} x_{0}, S T(S T)^{n} x_{0}, q t\right) \\
& =\mathrm{M}_{1}\left(\mathrm{ST}\left(\mathrm{x}_{\mathrm{n}-1}\right), \mathrm{STx}_{\mathrm{n}}, \mathrm{qt}\right) \\
& =\mathrm{M}_{1}\left(\mathrm{Sy}_{\mathrm{n}}, \mathrm{STx} \mathrm{n}, \mathrm{qt}\right) \\
& \geq \min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{Sy}_{\mathrm{n}}, \mathrm{t}\right) * \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{STx}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{Tx}_{\mathrm{n}}, \mathrm{t}\right)\right\} \text { (since by (2)) } \\
& =\min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right) * \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right)\right\} \\
& =\min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right)\right\} \\
& \geq M_{2}\left(y_{n}, y_{n+1}, t\right)
\end{aligned}
$$

## T. Veerapandi, T. Thiripura Sundari* and J. Paulraj Joseph/ SOME FIXED POINT THEOREMS IN TWO FUZZY METRIC SPACES/

Now

$$
\begin{aligned}
\mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right) & =\mathrm{M}_{2}\left(\mathrm{Tx}_{\mathrm{n}-1}, \mathrm{Tx}_{\mathrm{n}}, \mathrm{t}\right) \\
& =\mathrm{M}_{2}\left(\mathrm{Tx}_{\mathrm{n}-1}, T \mathrm{Sy}_{\mathrm{n}}, \mathrm{t}\right) \\
& \geq \min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, S \mathrm{y}_{\mathrm{n}}, \mathrm{t} / \mathrm{q}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{Tx}_{\mathrm{n}-1}, \mathrm{t} / \mathrm{q}\right) * \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, T \mathrm{TS}_{\mathrm{n}}, \mathrm{t} / \mathrm{q}\right)\right\} \quad \text { (since by (1)) } \\
& =\min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{x}_{\mathrm{n}}, \mathrm{t} / \mathrm{q}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}}, \mathrm{t} / \mathrm{q}\right) * \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t} / \mathrm{q}\right)\right\} \\
& \geq \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{x}_{\mathrm{n}}, \mathrm{t} \mathrm{q}\right)
\end{aligned}
$$

Hence

$$
\begin{aligned}
\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{qt}\right) & \geq \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right) \\
& \geq \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{x}_{\mathrm{n}}, \mathrm{t} / \mathrm{q}\right) \\
& \vdots \\
& \geq \mathrm{M}_{1}\left(\mathrm{x}_{0}, \mathrm{x}_{1}, \mathrm{t} / \mathrm{q}^{2 \mathrm{n}-1}\right) \rightarrow 1 \text { as } \mathrm{n} \rightarrow \infty(\text { since } \mathrm{q}<1)
\end{aligned}
$$

Thus $\left\{x_{n}\right\}$ is a Cauchy sequence in $X$. Since ( $X, M_{1}$, *) is complete, it converges to a point $z$ in $X$. Similarly, we can prove that the sequence $\left\{\mathrm{y}_{\mathrm{n}}\right\}$ is also a Cauchy sequence in Y and it converges to a point w in Y .

Now we prove $\mathrm{Tz}=\mathrm{w}$
Suppose Tz $\neq \mathrm{w}$.
We have

$$
\begin{aligned}
\mathrm{M}_{2}(\mathrm{Tz}, \mathrm{w}, \mathrm{qt}) & =\lim _{n \rightarrow \infty} \mathrm{M}_{2}\left(\mathrm{Tz}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{qt}\right) \\
& =\lim _{n \rightarrow \infty} \mathrm{M}_{2}\left(\operatorname{Tz}, \mathrm{TS} \mathrm{y}_{\mathrm{n}}, \mathrm{qt}\right) \\
& \geq \lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{1}\left(\mathrm{z}, \mathrm{Sy}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{Tz}, \mathrm{t}\right) * \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, T S \mathrm{y}_{\mathrm{n}}, \mathrm{t}\right)\right\} \\
& =\lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{1}\left(\mathrm{z}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{Tz}, \mathrm{t}\right) * \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right)\right\} \\
& =\min \left\{\mathrm{M}_{1}(\mathrm{z}, \mathrm{z}, \mathrm{t}), \mathrm{M}_{2}(\mathrm{w}, \mathrm{Tz}, \mathrm{t}) * \mathrm{M}_{2}(\mathrm{w}, \mathrm{w}, \mathrm{t})\right\} \\
& =\min \left\{1, \mathrm{M}_{2}(\mathrm{w}, \mathrm{Tz}, \mathrm{t}) * 1\right\} \\
& \geq \mathrm{M}_{2}(\mathrm{Tz}, \mathrm{w}, \mathrm{t})(\text { since } \mathrm{q}<1), \text { which is a contradiction. }
\end{aligned}
$$

Thus $\mathrm{Tz}=\mathrm{w}$.
Now we prove $\mathrm{Sw}=\mathrm{z}$.
Suppose Sw $\neq \mathrm{z}$.
We have

$$
\begin{aligned}
\mathrm{M}_{1}(\mathrm{Sw}, \mathrm{z}, \mathrm{qt}) & =\lim _{n \rightarrow \infty} \mathrm{M}_{1}\left(\mathrm{Sw}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{qt}\right) \\
& =\lim _{n \rightarrow \infty} \mathrm{M}_{1}\left(\mathrm{Sw}, \mathrm{STx}_{\mathrm{n}}, \mathrm{qt}\right) \\
& \geq \lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{Sw}, \mathrm{t}\right) * \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{STx}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{w}, \mathrm{Tx}_{\mathrm{n}}, \mathrm{t}\right)\right\} \\
& =\lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{Sw}, \mathrm{t}\right) * \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{w}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right)\right\} \\
& \left.=\min ^{2} \mathrm{M}_{1}(\mathrm{z}, \text { Sw}, \mathrm{t}) * 1,1\right\} \\
& \geq \mathrm{M}_{1}(\mathrm{z}, \mathrm{Sw}, \mathrm{t}) \quad(\text { since } \mathrm{q}<1), \text { which is a contradiction. }
\end{aligned}
$$

Thus Sw = z.
Therefore we have $\mathrm{STz}=\mathrm{Sw}=\mathrm{z}$ and $\mathrm{TS} \mathrm{w}=\mathrm{Tz}=\mathrm{w}$. Thus the point z is a fixed point of ST and the point w is a fixed point of TS.

Uniqueness: let $\mathrm{z}^{\prime}$ be another fixed point of ST such that $\mathrm{z}=\mathrm{z}^{\prime}$.
We have

$$
\begin{aligned}
\mathrm{M}_{1}\left(\mathrm{z}, \mathrm{z}^{\prime}, \mathrm{qt}\right) & =\mathrm{M}_{1}\left(\mathrm{STz}, \mathrm{STz} \mathrm{~S}^{\prime} \mathrm{qt}\right) \\
& \geq \min \left\{\mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{STz}, \mathrm{t}\right) * \mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{STz}^{\prime}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{Tz}, \mathrm{Tz}^{\prime}, \mathrm{t}\right)\right\} \\
& =\min \left\{\mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{z}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{Tz}, \mathrm{Tz}^{\prime}, \mathrm{t}\right)\right\} \\
& \geq \mathrm{M}_{2}\left(\mathrm{Tz}, \mathrm{Tz}^{\prime}, \mathrm{t}\right)
\end{aligned}
$$

Also we have

$$
\begin{aligned}
\mathrm{M}_{2}\left(\mathrm{Tz}, \mathrm{Tz}^{\prime}, \mathrm{t}\right) & =\mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{TSTz}^{\prime}, \mathrm{t}\right) \\
& \geq \min \left\{\mathrm{M}_{1}\left(\mathrm{z}, \mathrm{STz}^{\prime}, \mathrm{t} / \mathrm{q}\right), \mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{Tz}, \mathrm{t} / \mathrm{q}\right) * \mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{TSTz}^{\prime}, \mathrm{t} / \mathrm{q}\right)\right\}
\end{aligned}
$$

## T. Veerapandi, T. Thiripura Sundari* and J. Paulraj Joseph/ SOME FIXED POINT THEOREMS IN TWO FUZZY METRIC SPACES/

 IJMA- 3(8), August-2012.$$
\begin{aligned}
& =\min \left\{\mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{z}, \mathrm{t} / \mathrm{q}\right), \mathrm{M}_{2}\left(\mathrm{Tz}, \mathrm{Tz}^{\prime}, \mathrm{t} / \mathrm{q}\right)\right\} \\
& \geq \mathrm{M}_{1}\left(\mathrm{z}, \mathrm{z}^{\prime}, \mathrm{t} / \mathrm{q}\right)
\end{aligned}
$$

Hence

$$
\left.\mathrm{M}_{1}\left(\mathrm{z}, \mathrm{z}^{\prime}, \mathrm{qt}\right) \geq \mathrm{M}_{2}\left(\mathrm{Tz}, \mathrm{Tz}^{\prime}, \mathrm{t}\right) \geq \mathrm{M}_{1}\left(\mathrm{z}, \mathrm{z}^{\prime}, \mathrm{t} / \mathrm{q}\right) \text { (since } \mathrm{q}<1\right) \text {, }
$$

which is a contradiction.
Thus $\mathrm{z}=\mathrm{z}^{\prime}$.
So the point z is the unique fixed point of ST. Similarly, we prove the point w is also a unique fixed point of TS.
This completes the proof.
Remark 2.2: If ( $\mathrm{X}, \mathrm{M}_{1}$, *) and ( $\mathrm{Y}, \mathrm{M}_{2}$, *) are the same fuzzy metric spaces, then by the above theorem 2.1, we get the following theorem, as corollary.
 the following conditions:

$$
\begin{aligned}
& M(T x, T S y, q t) \geq \min \{M(x, S y, t), M(y, T x, t) * M(y, T S y, t)\} \\
& M(S y, S T x, q t) \geq \min \{M(x, S y, t) * M(x, S T x, t), M(y, T x, t)\}
\end{aligned}
$$

for all x , y in X where $\mathrm{q}<1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in X . Further $\mathrm{Tz}=\mathrm{w}$ and $\mathrm{Sw}=\mathrm{z}$.

Theorem 2.4: Let ( $\mathrm{X}, \mathrm{M}_{1}$, *) and ( $\mathrm{Y}, \mathrm{M}_{2}$, *) be two complete fuzzy metric spaces. If T is a mapping from X into Y and S is a mapping from Y into X , satisfying following conditions:

$$
\begin{align*}
& M_{2}(T x, T S y, q t) \geq \min \left\{M_{1}(x, S y, t), M_{2}(y, T x, t), M_{2}(y, T x, t) * M_{2}(y, T S y, t)\right\}  \tag{1}\\
& M_{1}(S y, S T x, q t) \geq \min \left\{M_{2}(y, T x, t), M_{1}(x, S y, t), M_{1}(x, S y, t) * M_{1}(x, S T x, t)\right\} \tag{2}
\end{align*}
$$

for all x in X and y in Y where $\mathrm{q}<1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y . Further $\mathrm{Tz}=\mathrm{w}$ and $\mathrm{Sw}=\mathrm{z}$.

Proof: Let $x_{0}$ be an arbitrary point in $X$. Define a sequence $\left\{x_{n}\right\}$ in $X$ and $\left\{y_{n}\right\}$ in $Y$, as follows:

$$
\mathrm{x}_{\mathrm{n}}=(\mathrm{ST})^{\mathrm{n}} \mathrm{x}_{0}, \mathrm{y}_{\mathrm{n}}=\mathrm{T}\left(\mathrm{x}_{\mathrm{n}-1}\right) \text { for } \mathrm{n}=1,2, \ldots
$$

we have

$$
\begin{aligned}
\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{qt}\right) & =\mathrm{M}_{1}\left((\mathrm{ST})^{\mathrm{n}} \mathrm{x}_{0},(\mathrm{ST})^{\mathrm{n}+1} \mathrm{x}_{0}, \mathrm{qt}\right) \\
& =\mathrm{M}_{1}\left(\mathrm{~S}\left(\mathrm{~T}(\mathrm{ST})^{\mathrm{n}-1} \mathrm{x}_{0}, \mathrm{ST}(\mathrm{ST})^{\mathrm{n}} \mathrm{x}_{0}, \mathrm{qt}\right)\right. \\
& =\mathrm{M}_{1}\left(\mathrm{ST}\left(\mathrm{x}_{\mathrm{n}-1}\right), \mathrm{STx} x_{\mathrm{n}}, \mathrm{qt}\right) \\
& =\mathrm{M}_{1}\left(\mathrm{Sy}_{\mathrm{n}}, \operatorname{STx_{n}}, \mathrm{qt}\right) \\
& \geq \min \left\{\mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, T x_{\mathrm{n}}, t\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{Sy}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, S y_{n}, \mathrm{t}\right) * \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{STx}_{\mathrm{n}}, \mathrm{t}\right)\right\} \\
& =\min \left\{\mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right) * \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{t}\right)\right\} \\
& =\min \left\{\mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right), 1,1 * \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{t}\right)\right\} \\
& \geq \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right)
\end{aligned}
$$

Now

$$
\begin{aligned}
\mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{qt}\right) & =\mathrm{M}_{2}\left(\mathrm{Tx}_{\mathrm{n}-1}, \mathrm{Tx}_{\mathrm{n}}, \mathrm{qt}\right) \\
& =\mathrm{M}_{2}\left(\mathrm{Tx}_{\mathrm{n}-1}, T S \mathrm{y}_{n}, \mathrm{qt}\right) \\
& \geq \min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{Sy}_{\mathrm{n}}, t\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{Tx}_{\mathrm{n}-1}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, T \mathrm{Tx}_{\mathrm{n}-1}, \mathrm{t}\right) * \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, T S y_{n}, \mathrm{t}\right)\right\} \\
& =\min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{x}_{\mathrm{n}}, t\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}}, \mathrm{t}\right) * \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, t\right)\right\} \\
& =\min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{x}_{\mathrm{n}}, t\right), 1,1 * \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, t\right)\right\} \\
& =\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right)
\end{aligned}
$$

## T. Veerapandi, T. Thiripura Sundari* and J. Paulraj Joseph/ SOME FIXED POINT THEOREMS IN TWO FUZZY METRIC SPACES/

 IJMA- 3(8), August-2012.$$
\begin{aligned}
\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{t}\right) \geq & \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right) \\
& \geq \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{xn}, \mathrm{t} / \mathrm{q}\right) \\
& \vdots \\
& \geq \mathrm{M}_{1}\left(\mathrm{x}_{0}, \mathrm{x}_{1}, \mathrm{t} / \mathrm{q}^{2 \mathrm{n}-1}\right) \rightarrow 0 \text { as } \mathrm{n} \rightarrow \infty(\text { since } \mathrm{q}<1)
\end{aligned}
$$

Thus $\left\{x_{n}\right\}$ is a Cauchy sequence in $X$. Since ( $X, M_{1}$, *) is complete, it converges to a point $z$ in $X$. Similarly, we can prove that the sequence $\left\{\mathrm{y}_{\mathrm{n}}\right\}$ is also a Cauchy sequence in Y and it converges to a point w in Y .

Now we prove $\mathrm{Tz}=\mathrm{w}$
Suppose $\mathrm{Tz} \neq \mathrm{w}$.
We have

$$
\begin{aligned}
\mathrm{M}_{2}(\mathrm{Tz}, \mathrm{w}, \mathrm{qt}) & =\lim _{n \rightarrow \infty} \mathrm{M}_{2}\left(\mathrm{Tz}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{qt}\right) \\
& =\lim _{n \rightarrow \infty} \mathrm{M}_{2}\left(\mathrm{Tz}, \mathrm{TS} \mathrm{y}_{\mathrm{n}}, \mathrm{qt}\right) \\
& \geq \lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{1}\left(\mathrm{z}, \mathrm{Sy}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{Tz}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{Tz}, \mathrm{t}\right) * \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{TS} \mathrm{y}_{\mathrm{n}}, \mathrm{t}\right)\right\} \\
& =\lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{1}\left(\mathrm{z}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{Tz}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{Tz}, \mathrm{t}\right) * \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right)\right\} \\
& =\min \left\{\mathrm{M}_{1}(\mathrm{z}, \mathrm{z}, \mathrm{t}), \mathrm{M}_{2}(\mathrm{w}, \mathrm{Tz}, \mathrm{t}), \mathrm{M}_{2}(\mathrm{w}, \mathrm{Tz} . \mathrm{t}) * \mathrm{M}_{2}(\mathrm{w}, \mathrm{w}, \mathrm{t})\right\} \\
& =\min \left\{1, \mathrm{M}_{2}(\mathrm{w}, \mathrm{Tz}, \mathrm{t}), \mathrm{M}_{2}(\mathrm{w}, \mathrm{Tz}, \mathrm{t}) * 1\right\} \\
& \left.\geq \mathrm{M}_{2}(\mathrm{w}, \mathrm{Tz}, \mathrm{t}) \text { (since } \mathrm{q}<1\right) \text { which is a contradiction }
\end{aligned}
$$

Thus Tz = w .

Now we prove $\mathrm{Sw}=\mathrm{z}$.
Suppose Sw $\neq \mathrm{z}$.
We have

$$
\begin{aligned}
\mathrm{M}_{1}(\mathrm{Sw}, \mathrm{z}, \mathrm{qt}) & =\lim _{n \rightarrow \infty} \mathrm{M}_{1}\left(\mathrm{Sw}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{qt}\right) \\
& =\lim _{n \rightarrow \infty} \mathrm{M}_{1}\left(\mathrm{Sw}_{2}, \mathrm{STx}_{\mathrm{n}}, \mathrm{qt}\right) \\
& \geq \lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{2}\left(\mathrm{w}, \mathrm{Tx}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{Sw}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{Sw}, \mathrm{t}\right) \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{STx}_{\mathrm{n}}, \mathrm{t}\right)\right\} \\
& =\lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{2}\left(\mathrm{w}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{Sw}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{Sw}, \mathrm{t}\right) * \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{t}\right)\right\} \\
& =\min ^{2}\left\{\mathrm{M}_{2}(\mathrm{w}, \mathrm{w}, \mathrm{t}), \mathrm{M}_{1}(\mathrm{z}, \mathrm{w}, \mathrm{t}), \mathrm{M}_{1}(\mathrm{z}, \mathrm{Sw}, \mathrm{t}) * \mathrm{M}_{1}(\mathrm{z}, \mathrm{z}, \mathrm{t})\right\} \\
& \geq \mathrm{M}_{1}(\mathrm{Sw}, \mathrm{z}, \mathrm{t})(\text { since } \mathrm{q}<1) \text { which is a contradiction. }
\end{aligned}
$$

Thus $\mathrm{Sw}=\mathrm{z}$.
Therefore we have $\mathrm{STz}=\mathrm{Sw}=\mathrm{z}$ and $\mathrm{TSw}=\mathrm{Tz}=\mathrm{w}$. Thus the point z is a fixed point of ST and the point w is a fixed point of TS.

Uniqueness: let $\mathrm{z}^{\prime}$ be another fixed point of ST such that $\mathrm{z}=\mathrm{z}^{\prime}$.
We have

$$
\begin{aligned}
\mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{z}, \mathrm{qt}\right) & =\mathrm{M}_{1}\left(\mathrm{STz}^{\prime}, \mathrm{STz}, \mathrm{qt}\right) \\
& \geq \min \left\{\mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{Tz}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{z}, \mathrm{STz}^{\prime}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{z}, \mathrm{STz}^{\prime}, \mathrm{t}\right) * \mathrm{M}_{1}(\mathrm{z} . \mathrm{STz}, \mathrm{t})\right\} \\
& \geq \min \left\{\mathrm{M}_{2}\left(\mathrm{Tz} z^{\prime}, \mathrm{Tz}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{z}, \mathrm{z}^{\prime}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{z}, \mathrm{z}^{\prime}, \mathrm{t}\right)\right\} \\
& \geq \mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{Tz}, \mathrm{t}\right)
\end{aligned}
$$

Also we have

$$
\begin{aligned}
\mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{Tz}, \mathrm{qt}\right) & =\mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{TSTz}, \mathrm{q}\right) \\
& \geq \min \left\{\mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{STz}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{Tz}, \mathrm{Tz} z^{\prime}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{Tz}, \mathrm{Tz}^{\prime}, \mathrm{t}\right) * \mathrm{M}_{2}(\mathrm{Tz}, \mathrm{TSTz}, \mathrm{t})\right\} \\
& =\min \left\{\mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{z}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{Tz}, \mathrm{Tz}^{\prime}, \mathrm{t}\right), \mathrm{M}_{2}(\mathrm{Tz}, \mathrm{Tz}, \mathrm{t})\right\} \\
& \geq \mathrm{M}_{1}\left(\mathrm{z}, \mathrm{z}^{\prime}, \mathrm{t}\right)
\end{aligned}
$$

(i.e) $\mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{Tz}, \mathrm{t}\right) \geq \mathrm{M}_{1}\left(\mathrm{z}, \mathrm{z}^{\prime}, \mathrm{t} / \mathrm{q}\right)$

## T. Veerapandi, T. Thiripura Sundari* and J. Paulraj Joseph/ SOME FIXED POINT THEOREMS IN TWO FUZZY METRIC SPACES/

Hence
$\mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{z}, \mathrm{qt}\right) \geq \mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{Tz}, \mathrm{t}\right) \geq \mathrm{M}_{1}\left(\mathrm{z}, \mathrm{z}^{\prime}, \mathrm{t} / \mathrm{q}\right)$ (since $\left.\mathrm{q}<1\right)$ which is a contradiction.
Thus $\mathrm{z}=\mathrm{z}^{\prime}$.
So the point z is the unique fixed point of ST. Similarly, we prove the point w is also a unique fixed point of TS. This completes the proof.

Remark 2.5: If ( $\mathrm{X}, \mathrm{M}_{1}$, *) and $\left(\mathrm{Y}, \mathrm{M}_{2}\right.$, *) are the same fuzzy metric spaces, then by the above theorem 2.4 , we get the following theorem, as corollary.

Corollary 2.6: Let ( $\mathrm{X}, \mathrm{M}, *$ *) be a complete fuzzy metric space. If S and T are mappings from X into itself satisfying the following conditions:

```
M(Tx,TSy, qt)\geqmin {M(x,Sy, t),M(y,Tx, t),M(y,Tx, t)*M(y,TSy, t)}
M(Sy,STx, qt)\geqmin {M(y,Tx, t),M(x,Sy, t),M(x,Sy, t) *M(x,STx, t)}
```

for all x , y in X where $\mathrm{q}<1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in X . Further $\mathrm{Tz}=\mathrm{w}$ and $\mathrm{Sw}=\mathrm{z}$.

Theorem 2.7: Let ( $X, M_{1}, *$ ) and $\left(Y, M_{2}, *\right)$ be two complete fuzzy metric spaces. If $T$ is a mapping from $X$ into $Y$ and $S$ is a mapping from $Y$ into $X$, satisfying the following conditions:

$$
\begin{align*}
& \mathrm{M}_{2}(T x, T S y, q t) \geq \min \left\{\mathrm{M}_{1}(\mathrm{x}, \mathrm{Sy}, \mathrm{t}), \mathrm{M}_{2}(\mathrm{y}, \mathrm{Tx}, \mathrm{t}), \mathrm{M}_{2}(\mathrm{y}, \mathrm{TSy}, \mathrm{t}), \mathrm{M}_{1}(\mathrm{x}, \mathrm{STx}, \mathrm{t})\right\}  \tag{1}\\
& \mathrm{M}_{1}(\mathrm{Sy}, \mathrm{STx}, \mathrm{qt}) \geq \min \left\{\mathrm{M}_{2}(\mathrm{y}, \mathrm{Tx}, \mathrm{t}), \mathrm{M}_{1}(\mathrm{x}, \mathrm{Sy}, \mathrm{t}), \mathrm{M}_{1}(\mathrm{x}, \mathrm{STx}, \mathrm{t}), \mathrm{M}_{2}(\mathrm{~T} x, T S y, t)\right\} \tag{2}
\end{align*}
$$

for all x , y in X where $\mathrm{q}<1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y . Further $\mathrm{Tz}=\mathrm{w}$ and $\mathrm{Sw}=\mathrm{z}$.

Proof: Let $x_{0}$ be an arbitrary point in $X$. Define a sequence $\left\{x_{n}\right\}$ in $X$ and $\left\{y_{n}\right\}$ in $Y$, as follows:

$$
\mathrm{x}_{\mathrm{n}}=(\mathrm{ST})^{\mathrm{n}} \mathrm{x}_{0}, \mathrm{y}_{\mathrm{n}}=\mathrm{T}\left(\mathrm{x}_{\mathrm{n}-1}\right) \text { for } \mathrm{n}=1,2, \ldots
$$

we have

$$
\begin{aligned}
& M_{1}\left(x_{n}, x_{n+1}, q t\right)=M_{1}\left((S T)^{n} x_{0},(S T)^{n+1} x_{0}, q t\right) \\
& =M_{1}\left(S\left(T(S T)^{n-1} x_{0}, S T(S T)^{n} x_{0}, q t\right)\right. \\
& =\mathrm{M}_{1}\left(\mathrm{ST}\left(\mathrm{x}_{\mathrm{n}-1}\right), \mathrm{STx}_{\mathrm{n}}, \mathrm{qt}\right) \\
& =M_{1}\left(S y_{n}, S T x_{n}, q t\right) \\
& \geq \min \left\{\mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, T x_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, S \mathrm{y}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{STx}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{Tx}_{\mathrm{n}}, T S y_{\mathrm{n}}, \mathrm{t}\right)\right\} \\
& =\min \left\{M_{2}\left(y_{n}, y_{n+1}, t\right), M_{1}\left(x_{n}, x_{n}, t\right), M_{1}\left(x_{n}, x_{n+1}, t\right), M_{2}\left(y_{n+1}, y_{n+1}, t\right)\right\} \\
& \geq \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right)
\end{aligned}
$$

We have

$$
\begin{aligned}
\mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{qt}\right) & =\mathrm{M}_{2}\left(\mathrm{Tx}_{\mathrm{n}-1}, \mathrm{Tx}_{\mathrm{n}}, \mathrm{qt}\right) \\
& =\mathrm{M}_{2}\left(\mathrm{Tx}_{\mathrm{n}-1}, \mathrm{TSy}_{\mathrm{n}}, \mathrm{qt}\right) \\
& \geq \min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{Sy}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{Tx}_{\mathrm{n}-1}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, T \mathrm{TS}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{STx}_{\mathrm{n}-1}, \mathrm{t}\right)\right\} \\
& =\min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{x}_{\mathrm{n}}, t\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right)\right\} \\
& =\min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right), 1, \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right)\right\} \\
& \geq \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right)
\end{aligned}
$$

(i.e), $\mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right) \geq \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{x}_{\mathrm{n}}, \mathrm{t} / \mathrm{q}\right)$

Hence

$$
\begin{aligned}
\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{qt}\right) & \geq \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right) \\
& \geq \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{x}_{\mathrm{n}}, \mathrm{t} / \mathrm{q}\right) \\
& \\
& \geq \mathrm{M}_{1}\left(\mathrm{x}_{0}, \mathrm{x}_{1}, \mathrm{t} / \mathrm{q}^{2 \mathrm{n}-1}\right) \rightarrow 1 \text { as } \mathrm{n} \rightarrow \infty(\text { since } \mathrm{q}<1)
\end{aligned}
$$

Thus $\left\{x_{n}\right\}$ is a Cauchy sequence in $X$. Since ( $X, M_{1}$, *) is complete, it converges to a point $z$ in $X$. Similarly, we can prove that the sequence $\left\{y_{n}\right\}$ is also a Cauchy sequence in $Y$ and it converges to a point $w$ in $Y$.

Now we prove $\mathrm{Tz}=\mathrm{w}$

## T. Veerapandi, T. Thiripura Sundari* and J. Paulraj Joseph/ SOME FIXED POINT THEOREMS IN TWO FUZZY METRIC SPACES/

 IJMA- 3(8), August-2012.Suppose Tz $\neq \mathrm{w}$.
We have

$$
\begin{aligned}
\mathrm{M}_{2}(\mathrm{Tz}, \mathrm{w}, \mathrm{qt}) & =\lim _{n \rightarrow \infty} \mathrm{M}_{2}\left(\mathrm{Tz}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{qt}\right) \\
& =\lim _{n \rightarrow \infty} \mathrm{M}_{2}\left(\mathrm{Tz}, \mathrm{TS} \mathrm{y}_{\mathrm{n}}, \mathrm{qt}\right) \\
& \geq \lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{1}\left(\mathrm{z}, \mathrm{Sy}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{Tz}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{TSy}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t})\right\} \\
& =\lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{1}\left(\mathrm{z}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{Tz}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t})\right\} \\
& =\min \left\{\mathrm{M}_{1}(\mathrm{z}, \mathrm{z}, \mathrm{t}), \mathrm{M}_{2}(\mathrm{w}, \mathrm{Tz}, \mathrm{t}), \mathrm{M}_{2}(\mathrm{w}, \mathrm{w}, \mathrm{t}), \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t})\right\} \\
& =\min _{11}\left\{1, \mathrm{M}_{2}(\mathrm{w}, \mathrm{Tz}, \mathrm{t}), 1, \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t})\right\} \\
& \geq \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t})
\end{aligned}
$$

Also

$$
\begin{aligned}
\mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{qt}) & =\lim _{n \rightarrow \infty} \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{STz}, \mathrm{qt}\right) \\
& =\lim _{n \rightarrow \infty} \mathrm{M}_{1}\left(\mathrm{Sy}_{\mathrm{n}}, \mathrm{STz}, \mathrm{qt}\right) \\
& \geq \lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \operatorname{Tz}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{z}, \mathrm{Sy}_{\mathrm{n}}\right), \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t}), \mathrm{M}_{2}\left(\operatorname{Tz}, \mathrm{TSy}_{\mathrm{n}}, \mathrm{t}\right)\right\} \\
& =\lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \operatorname{Tz}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{z}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t}), \mathrm{M}_{2}\left(\operatorname{Tz}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right)\right\} \\
& =\min \left\{\mathrm{M}_{2}(\mathrm{w}, \operatorname{Tz}, \mathrm{t}), \mathrm{M}_{1}(\mathrm{z}, \mathrm{z}, \mathrm{t}), \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t}), \mathrm{M}_{2}(\operatorname{Tz}, \mathrm{w}, \mathrm{t})\right\} \\
& =\min _{2}\left\{\mathrm{M}_{2}(\mathrm{w}, \operatorname{Tz}, \mathrm{t}), 1, \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t}), \mathrm{M}_{2}(\mathrm{Tz}, \mathrm{w}, \mathrm{t})\right\} \\
& \geq \mathrm{M}_{2}(\operatorname{Tz}, \mathrm{w}, \mathrm{t})
\end{aligned}
$$

$$
\text { (i.e), } \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t}) \geq \mathrm{M}_{2}(\mathrm{Tz}, \mathrm{w}, \mathrm{t} / \mathrm{q})
$$

hence
$M_{2}(T z, w, q t) \geq M_{2}(T z, w, t / q)$, which is a contradiction. (Since $q<1$ )
Thus $\mathrm{Tz}=\mathrm{w}$.
Now we prove $\mathrm{Sw}=\mathrm{z}$.
Suppose Sw $\neq \mathrm{z}$.
Then we have

Hence

$$
\mathrm{M}_{1}(\mathrm{Sw}, \mathrm{z}, \mathrm{q}) \geq \mathrm{M}_{2}(\mathrm{w}, \mathrm{TSw}, \mathrm{t}) \geq \mathrm{M}_{1}(\mathrm{Sw}, \mathrm{z}, \mathrm{t} / \mathrm{q})
$$

Which is a contradiction. (since $\mathrm{q}<1$ )

$$
\begin{aligned}
& \mathrm{M}_{1}(\mathrm{Sw}, \mathrm{z}, \mathrm{qt})=\lim _{n \rightarrow \infty} \mathrm{M}_{1}\left(\mathrm{Sw}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{qt}\right) \\
& =\lim _{n \rightarrow \infty} \mathrm{M}_{1}\left(\mathrm{Sw}, \mathrm{STx}_{\mathrm{n}}, \mathrm{qt}\right) \\
& \geq \lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{2}\left(\mathrm{w}, \mathrm{Tx}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{Sw}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{STx}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{Tx}_{\mathrm{n}}, \mathrm{TSw}, \mathrm{t}\right)\right\} \\
& =\lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{2}\left(\mathrm{w}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{Sw}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}+1}, \mathrm{TSw}, \mathrm{t}\right)\right\} \\
& =\min \left\{\mathrm{M}_{2}(\mathrm{w}, \mathrm{w}, \mathrm{t}), \mathrm{M}_{1}(\mathrm{z}, \mathrm{Sw}, \mathrm{t}), \mathrm{M}_{1}(\mathrm{z}, \mathrm{z}, \mathrm{t}), \mathrm{M}_{2}(\mathrm{w}, \mathrm{TSw}, \mathrm{t})\right\} \\
& =\min \left\{1, \mathrm{M}_{1}(\mathrm{z}, \mathrm{Sw}, \mathrm{t}), 1, \mathrm{M}_{2}(\mathrm{w}, \mathrm{TSw}, \mathrm{t})\right\} \\
& \geq \mathrm{M}_{2} \text { (w,TSw,t) } \\
& \mathrm{M}_{2}(\mathrm{w}, \mathrm{TSw}, \mathrm{qt})=\lim _{n \rightarrow \infty} \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}+1}, \mathrm{TS} w, \mathrm{qt}\right) \\
& =\lim _{n \rightarrow \infty} \mathrm{M}_{2}\left(\mathrm{Tx}_{\mathrm{n}}, \mathrm{TSw}, \mathrm{qt}\right) \\
& \geq \lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, S \mathrm{Sw}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{w}, T \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}(\mathrm{w}, \mathrm{TSw}, \mathrm{t}), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{STx}_{\mathrm{n}}, \mathrm{t}\right)\right\} \\
& =\min \left\{\mathrm{M}_{1}(\mathrm{z}, \mathrm{Sw}, \mathrm{t}), 1, \mathrm{M}_{2}(\mathrm{w}, \mathrm{TS} w, \mathrm{t}), 1\right\} \\
& \geq M_{1}(z, S w, t) \\
& \text { (i.e), } M_{2}(w, T S w, t) \geq M_{1}(S w, z, t / q)
\end{aligned}
$$

## T. Veerapandi, T. Thiripura Sundari* and J. Paulraj Joseph/ SOME FIXED POINT THEOREMS IN TWO FUZZY METRIC SPACES/

 IJMA- 3(8), August-2012.Thus Sw = z.
Therefore we have $\mathrm{STz}=\mathrm{Sw}=\mathrm{z}$ and $\mathrm{TSw}=\mathrm{Tz}=\mathrm{w}$. Thus the point z is a fixed point of ST and the point w is a fixed point of TS.

Uniqueness: Let $\mathrm{z}^{\prime}$ be the another fixed point of ST such that $\mathrm{z} \neq \mathrm{z}^{\prime}$.
Now

$$
\begin{aligned}
\mathrm{M}_{1}\left(\mathrm{z}, \mathrm{z}^{\prime}, \mathrm{qt}\right) & =\lim _{n \rightarrow \infty} \mathrm{M}_{1}\left(\mathrm{Sw}, \mathrm{STz}^{\prime}, \mathrm{qt}\right) \\
& \geq \lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{2}\left(\mathrm{w}, \mathrm{Tz}^{\prime}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{Sw}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{STz}^{\prime}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{w}, \mathrm{t}\right)\right\} \\
& =\min \left\{\mathrm{M}_{2}\left(\mathrm{w}, \mathrm{Tz}^{\prime}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{z}, \mathrm{t}\right), 1, \mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{w}, \mathrm{t}\right)\right\} \\
& \geq \mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{w}, \mathrm{t}\right)
\end{aligned}
$$

Also

$$
\begin{aligned}
\mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{w}, \mathrm{qt}\right) & =\lim _{n \rightarrow \infty} \mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{qt}\right) \\
& =\lim _{n \rightarrow \infty} \mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{TSy}, \mathrm{qt}\right) \\
& \geq \lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{Sy}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{Tz}^{\prime}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{TSy}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{STz}^{\prime}, \mathrm{t}\right)\right\} \\
& \left.=\min ^{2}\left\{\mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{z}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{w}, \mathrm{Tz}^{\prime}, \mathrm{t}\right), 1,1\right)\right\} \\
& \geq \mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{z}, \mathrm{t}\right)
\end{aligned}
$$

(i.e), $\mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{w}, \mathrm{t}\right) \geq \mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{z}, \mathrm{t} / \mathrm{q}\right)$

Therefore we have

$$
\mathrm{M}_{1}\left(\mathrm{z}, \mathrm{z}^{\prime}, \mathrm{qt}\right) \geq \mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{w}, \mathrm{t}\right) \geq \mathrm{M}_{1}\left(\mathrm{z}, \mathrm{z}^{\prime}, \mathrm{t}\right)
$$

Which is a contradiction. $\quad$ (since $\mathrm{q}<1$ )
Thus $\mathrm{z}=\mathrm{z}^{\prime}$.
So the point z is the unique fixed point of ST. Similarly, we prove the point w is also a unique fixed point of TS. .
This completes the proof.
Remark 2.8: If $\left(\mathrm{X}, \mathrm{M}_{1}, *\right)$ and $\left(\mathrm{Y}, \mathrm{M}_{2}, *\right)$ are the same fuzzy metric spaces, then by the above theorem 2.7 , we get the following theorem, as corollary.

Corollary 2.9: Let ( $\mathrm{X}, \mathrm{M}$, *) be a complete fuzzy metric space. If S and T are mappings from X into itself satisfying the following conditions:

```
M(Tx,TSy,qt) \geq min {M(x,Sy,t), M(y,Tx,t), M(y,TSy,t), M(x,STx,t)}
M(Sy,STx,qt)\geqmin {M(y,Tx,t),M(x,Sy,t),M(x,STx,t),M(Tx,TSy,t)}
```

for all x , y in X where $\mathrm{q}<1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y . Further $\mathrm{Tz}=\mathrm{w}$ and $\mathrm{Sw}=\mathrm{z}$.

Theorem 2.10: Let ( $\mathrm{X}, \mathrm{M}_{1}, *$ *) and $\left(\mathrm{Y}, \mathrm{M}_{2}\right.$, *) be two complete fuzzy metric spaces. If T is a mapping from X into Y and $S$ is a mapping from $Y$ into $X$, satisfying following conditions:

$$
\begin{align*}
& M_{2}(T x, T S y, q t) \geq \min \left\{M_{1}(x, S y, t), M_{2}(y, T x, t), M_{2}(y, T S y, t), M_{1}(x, S T x, t), M_{1}(S y, S T x, t)\right\}  \tag{1}\\
& M_{1}(S y, S T x, q t) \geq \min \left\{M_{2}(y, T x, t), M_{1}(x, S y, t), M_{1}(x, S T x, t), M_{2}(T x, T S y, t), M_{2}(y, T S y, t)\right\} \tag{2}
\end{align*}
$$

for all x in X and y in Y where $\mathrm{q}<1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y . Further $\mathrm{Tz}=\mathrm{w}$ and $\mathrm{Sw}=\mathrm{z}$.

Proof: Let $x_{0}$ be an arbitrary point in $X$. Define a sequence $\left\{x_{n}\right\}$ in $X$ and $\left\{y_{n}\right\}$ in $Y$, as follows:

$$
\mathrm{x}_{\mathrm{n}}=(\mathrm{ST})^{\mathrm{n}} \mathrm{x}_{0}, \mathrm{y}_{\mathrm{n}}=\mathrm{T}\left(\mathrm{x}_{\mathrm{n}-1}\right) \text { for } \mathrm{n}=1,2, \ldots
$$

## T. Veerapandi, T. Thiripura Sundari* and J. Paulraj Joseph/ SOME FIXED POINT THEOREMS IN TWO FUZZY METRIC SPACES/

 IJMA- 3(8), August-2012.we have

$$
\begin{aligned}
& \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{qt}\right)=\mathrm{M}_{1}\left((\mathrm{ST})^{\mathrm{n}} \mathrm{x}_{0},(\mathrm{ST})^{\mathrm{n}+1} \mathrm{x}_{0}, \mathrm{qt}\right) \\
& =\mathrm{M}_{1}\left(\mathrm{~S}\left(\mathrm{~T}(\mathrm{ST})^{\mathrm{n}-1} \mathrm{x}_{0}, \mathrm{ST}(\mathrm{ST})^{\mathrm{n}} \mathrm{x}_{0}, \mathrm{qt}\right)\right. \\
& =\mathrm{M}_{1}\left(\mathrm{ST}\left(\mathrm{x}_{\mathrm{n}-1}\right), \mathrm{STx}_{\mathrm{n}}, \mathrm{qt}\right) \\
& =\mathrm{M}_{1}\left(\mathrm{Sy}_{\mathrm{n}}, \mathrm{STx} \mathrm{x}_{\mathrm{n}}, \mathrm{qt}\right) \\
& \geq \min \left\{\mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, T x_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, S \mathrm{y}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, S T x_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{Tx}_{\mathrm{n}}, T S y_{\mathrm{n}}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, T S y_{\mathrm{n}}, \mathrm{t}\right)\right\} \\
& =\min \left\{\mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}+1}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right)\right\} \\
& =\min \left\{\mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right), 1, \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{t}\right), 1, \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right)\right\} \\
& \geq \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right)
\end{aligned}
$$

Also we have

$$
\begin{aligned}
& \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{qt}\right)=\mathrm{M}_{2}\left(\mathrm{Tx}_{\mathrm{n}-1}, \mathrm{Tx}_{\mathrm{n}}, \mathrm{qt}\right) \\
& =\mathrm{M}_{2}\left(\mathrm{Tx}_{\mathrm{n}-1}, \mathrm{TSy}_{\mathrm{n}}, \mathrm{qt}\right) \\
& \geq \min \left\{M_{1}\left(x_{n-1}, \operatorname{Sy}_{n}, t\right), M_{2}\left(y_{n}, T x_{n-1}, t\right), M_{2}\left(y_{n}, T S y_{n}, t\right), M_{1}\left(x_{n-1}, S T x_{n-1}, t\right), M_{1}\left(S y_{n}, S T x_{n-1}, t\right)\right\} \\
& =\min \left\{M_{1}\left(x_{n-1}, x_{n}, t\right), M_{2}\left(y_{n}, y_{n}, t\right), M_{2}\left(y_{n}, y_{n+1}, t\right), M_{1}\left(x_{n-1}, x_{n}, t\right), M_{1}\left(x_{n-1}, x_{n}, t\right\}\right\} \\
& =\min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{X}_{\mathrm{n}}, \mathrm{t}\right), 1, \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{X}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{X}_{\mathrm{n}}, \mathrm{t}\right\}\right\} \\
& \geq \mathrm{M}_{1}\left(\mathrm{X}_{\mathrm{n}-1}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right)
\end{aligned}
$$

Now

$$
\begin{aligned}
\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{qt}\right) & \geq \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right) \\
& \geq \mathrm{M}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{x}_{\mathrm{n}}, \mathrm{t} / \mathrm{q}\right) \\
& \vdots \\
& \geq \mathrm{M}_{1}\left(\mathrm{x}_{0}, \mathrm{x}_{1}, \mathrm{t} / \mathrm{q}^{2 \mathrm{n}-1}\right) \rightarrow 1 \text { as } \mathrm{n} \rightarrow \infty \quad(\text { since } \mathrm{q}<1)
\end{aligned}
$$

Thus $\left\{X_{n}\right\}$ is a Cauchy sequence in $X$. Since ( $X, M_{1}, *$ ) is complete, it converges to a point $z$ in $X$. Similarly, we can prove that the sequence $\left\{\mathrm{y}_{\mathrm{n}}\right\}$ is also a Cauchy sequence in Y and it converges to a point w in Y .

## Now we prove $\mathrm{Tz}=\mathrm{w}$

We have

$$
\begin{aligned}
& \mathrm{M}_{2}(\mathrm{Tz}, \mathrm{w}, \mathrm{qt})=\lim _{n \rightarrow \infty} \mathrm{M}_{2}\left(\mathrm{Tz}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{qt}\right) \\
& =\lim _{n \rightarrow \infty} \mathrm{M}_{2}\left(\mathrm{Tz}, \mathrm{TSy}_{\mathrm{n}}, \mathrm{qt}\right) \\
& \geq \lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{1}(\mathrm{z}, \mathrm{Sy}, \mathrm{t}, \mathrm{t}), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, T z, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, T S \mathrm{y}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t}), \mathrm{M}_{1}\left(\mathrm{Sy}_{\mathrm{n}}, \mathrm{STz}, \mathrm{t}\right)\right\} \\
& =\lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{1}\left(\mathrm{z}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{Tz}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t}), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{STz}, \mathrm{t}\right)\right\} \\
& =\min \left\{1, \mathrm{M}_{2}(\mathrm{w}, \mathrm{Tz}, \mathrm{t}), 1, \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t}), \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t})\right\} \\
& \geq \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t}) \\
& \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{qt})=\lim _{n \rightarrow \infty} \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{STz}, \mathrm{qt}\right) \\
& =\lim _{n \rightarrow \infty} \mathrm{M}_{1}\left(\mathrm{Sy}_{\mathrm{n}}, \mathrm{STz}, \mathrm{qt}\right) \\
& \geq \lim _{n \rightarrow \infty} \min \left\{M_{2}\left(y_{n}, T z, t\right), M_{1}\left(z, S y_{n}, t\right), M_{1}(z, S T z, t), M_{2}\left(T z, T S y_{n}, t\right), M_{2}\left(y_{n}, T S y_{n}, t\right)\right\} \\
& =\lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \operatorname{Tz}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{z}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t}), \mathrm{M}_{2}\left(\mathrm{Tz}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right)\right\} \\
& =\min \left\{\mathrm{M}_{2}(\mathrm{w}, \mathrm{Tz}, \mathrm{t}), 1, \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t}), \mathrm{M}_{2}(\mathrm{Tz}, \mathrm{w}, \mathrm{t}), 1\right\} \\
& \geq \mathrm{M}_{2}(\mathrm{Tz}, \mathrm{w}, \mathrm{t})
\end{aligned}
$$

Hence

$$
\mathrm{M}_{2}(\mathrm{Tz}, \mathrm{w}, \mathrm{qt}) \geq \mathrm{M}_{2}(\mathrm{Tz}, \mathrm{w}, \mathrm{t} / \mathrm{q})
$$

Thus Tz = w.
Now we prove $\mathrm{Sw}=\mathrm{z}$.
Suppose $\mathrm{Sw}_{\mathrm{w}} \neq \mathrm{z}$.

$$
\begin{aligned}
\mathrm{M}_{1}(\mathrm{Sw}, \mathrm{z}, \mathrm{qt}) & =\lim _{n \rightarrow \infty} \mathrm{M}_{1}\left(\mathrm{Sw}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{qt}\right) \\
& =\lim _{n \rightarrow \infty} \mathrm{M}_{1}\left(\mathrm{Sw}, \mathrm{STx}_{\mathrm{n}}, \mathrm{qt}\right)
\end{aligned}
$$

## T. Veerapandi, T. Thiripura Sundari* and J. Paulraj Joseph/ SOME FIXED POINT THEOREMS IN TWO FUZZY METRIC SPACES/

 IJMA- 3(8), August-2012.$$
\begin{aligned}
& \geq \lim _{n \rightarrow \infty} \min \left\{M_{2}\left(w, T x_{n}, t\right), M_{1}\left(x_{n}, S w, t\right), M_{1}\left(x_{n}, S T x_{n}, t\right), M_{2}\left(T x_{n}, T S w, t\right), M_{2}\left(y_{n}, T S x_{n}, t\right)\right\} \\
& =\lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{2}\left(\mathrm{w}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{Sw}_{\mathrm{w}}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{X}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{y}_{\mathrm{n}+1}, \mathrm{TS}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right)\right\} \\
& =\min \left\{1, \mathrm{M}_{1}(\mathrm{z}, \mathrm{Sw}, \mathrm{t}), 1, \mathrm{M}_{1}(\mathrm{w}, \mathrm{TSw}, \mathrm{t}), 1\right\} \\
& \geq \mathrm{M}_{2} \text { (w,TSw,t) }
\end{aligned}
$$

Now

$$
\begin{aligned}
& \mathrm{M}_{2}(\mathrm{w}, \mathrm{TSw}, \mathrm{qt})=\lim _{n \rightarrow \infty} \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}+1}, \mathrm{TSw}, \mathrm{qt}\right) \\
& =\lim _{n \rightarrow \infty} \mathrm{M}_{2}\left(\mathrm{Tx}_{\mathrm{n}}, \mathrm{TSw}, \mathrm{qt}\right) \\
& \geq \lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{Sw}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{w}, T x_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}(\mathrm{w}, \mathrm{TSw}, \mathrm{t}), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, S T x_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{Sw}_{\mathrm{w}}, \mathrm{STx}_{\mathrm{n}}, \mathrm{t}\right)\right\} \\
& =\lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, S \mathrm{~S}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{w}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{2}(\mathrm{w}, \mathrm{TSw}, \mathrm{t}), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{Sw}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{t}\right)\right\} \\
& =\min \left\{\mathrm{M}_{1}(\mathrm{z}, \mathrm{Sw}, \mathrm{t}), 1, \mathrm{M}_{2}(\mathrm{w}, \mathrm{TS} \mathrm{w}, \mathrm{t}), 1, \mathrm{M}_{1}(\mathrm{Sw}, \mathrm{z}, \mathrm{t})\right\} \\
& \geq M_{1}(S w, z, t)
\end{aligned}
$$

Hence

$$
\begin{aligned}
\mathrm{M}_{1}(\mathrm{Sw}, \mathrm{z}, \mathrm{qt}) & \geq \mathrm{M}_{2}(\mathrm{w}, \mathrm{TSw}, \mathrm{t}) \\
& \geq \mathrm{M}_{1}(\mathrm{Sw}, \mathrm{z}, \mathrm{t} / \mathrm{q})
\end{aligned}
$$

Which is a contradiction. (since $\mathrm{q}<1$ )
Thus Sw = z.
Therefore we have $\mathrm{STz}=\mathrm{Sw}=\mathrm{z}$ and $\mathrm{TSw}=\mathrm{Tz}=\mathrm{w}$. Thus the point z is a fixed point of ST and the point w is a fixed point of TS.

Uniqueness: Let $\mathrm{z}^{\prime}$ be the another fixed point of ST such that $\mathrm{z} \neq \mathrm{z}^{\prime}$.
Now

$$
\begin{aligned}
& \mathrm{M}_{1}\left(\mathrm{z}, \mathrm{z}^{\prime}, \mathrm{qt}\right)=\mathrm{M}_{1}\left(\mathrm{Sw}, \text { STz }^{\prime}, \mathrm{qt}\right) \\
& \geq \min \left\{\mathrm{M}_{2}\left(\mathrm{w}, \mathrm{Tz}^{\prime}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{Sw}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{z}^{\prime}, S T z^{\prime}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{TSw}, \mathrm{t}\right), \mathrm{M}_{2}(\mathrm{w}, \mathrm{TSw}, \mathrm{t})\right\} \\
& =\min \left\{\mathrm{M}_{2}\left(\mathrm{w}, \mathrm{Tz}^{\prime}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{z}, \mathrm{t}\right), 1, \mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{w}, \mathrm{t}\right), 1\right\} \\
& \geq \mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{w}, \mathrm{t}\right) \\
& \mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{w}, \mathrm{qt}\right)=\mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{TSw}, \mathrm{t}\right) \\
& \geq \min \left\{\mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{Sw}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{w}, \mathrm{Tz}^{\prime}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{TSz}^{\prime}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{z}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{z}, \mathrm{STz}{ }^{\prime}, \mathrm{t}\right)\right\} \\
& =\min \left\{\mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{z}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{w}, \operatorname{Tz} z^{\prime}, \mathrm{t}\right), 1, \mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{z}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{z}, \mathrm{z}^{\prime}, \mathrm{t}\right)\right\} \\
& \geq \mathrm{M}_{1}\left(\mathrm{z}, \mathrm{z}^{\prime}, \mathrm{t}\right)
\end{aligned}
$$

Hence

$$
\begin{aligned}
\mathrm{M}_{1}\left(\mathrm{z}, \mathrm{z}^{\prime}, \mathrm{qt}\right) & \geq \mathrm{M}_{2}(\mathrm{Tz}, \mathrm{w}, \mathrm{t}) \\
& \geq \mathrm{M}_{1}\left(\mathrm{z}, \mathrm{z}^{\prime}, \mathrm{t} / \mathrm{q}\right)
\end{aligned}
$$

Which is a contradiction. (since $\mathrm{q}<1$ )
Thus $\mathrm{z}=\mathrm{z}^{\prime}$.
So the point z is the unique fixed point of ST. Similarly, we prove the point w is also a unique fixed point of TS.
This completes the proof.
Remark 2.11: If $\left(X, M_{1}, *\right)$ and $\left(Y, M_{2}, *\right)$ are the same fuzzy metric spaces, then by the above theorem 2.10, we get the following theorem, as corollary.

Corollary 2.12: Let ( $\mathrm{X}, \mathrm{M}, *$ *) be a complete fuzzy metric space. If S and T are mappings from X into itself satisfying the following conditions:
$M(T x, T S y, q t) \geq \min \{M(x, S y, t), M(y, T x, t), M(y, T S y, t), M(x, S T x, t), M(S y, S T x, t)\}$
$M(S y, S T x, q t) \geq \min \{M(y, T x, t), M(x, S y, t), M(x, S T x, t), M(T x, T S y, t), M(y, T S y, t)\}$

## T. Veerapandi, T. Thiripura Sundari* and J. Paulraj Joseph/ SOME FIXED POINT THEOREMS IN TWO FUZZY METRIC SPACES/

 IJMA- 3(8), August-2012.for all x , y in X where $\mathrm{q}<1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y . Further $\mathrm{Tz}=\mathrm{w}$ and $\mathrm{Sw}=\mathrm{z}$.

Theorem 2.13: Let $\left(X, M_{1}, *\right)$ and $\left(Y, M_{2}, *\right)$ be two complete fuzzy metric spaces. If $T$ is a mapping from $X$ into $Y$ and $S$ is a mapping from $Y$ into $X$, satisfying the following conditions:

$$
\begin{equation*}
M_{2}(T x, T S y, t) \geq \min \left\{M_{1}(x, S y, t), M_{1}(S y, S T x, t), M_{2}(y, T x, t){ }^{*} M_{2}(y, T S y, t), M_{1}(x, S T x, t)\right\} \tag{1}
\end{equation*}
$$

$M_{1}(S y, S T x, t) \geq \min \left\{M_{1}(x, S y, t){ }^{*} M_{1}(x, S T x, t), M_{2}(y, T S y, t), M_{2}(y, T x, t), M_{2}(T x, T S y, t)\right\}$
for all x in X and y in Y where $\mathrm{q}<1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y .
Further $\mathrm{Tz}=\mathrm{w}$ and $\mathrm{Sw}=\mathrm{z}$.
Proof. Let $x_{0}$ be an arbitrary point in $X$. Define a sequence $\left\{x_{n}\right\}$ in $X$ and $\left\{y_{n}\right\}$ in $Y$, as follows:

$$
\mathrm{x}_{\mathrm{n}}=(\mathrm{ST})^{\mathrm{n}} \mathrm{x}_{0}, \mathrm{y}_{\mathrm{n}}=\mathrm{T}\left(\mathrm{x}_{\mathrm{n}-1}\right) \text { for } \mathrm{n}=1,2, \ldots
$$

We have

$$
\begin{aligned}
& M_{1}\left(x_{n}, x_{n+1}, q t\right)=M_{1}\left((S T)^{n} x_{0},(S T)^{n+1} x_{0}, q t\right) \\
& \left.=M_{1}(S(T) S T)^{n-1} x_{0}, S T(S T)^{n} x_{0}, q t\right) \\
& =M_{1}\left(S T\left(x_{n-1}\right), \text { STx }_{n}, q t\right) \\
& =\mathrm{M}_{1}\left(\mathrm{Sy}_{\mathrm{n}}, \mathrm{STx}_{\mathrm{n}}, \mathrm{qt}\right) \\
& \geq \min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, S \mathrm{y}_{\mathrm{n}}, \mathrm{t}\right) *{ }^{*} \mathrm{M}_{\left.\left(\mathrm{x}_{\mathrm{n}}, S T x_{n}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, T S y_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, T \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{Tx}_{\mathrm{n}}, T S \mathrm{y}_{\mathrm{n}}, \mathrm{t}\right)\right\}}\right. \\
& =\min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right) * \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}+1}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right)\right\} \\
& =\min \left\{1 * M_{1}\left(x_{n}, x_{n+1}, t\right), M_{2}\left(y_{n}, y_{n+1}, t\right), M_{2}\left(y_{n}, y_{n+1}, t\right), 1\right\} \\
& \geq \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right)
\end{aligned}
$$

```
Now
\[
\mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{qt}\right)=\mathrm{M}_{2}\left(\mathrm{Tx}_{\mathrm{n}-1}, T \mathrm{x}_{\mathrm{n}}, \mathrm{qt}\right)
\]
\[
=\mathrm{M}_{2}\left(\mathrm{Tx}_{\mathrm{n}-1}, \mathrm{TS} \mathrm{y}_{\mathrm{n}}, \mathrm{qt}\right)
\]
\[
\geq \min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{Sy}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{Sy}_{\mathrm{n}}, \mathrm{STx}_{\mathrm{n}-1}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{Tx}_{\mathrm{n}-1}, \mathrm{t}\right) * \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{TSy}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{STx}_{\mathrm{n}-1}, \mathrm{t}\right)\right\}
\]
\[
=\min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right), 1,1 * \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right)\right\}
\]
\[
\geq \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right)
\]
```

Hence

$$
\begin{aligned}
\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{qt}\right) & \geq \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right) \\
& \geq \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}-1}, \mathrm{x}_{\mathrm{n}}, \mathrm{t} / \mathrm{q}\right) \\
& \vdots \\
& \geq \mathrm{M}_{1}\left(\mathrm{x}_{0}, \mathrm{x}_{1}, \mathrm{t} / \mathrm{q}^{2 \mathrm{n}-1}\right) \rightarrow 1 \text { as } \mathrm{n} \rightarrow \infty \quad(\text { since } \mathrm{q}<1)
\end{aligned}
$$

Thus $\left\{x_{n}\right\}$ is a Cauchy sequence in ( $X, M_{1}, *$, . Since ( $X, M_{1}, *$ ) is complete, it converges to a point $z$ in $X$. Similarly, we can prove that the sequence $\left\{\mathrm{y}_{\mathrm{n}}\right\}$ is also a Cauchy sequence in $\left(\mathrm{Y}, \mathrm{M}_{2}, *\right.$, Since $\left(\mathrm{Y} \mathrm{M}_{2}\right.$, *) is complete, it converges to a point w in Y .

Now we prove $\mathrm{Tz}=\mathrm{w}$.
Suppose $\mathrm{Tz} \neq \mathrm{w}$.
We have

$$
\begin{aligned}
\mathrm{M}_{2}(\mathrm{Tz}, \mathrm{w}, \mathrm{qt}) & =\lim _{n \rightarrow \infty} \mathrm{M}_{2}\left(\mathrm{Tz}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{qt}\right) \\
& =\lim _{n \rightarrow \infty} \mathrm{M}_{2}\left(\operatorname{Tz}, \mathrm{TSy} \mathrm{y}_{\mathrm{n}}, \mathrm{qt}\right) \\
& \geq \lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{1}\left(\mathrm{z}, \mathrm{Sy}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{Sy}_{\mathrm{n}}, S T z, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{Tz}, \mathrm{t}\right) * \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{TS} \mathrm{y}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t})\right\} \\
& =\lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{1}\left(\mathrm{z}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{STz}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, T \mathrm{Tz}, \mathrm{t}\right) * \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t})\right\} \\
& \geq \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t})
\end{aligned}
$$

## T. Veerapandi, T. Thiripura Sundari* and J. Paulraj Joseph/ SOME FIXED POINT THEOREMS IN TWO FUZZY METRIC SPACES/

 IJMA- 3(8), August-2012.Now

$$
\begin{aligned}
\mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{qt}) & =\lim _{n \rightarrow \infty} \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{STz}, \mathrm{qt}\right) \\
& =\lim _{n \rightarrow \infty} \mathrm{M}_{1}\left(\mathrm{Sy}_{\mathrm{n}}, \mathrm{STz}, \mathrm{qt}\right) \\
& \geq \lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{1}(\mathrm{z}, \mathrm{Sy}, \mathrm{t}) * \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t}), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{TS} \mathrm{y}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{Tz}, \mathrm{t}\right), \mathrm{M}_{2}\left(\operatorname{Tz}, \mathrm{TS} \mathrm{y}_{\mathrm{n}}, \mathrm{t}\right)\right\} \\
& =\lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{1}\left(\mathrm{z}, \mathrm{x}_{\mathrm{n}}, \mathrm{t}\right) * \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t}), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}}, \mathrm{Tz}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{Tz}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right)\right\} \\
& =\min _{1}\left\{1^{*} \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t}), 1, \mathrm{M}_{2}(\mathrm{w}, \mathrm{Tz}, \mathrm{t}), \mathrm{M}_{2}(\operatorname{Tz}, \mathrm{w}, \mathrm{t})\right\} \\
& \geq \mathrm{M}_{2}(\mathrm{Tz}, \mathrm{w}, \mathrm{t})
\end{aligned}
$$

Hence
$\mathrm{M}_{2}(\mathrm{Tz}, \mathrm{w}, \mathrm{qt}) \geq \mathrm{M}_{1}(\mathrm{z}, \mathrm{STz}, \mathrm{t}) \geq \mathrm{M}_{2}(\mathrm{Tz}, \mathrm{w}, \mathrm{t} / \mathrm{q})$ (since $\left.\mathrm{q}<1\right)$ which is a contradiction.
Thus $\mathrm{Tz}=\mathrm{w}$.
Now we prove $\mathrm{Sw}=\mathrm{z}$.
Suppose Sw $\neq \mathrm{z}$.
We have

$$
\begin{aligned}
\mathrm{M}_{1}(\mathrm{Sw}, \mathrm{z}, \mathrm{qt}) & =\lim _{n \rightarrow \infty} \mathrm{M}_{1}\left(\mathrm{Sw}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{qt}\right) \\
& =\lim _{n \rightarrow \infty} \mathrm{M}_{1}\left(\mathrm{Sw}, \mathrm{STx}_{\mathrm{n}}, \mathrm{qt}\right) \\
& \geq \lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{Sw}, \mathrm{t}\right) * \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{STx}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}(\mathrm{w}, \mathrm{TSw}, \mathrm{t}), \mathrm{M}_{2}\left(\mathrm{w}, \mathrm{Tx}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{Tx}_{\mathrm{n}}, \mathrm{TSw}, \mathrm{t}\right)\right\} \\
& =\lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{Sw}, \mathrm{t}\right) * \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{2}(\mathrm{w}, \mathrm{TSw}, \mathrm{t}), \mathrm{M}_{2}\left(\mathrm{w}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}+1}, \mathrm{w}, \mathrm{t}\right)\right\} \\
& \geq \mathrm{M}_{2}(\mathrm{w}, \mathrm{TSw}, \mathrm{t})
\end{aligned}
$$

Now

$$
\begin{aligned}
\mathrm{M}_{2}(\mathrm{w}, \mathrm{TSw}, \mathrm{qt}) & =\lim _{n \rightarrow \infty} \mathrm{M}_{2}\left(\mathrm{y}_{\mathrm{n}+1}, \mathrm{TSw}, \mathrm{qt}\right) \\
& =\lim _{n \rightarrow \infty} \mathrm{M}_{2}\left(\mathrm{Tx}_{\mathrm{n}}, \mathrm{TSw}, \mathrm{qt}\right) \\
& \geq \lim _{n \rightarrow \infty} \min \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, S \mathrm{Sw}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{Sw}, \mathrm{STx}_{\mathrm{n}}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{w}, \mathrm{Tx}_{\mathrm{n}}, \mathrm{t}\right) * \mathrm{M}_{2}(\mathrm{w}, \mathrm{TSw}), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, T \mathrm{Tx}_{\mathrm{n}}, \mathrm{t}\right)\right\} \\
& =\lim _{n \rightarrow \infty} \max \left\{\mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, S \mathrm{Sw}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{Sw}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{w}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right) * \mathrm{M}_{2}(\mathrm{w}, \mathrm{TSw}, \mathrm{t}), \mathrm{M}_{1}\left(\mathrm{x}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}\right)\right\} \\
& \geq \mathrm{M}_{1}(\mathrm{z}, \mathrm{Sw}, \mathrm{t})
\end{aligned}
$$

Hence

$$
\left.\mathrm{M}_{1}(\mathrm{Sw}, \mathrm{z}, \mathrm{qt}) \geq \mathrm{M}_{2}(\mathrm{w}, \mathrm{TSw}, \mathrm{t}) \geq \mathrm{M}_{1}(\mathrm{z}, \mathrm{Sw}, \mathrm{t} / \mathrm{q}) \text { (since } \mathrm{q}<1\right) \text { which is a contradiction. }
$$

Thus $\mathrm{Sw}=\mathrm{z}$.
We have $\mathrm{STz}=\mathrm{Sw}=\mathrm{z}$ and $\mathrm{TS} \mathrm{w}=\mathrm{Tz}=\mathrm{w}$. Thus the point z is a fixed point of ST in X and the point w is a fixed point of TS in Y.

Uniqueness: Let $\mathrm{z}^{\prime} \neq \mathrm{z}$ be the another fixed point of ST in X .
We have

$$
\begin{aligned}
\mathrm{M}_{1}\left(\mathrm{z}, \mathrm{z}^{\prime}, \mathrm{qt}\right) & =\mathrm{M}_{1}\left(\mathrm{Sw}, \mathrm{STz}^{\prime}, \mathrm{qt}\right) \\
& \geq \min \left\{\mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{Sw}, \mathrm{t}\right) * \mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{STz}^{\prime}, \mathrm{t}\right), \mathrm{M}_{2}(\mathrm{w}, \mathrm{TSw}, \mathrm{t}), \mathrm{M}_{2}\left(\mathrm{w}, \mathrm{Tz}^{\prime}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{TSw}, \mathrm{t}\right)\right\} \\
& =\min \left\{\mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{z}, \mathrm{t}\right)^{*} 1,1, \mathrm{M}_{2}\left(\mathrm{w}, \mathrm{Tz}^{\prime}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{w}, \mathrm{t}\right)\right\} \\
& \geq \mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{w}, \mathrm{t}\right)
\end{aligned}
$$

```
Now
    \(\mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{w}, \mathrm{qt}\right)=\mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{TSw}, \mathrm{qt}\right)\)
                \(\geq \min \left\{\mathrm{M}\left(\mathrm{z}^{\prime}, \mathrm{Sw}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{Sw}, \mathrm{STz}^{\prime}, \mathrm{t}\right), \mathrm{M}_{2}\left(\mathrm{w}, \mathrm{Tz}^{\prime}, \mathrm{t}\right) * \mathrm{M}_{2}(\mathrm{w}, \mathrm{TSw}, \mathrm{t}), \mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{STz}^{\prime}, \mathrm{t}\right)\right\}\)
                \(=\min \left\{\mathrm{M}_{1}\left(\mathrm{z}^{\prime}, \mathrm{z}, \mathrm{t}\right), \mathrm{M}_{1}\left(\mathrm{z}, \mathrm{z}^{\prime}, \mathrm{t}\right), \mathrm{M}_{2}(\mathrm{w}, \mathrm{Tz}, \mathrm{t})^{*} 1,1\right\}\)
        \(\geq \mathrm{M}_{1}\left(\mathrm{z}, \mathrm{z}^{\prime}, \mathrm{t}\right)\)
```

Hence

$$
\mathrm{M}_{1}\left(\mathrm{z}, \mathrm{z}^{\prime}, \mathrm{qt}\right) \geq \mathrm{M}_{2}\left(\mathrm{Tz}^{\prime}, \mathrm{w}, \mathrm{t}\right) \geq \mathrm{M}_{1}\left(\mathrm{z}, \mathrm{z}^{\prime}, \mathrm{t} / \mathrm{q}\right) \text { which is a contradiction. }
$$

Thus $\mathrm{z}=\mathrm{z}^{\prime}$.
So the point z is a unique fixed point of ST. Similarly, we prove the point w is also a unique point of TS.
This completes the proof.
Remark 2.14: If ( $\mathrm{X}, \mathrm{M}_{1}$, *) and ( $\mathrm{Y}, \mathrm{M}_{2}$, *) are the same fuzzy metric spaces, then by the above theorem 2.13, we get the following theorem, as corollary.

Corollary 2.15: Let ( $\mathrm{X}, \mathrm{M}, *$ *) be a complete fuzzy metric space. If S and T are mappings from X into itself satisfying the following conditions:

$$
\begin{aligned}
& \mathrm{M}(T x, T S y, t) \geq \min \{\mathrm{M}(\mathrm{x}, \mathrm{Sy}, \mathrm{t}), \mathrm{M}(\mathrm{Sy}, S T \mathrm{x}, \mathrm{t}), \mathrm{M}(\mathrm{y}, \mathrm{Tx}, \mathrm{t}) * \mathrm{M}(\mathrm{y}, \mathrm{TSy}, \mathrm{t}), \mathrm{M}(\mathrm{x}, \mathrm{STx}, \mathrm{t})\} \\
& \mathrm{M}(\mathrm{Sy}, \mathrm{STx}, \mathrm{t}) \geq \min \left\{\mathrm{M}(\mathrm{x}, S y, \mathrm{t}) * \mathrm{M}\left(\mathrm{x}, \mathrm{STx}_{\mathrm{x}}, \mathrm{t}\right), \mathrm{M}(\mathrm{y}, \mathrm{TSy}, \mathrm{t}), \mathrm{M}(\mathrm{y}, \mathrm{Tx}, \mathrm{t}), \mathrm{M}(\mathrm{Tx}, \mathrm{TSy}, \mathrm{t})\right\}
\end{aligned}
$$

for all x , y in X where $\mathrm{q}<1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y . Further $\mathrm{Tz}=\mathrm{w}$ and $\mathrm{Sw}=\mathrm{z}$.

## REFERENCES

[1] Cho Y.J., Fixed points in fuzzy metric spaces, J. Math. Vol.5, No.4, (1997), 949-962.
[2] Deng Z., Fuzzy pseudo metric space, J. Math. Anal. Appl., 86 (1982), 74-95.
[3] Erceg M.A, Metric space in fuzzy set theory, J. Math. Anal. Appl., (1979),205-230.
[4] Fang J.X., On fixed point theorems in fuzzy metric spaces, Fuzzy sets and systems, 46 (1992), 107-113.
[5] George A., Veeramani P., On some results in fuzzy metric spaces, fuzzy sets, and systems, 64 (1994), 395-399.
[6] Grabiec M., Fixed point in fuzzy metric space, Fuzzy sets and systems, 27(1988), 385-389.
[7] Kramosil I., Michalek J., Fuzzy metric and statistical metric spaces, Kybernetica, 11(1975), 326-334.
[8] Veerapandi T., Thiripura Sundari T., Paulraj Joseph J., Some fixed point theorems two metric spaces, International journal of Mathematical Archive-3(3) (2012), 826-837.
[9] Zadeh L.A., Fuzzy sets, Inform. and Control, 8 (1965), 338-353.
[10] Zaheer K. Ansari, Rajesh Shrivastava, Gunjan Ansari, Arun Garg, Some fixed point theorems in fuzzy 2metric and fuzzy 3-metric spaces, Int. J.Contemp. Math, Sciences, Vol.6, 2011, no. 46, 2291-2301.

Source of support: Nil, Conflict of interest: None Declared

