International Journal of Mathematical Archive-3(8), 2012, 3143-3155
IJMA Available online through www.ijma.info ISSN 2229 – 5046

SOME FIXED POINT THEOREMS IN TWO FUZZY METRIC SPACES

T. Veerapandi
Associate Professor of Mathematics, P. M. T College, Melaneelithanallur-627953, India

T. Thiripura Sundari*
Department of Mathematics, Sri K. G. S Arts College, Srivaikuntam, India

J. Paulraj Joseph
Associate Professor of Mathematics, Manonmaniam Sundaranar University, Tirunelveli, India

(Received on: 19-07-12; Revised & Accepted on: 11-08-12)

ABSTRACT

In this paper we prove some fixed point theorems for generalized contraction mappings in two complete fuzzy metric spaces.

Key words and Phrases: *fixed point, common fixed point and complete fuzzy metric space.*

AMS Mathematics Subject Classification: *47H10, 54H25.*

1. INTRODUCTION.

Fuzzy set was defined by Zadeh [9] in 1965, has lead to a rich growth of fuzzy mathematics. Kramosil and Michalek [7] introduced fuzzy metric space, George and Veeramani [5] modified the notion of fuzzy metric spaces with the help of continuous t-norms. Many authors namely Deng [2], Erceg [3] used the concept of fuzzy mathematics in different ways. Recently there are several authors proved many kinds of fixed point theorems in fuzzy metric spaces viz. [4] In this paper we prove some fixed point theorems in two complete fuzzy metric spaces for contractive type mappings and non-expansive mappings by generalizing the results of Veerapandi et al [8] on fuzzy metric space.
Now we begin with some known definitions and preliminary concepts.

Definition1.2: A fuzzy set A in X is a function with domain X and values in [0,1].

Definition1.3 [10]: A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0,1]$ is a continuous t-norm if it satisfies the following conditions:
(i) $*$ is commutative and associative,
(ii) $*$ is continuous,
(iii) $a * 1 = a$ for all $a \in [0,1]$,
(iv) $a * b \le c * d$ whenever $a \le c$ and $b \le d$, for each $a, b, c, d \in [0,1]$.

Examples of t-norm are $a * b = ab$ and $a * b = \min(a,b)$.

Definition1.4 [7]: A 3-tuple (X, M, $*$) is said to be a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times [0, \infty)$ satisfying the following conditions, for all $x, y, z \in X$ and $s, t > 0$,
(f-1) $M(x, y, t) > 0$,
(f-2) $M(x, y, t) = 1$ if and only if $x = y$,
(f-3) $M(x, y, t) = M(y, x, t)$,
(f- 4) $M(x, y, t) * M(y, z, s) \le M(x, z, t+s)$,
(f-5) $M(x, y, .) : [0, \infty) \rightarrow [0,1]$ is left continuous $\forall$ $x, y, z \in X$ and $t, s > 0$.

Then M is called a fuzzy metric on X and M(x, y, t) denotes the degree of nearness between x and y with respect to t.

Example1.5 [5]: (Induced fuzzy metric): Let (X, d) be a metric space, define $a * b = \min\{a,b\}$ for all $a, b \in [0,1]$ and M_d be fuzzy set on $X^2 \times [0, \infty)$ defined as,

Corresponding author: *T. Thiripura Sundari*,*
Department of Mathematics, Sri K. G. S Arts College, Srivaikuntam, India

$$M_d(x, y, t) = \frac{t}{t + d(x, y)}$$

for all x, y $\in$ X and t > 0. Then (X, M_d, ✻) is a fuzzy metric space. We call this fuzzy metric M_d as the standard intuitionistic fuzzy metric.

Definition 1.6[6]: Let (X, M, ✻) be a fuzzy metric space. Then

(i) A sequence $\{x_n\}$ in X is said to be convergent to a point x $\in$ X, if

$$\lim_{n\to\infty} M(x_n, x, t) = 1 \text{ for all } t > 0.$$

(ii) A sequence $\{x_n\}$ in X is said to be a Cauchy sequence if

$$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1 \text{ for all } t > 0 \text{ and } p > 0.$$

(iii) A fuzzy metric space is said to be complete if every Cauchy sequence is convergent to a point in it.

Remark 1.7: Since ✻ is continuous, it follows from (f-4), that the limit of the sequence in fuzzy metric space is uniquely determined.

Let (X, M, ✻) be a fuzzy metric space with the following conditions.

(f-6) $\lim_{t\to\infty} M(x, y, t) = 1$ for all x, y $\in$ X

Lemma 1.8[6]: For all x, y $\in$ X, M(x, y,.) is non-decreasing.

Lemma 1.9[1]: Let $\{x_n\}$ be a sequence in a fuzzy metric space (X, M, ✻) with the condition (f-6). If there exists a number q $\in$(0,1) such that

$$M(x_{n+2}, x_{n+1}, qt) \geq M(x_{n+1}, x_n, t) \quad \text{------------------- (f-7)}$$

for all t > 0 and n = 1,2,......, then $\{x_n\}$ is a cauchy sequence in X.

Lemma 1.10: If for all x, y$\in$X, t > 0 and for a number q $\in$ (0, 1),

$$M(x, y, qt) \geq M(x, y, t), \text{ then } x = y.$$

2. MAIN RESULTS

Theorem2.1. Let (X, M_1, ✻) and (Y, M_2, ✻) be two complete fuzzy metric spaces. If T is a mapping from X into Y and S is a mapping from Y into X , satisfying the following conditions:

$$M_2(Tx, TSy, qt) \geq \min\{M_1(x, Sy, t), M_2(y, Tx, t) ✻ M_2(y, TSy, t)\} \quad (1)$$

$$M_1(Sy, STx, qt) \geq \min\{M_1(x, Sy, t) ✻ M_1(x, STx, t), M_2(y, Tx, t)\} \quad (2)$$

for all x in X and y in Y where q < 1, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y. Further Tz = w and Sw = z.

Proof: Let x_0 be an arbitrary point in X. Define a sequence $\{x_n\}$ in X and $\{y_n\}$ in Y, as follows:

$$x_n = (ST)^n x_0, \; y_n = T(x_{n-1}) \text{ for } n = 1,2,\ldots .$$

We have

$$\begin{aligned}
M_1(x_n, x_{n+1}, qt) &= M_1((ST)^n x_0, (ST)^{n+1} x_0, qt) \\
&= M_1(S(T(ST)^{n-1} x_0, ST(ST)^n x_0, qt) \\
&= M_1(ST(x_{n-1}), STx_n, qt) \\
&= M_1(Sy_n, STx_n, qt) \\
&\geq \min\{M_1(x_n, Sy_n, t) ✻ M_1(x_n, STx_n, t), M_2(y_n, Tx_n, t)\} \text{ (since by (2))} \\
&= \min\{M_1(x_n, x_n, t) ✻ M_1(x_n, x_{n+1}, t), M_2(y_n, y_{n+1}, t)\} \\
&= \min\{M_1(x_n, x_{n+1}, t), M_2(y_n, y_{n+1}, t)\} \\
&\geq M_2(y_n, y_{n+1}, t)
\end{aligned}$$

Now

$$\begin{aligned} M_2(y_n, y_{n+1}, t) &= M_2(Tx_{n-1}, Tx_n, t) \\ &= M_2(Tx_{n-1}, TSy_n, t) \\ &\geq \min\{M_1(x_{n-1}, Sy_n, t/q), M_2(y_n, Tx_{n-1}, t/q) * M_2(y_n, TSy_n, t/q)\} \quad \text{(since by (1))} \\ &= \min\{M_1(x_{n-1}, x_n, t/q), M_2(y_n, y_n, t/q) * M_2(y_n, y_{n+1}, t/q)\} \\ &\geq M_1(x_{n-1}, x_n, t/q) \end{aligned}$$

Hence

$$\begin{aligned} M_1(x_n, x_{n+1}, qt) &\geq M_2(y_n, y_{n+1}, t) \\ &\geq M_1(x_{n-1}, x_n, t/q) \\ &\vdots \\ &\geq M_1(x_0, x_1, t/q^{2n-1}) \to 1 \text{ as } n\to\infty \text{ (since } q<1) \end{aligned}$$

Thus $\{x_n\}$ is a Cauchy sequence in X. Since $(X, M_1, *)$ is complete, it converges to a point z in X. Similarly, we can prove that the sequence $\{y_n\}$ is also a Cauchy sequence in Y and it converges to a point w in Y.

Now we prove $Tz = w$

Suppose $Tz \neq w$.

We have

$$\begin{aligned} M_2(Tz, w, qt) &= \lim_{n\to\infty} M_2(Tz, y_{n+1}, qt) \\ &= \lim_{n\to\infty} M_2(Tz, TSy_n, qt) \\ &\geq \lim_{n\to\infty} \min\{M_1(z, Sy_n, t), M_2(y_n, Tz, t) * M_2(y_n, TSy_n, t)\} \\ &= \lim_{n\to\infty} \min\{M_1(z, x_n, t), M_2(y_n, Tz, t) * M_2(y_n, y_{n+1}, t)\} \\ &= \min\{M_1(z, z, t), M_2(w, Tz, t) * M_2(w, w, t)\} \\ &= \min\{1, M_2(w, Tz, t) * 1\} \\ &\geq M_2(Tz, w, t) \text{ (since } q<1\text{), which is a contradiction.} \end{aligned}$$

Thus $Tz = w$.

Now we prove $Sw = z$.

Suppose $Sw \neq z$.

We have

$$\begin{aligned} M_1(Sw, z, qt) &= \lim_{n\to\infty} M_1(Sw, x_{n+1}, qt) \\ &= \lim_{n\to\infty} M_1(Sw, STx_n, qt) \\ &\geq \lim_{n\to\infty} \min\{M_1(x_n, Sw, t) * M_1(x_n, STx_n, t), M_2(w, Tx_n, t)\} \\ &= \lim_{n\to\infty} \min\{M_1(x_n, Sw, t) * M_1(x_n, x_{n+1}, t), M_2(w, y_{n+1}, t)\} \\ &= \min\{M_1(z, Sw, t) * 1, 1\} \\ &\geq M_1(z, Sw, t) \text{ (since } q<1\text{), which is a contradiction.} \end{aligned}$$

Thus $Sw = z$.

Therefore we have $STz = Sw = z$ and $TSw = Tz = w$. Thus the point z is a fixed point of ST and the point w is a fixed point of TS.

Uniqueness: let z' be another fixed point of ST such that $z = z'$.

We have

$$\begin{aligned} M_1(z, z', qt) &= M_1(STz, STz' qt) \\ &\geq \min\{M_1(z', STz, t) * M_1(z', STz', t), M_2(Tz, Tz', t)\} \\ &= \min\{M_1(z', z, t), M_2(Tz, Tz', t)\} \\ &\geq M_2(Tz, Tz', t) \end{aligned}$$

Also we have

$$\begin{aligned} M_2(Tz, Tz', t) &= M_2(Tz', TSTz', t) \\ &\geq \min\{M_1(z, STz', t/q), M_2(Tz', Tz, t/q) * M_2(Tz', TSTz', t/q)\} \end{aligned}$$

$= \min \{ M_1(z',z,t/q) , M_2(Tz, Tz',t/q)\}$
$\geq M_1(z, z',t/q)$

Hence

$$M_1(z,z',qt) \geq M_2(Tz,Tz',t) \geq M_1(z, z',t/q) \text{ (since } q<1),$$

which is a contradiction.

Thus $z = z'$.

So the point z is the unique fixed point of ST. Similarly, we prove the point w is also a unique fixed point of TS.

This completes the proof.

Remark 2.2: If $(X, M_1, *)$ and $(Y, M_2, *)$ are the same fuzzy metric spaces, then by the above theorem 2.1, we get the following theorem, as corollary.

Corollary 2.3: Let $(X,M, *)$ be a complete fuzzy metric space. If S and T are mappings from X into itself satisfying the following conditions:

$$M (Tx,TSy, qt) \geq \min\{M(x, Sy, t), M(y,Tx, t) * M(y,TSy, t)\}$$

$$M (Sy, STx, qt) \geq \min \{M(x, Sy, t) * M(x, STx, t), M(y,Tx, t)\}$$

for all x, y in X where $q < 1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in X. Further $Tz = w$ and $Sw = z$.

Theorem 2.4: Let $(X, M_1, *)$ and $(Y, M_2, *)$ be two complete fuzzy metric spaces. If T is a mapping from X into Y and S is a mapping from Y into X, satisfying following conditions:

$$M_2(Tx,TSy, qt) \geq \min\{M_1(x, Sy, t), M_2(y,Tx, t), M_2(y,Tx, t) * M_2(y,TSy, t)\} \quad (1)$$

$$M_1(Sy,STx, qt) \geq \min\{M_2(y,Tx, t), M_1(x,Sy, t), M_1(x,Sy, t) * M_1(x,STx, t)\} \quad (2)$$

for all x in X and y in Y where $q < 1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y. Further $Tz = w$ and $Sw = z$.

Proof: Let x_0 be an arbitrary point in X. Define a sequence $\{x_n\}$in X and $\{y_n\}$ in Y, as follows:

$$x_n = (ST)^n x_0, \quad y_n = T(x_{n-1}) \text{ for } n = 1,2,\dots .$$

we have

$$\begin{aligned} M_1(x_n ,x_{n+1},qt) &= M_1 ((ST)^n x_0 , (ST)^{n+1} x_0 ,qt) \\ &= M_1(S(T(ST)^{n-1} x_0 , ST(ST)^n x_0 ,qt) \\ &= M_1(ST(x_{n-1}) , STx_n, qt) \\ &= M_1(Sy_n , STx_n ,qt) \\ &\geq \min\{M_2(y_n,Tx_n,t), M_1(x_n, Sy_n,t), M_1(x_n,Sy_n,t) * M_1(x_n,STx_n,t)\} \\ &= \min\{M_2(y_n,y_{n+1},t), M_1(x_n , x_n,t) , M_1(x_n,x_n,t) * M_1(x_n,x_{n+1},t)\} \\ &= \min\{M_2(y_n,y_{n+1},t), 1,1 * M_1(x_n , x_{n+1},t)\} \\ &\geq M_2(y_n,y_{n+1},t) \end{aligned}$$

Now

$$\begin{aligned} M_2(y_n,y_{n+1},qt) &= M_2(Tx_{n-1},Tx_n,qt) \\ &= M_2(Tx_{n-1},TSy_n,qt) \\ &\geq \min\{M_1(x_{n-1},Sy_n, t),M_2(y_n,Tx_{n-1}, t),M_2(y_n,Tx_{n-1}, t) * M_2(y_n,TSy_n, t)\} \\ &= \min\{M_1(x_{n-1},x_n, t), M_2(y_n,y_n, t), M_2(y_n,y_n, t)*M_2(y_n ,y_{n+1}, t)\} \\ &= \min\{ M_1(x_{n-1},x_n,t), 1 , 1 * M_2(y_n ,y_{n+1},t)\} \\ &= M_1(x_{n-1}, x_n,t) \end{aligned}$$

$M_1(x_n, x_{n+1}, t) \geq M_2(y_n, y_{n+1}, t)$
$\geq M_1(x_{n-1}, xn, t/q)$
$\vdots$
$\geq M_1(x_0, x_1, t/q^{2n-1}) \to 0$ as $n \to \infty$ (since $q<1$)

Thus $\{x_n\}$ is a Cauchy sequence in X. Since $(X, M_1, *)$ is complete, it converges to a point z in X. Similarly, we can prove that the sequence $\{y_n\}$ is also a Cauchy sequence in Y and it converges to a point w in Y.

Now we prove Tz = w

Suppose $Tz \neq w$.

We have

$$\begin{aligned} M_2(Tz, w, qt) &= \lim_{n\to\infty} M_2(Tz, y_{n+1}, qt) \\ &= \lim_{n\to\infty} M_2(Tz, TSy_n, qt) \\ &\geq \lim_{n\to\infty} \min\{M_1(z,Sy_n,t), M_2(y_n,Tz,t), M_2(y_n,Tz,t) * M_2(y_n,TSy_n,t)\} \\ &= \lim_{n\to\infty} \min\{M_1(z,x_n,t), M_2(y_n,Tz,t), M_2(y_n,Tz,t) * M_2(y_n,y_{n+1},t)\} \\ &= \min\{M_1(z,z,t), M_2(w,Tz,t), M_2(w,Tz.t) * M_2(w,w,t)\} \\ &= \min\{1, M_2(w,Tz,t), M_2(w,Tz,t) * 1\} \\ &\geq M_2(w,Tz,t) \text{ (since } q<1) \text{ which is a contradiction} \end{aligned}$$

Thus Tz = w .

Now we prove Sw = z.

Suppose $Sw \neq z$.

We have

$$\begin{aligned} M_1(Sw,z,qt) &= \lim_{n\to\infty} M_1(Sw,x_{n+1},qt) \\ &= \lim_{n\to\infty} M_1(Sw,STx_n,qt) \\ &\geq \lim_{n\to\infty} \min\{M_2(w,Tx_n,t), M_1(x_n,Sw,t), M_1(x_n,Sw,t) M_1(x_n,STx_n,t)\} \\ &= \lim_{n\to\infty} \min\{M_2(w,y_{n+1},t), M_1(x_n,Sw,t), M_1(x_n,Sw,t) * M_1(x_n,x_{n+1},t)\} \\ &= \min\{M_2(w,w,t), M_1(z,w,t), M_1(z,Sw,t) * M_1(z,z,t)\} \\ &\geq M_1(Sw,z,t) \text{ (since } q<1) \text{ which is a contradiction.} \end{aligned}$$

Thus Sw = z.

Therefore we have STz = Sw = z and TSw = Tz = w. Thus the point z is a fixed point of ST and the point w is a fixed point of TS.

Uniqueness: let z′ be another fixed point of ST such that z = z′.

We have

$$\begin{aligned} M_1(z',z,qt) &= M_1(STz', STz,qt) \\ &\geq \min\{M_2(Tz',Tz,t), M_1(z,STz',t), M_1(z,STz',t) * M_1(z.STz,t)\} \\ &\geq \min\{M_2(Tz',Tz,t), M_1(z, z',t), M_1(z, z',t)\} \\ &\geq M_2(Tz',Tz,t) \end{aligned}$$

Also we have

$$\begin{aligned} M_2(Tz',Tz,qt) &= M_2(Tz',TSTz,qt) \\ &\geq \min\{M_1(z',STz,t), M_2(Tz,Tz',t), M_2(Tz,Tz',t) * M_2(Tz,TSTz,t)\} \\ &= \min\{M_1(z',z,t), M_2(Tz,Tz',t), M_2(Tz,Tz',t)\} \\ &\geq M_1(z,z',t) \end{aligned}$$

(i.e) $M_2(Tz',Tz,t) \geq M_1(z,z',t/q)$

Hence

$M_1(z',z, qt) \geq M_2(Tz',Tz, t) \geq M_1(z,z', t/q)$ (since $q<1$) which is a contradiction.

Thus $z = z'$.

So the point z is the unique fixed point of ST. Similarly, we prove the point w is also a unique fixed point of TS. This completes the proof.

Remark 2.5: If $(X, M_1, *)$ and $(Y,M_2, *)$ are the same fuzzy metric spaces, then by the above theorem 2.4, we get the following theorem, as corollary.

Corollary 2.6: Let $(X, M, *)$ be a complete fuzzy metric space. If S and T are mappings from X into itself satisfying the following conditions:

$$M(Tx,TSy, qt) \geq \min\{M(x,Sy, t), M(y,Tx, t), M(y,Tx, t) * M(y,TSy, t)\}$$
$$M(Sy,STx, qt) \geq \min\{M(y,Tx, t), M(x,Sy, t), M(x,Sy, t) * M(x,STx, t)\}$$

for all x, y in X where $q < 1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in X. Further $Tz = w$ and $Sw = z$.

Theorem 2.7: Let $(X, M_1, *)$ and $(Y,M_2, *)$ be two complete fuzzy metric spaces. If T is a mapping from X into Y and S is a mapping from Y into X, satisfying the following conditions:

$$M_2(Tx,TSy,qt) \geq \min\{M_1(x,Sy,t), M_2(y,Tx,t), M_2(y,TSy,t), M_1(x,STx,t)\} \quad (1)$$
$$M_1(Sy,STx,qt) \geq \min\{M_2(y,Tx,t), M_1(x,Sy,t), M_1(x,STx,t), M_2(Tx,TSy,t)\} \quad (2)$$

for all x, y in X where $q < 1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y. Further $Tz = w$ and $Sw = z$.

Proof: Let x_0 be an arbitrary point in X. Define a sequence $\{x_n\}$in X and $\{y_n\}$ in Y, as follows:

$$x_n = (ST)^n x_0 ,\ y_n = T(x_{n-1}) \text{ for } n = 1,2,\dots .$$

we have

$$\begin{aligned} M_1(x_n ,x_{n+1},qt) &= M_1 ((ST)^n x_0 , (ST)^{n+1} x_0 ,qt) \\ &= M_1(S(T(ST)^{n-1} x_0 , ST(ST)^n x_0 ,qt) \\ &= M_1(ST(x_{n-1}) , STx_n, qt) \\ &= M_1(Sy_n , STx_n ,qt) \\ &\geq \min\{M_2(y_n,Tx_n,t), M_1(x_n,Sy_n,t), M_1(x_n,STx_n,t), M_2(Tx_n,TSy_n,t)\} \\ &= \min\{M_2(y_n, y_{n+1},t) , M_1(x_n, x_n,t), M_1(x_n, x_{n+1},t), M_2(y_{n+1}, y_{n+1},t)\} \\ &\geq M_2(y_n , y_{n+1},t) \end{aligned}$$

We have

$$\begin{aligned} M_2(y_n ,y_{n+1},qt) &= M_2(Tx_{n-1},Tx_n,qt) \\ &= M_2(Tx_{n-1},TSy_n,qt) \\ &\geq \min\{M_1(x_{n-1},Sy_n,t),M_2(y_n,Tx_{n-1},t),M_2(y_n,TSy_n,t),M_1(x_{n-1},STx_{n-1},t)\} \\ &= \min\{M_1(x_{n-1},x_n,t), M_2(y_n,y_n,t), M_2(y_n ,y_{n+1},t), M_1(x_{n-1},x_n,t)\} \\ &= \min\{ M_1(x_{n-1},x_n,t), 1, M_2(y_n ,y_{n+1},t), M_1(x_{n-1},x_n,t)\} \\ &\geq M_1(x_{n-1}, x_n,t) \end{aligned}$$

(i.e), $M_2(y_n ,y_{n+1},t) \geq M_1(x_{n-1}, x_n,t/q)$

Hence

$$\begin{aligned} M_1(x_n,x_{n+1},qt) &\geq M_2 (y_n,y_{n+1},t) \\ &\geq M_1 (x_{n-1}, x_n,t/q) \\ &\vdots \\ &\geq M_1(x_0, x_1,t/q^{2n-1}) \to 1 \text{ as } n\to\infty \text{ (since } q<1) \end{aligned}$$

Thus $\{x_n\}$ is a Cauchy sequence in X. Since $(X, M_1, *)$ is complete, it converges to a point z in X. Similarly, we can prove that the sequence $\{y_n\}$ is also a Cauchy sequence in Y and it converges to a point w in Y.

Now we prove $Tz = w$

Suppose $Tz \neq w$.

We have

$$M_2(Tz, w, qt) = \lim_{n\to\infty} M_2(Tz, y_{n+1}, qt)$$
$$= \lim_{n\to\infty} M_2(Tz, TSy_n, qt)$$
$$\geq \lim_{n\to\infty} \min\{M_1(z,Sy_n,t), M_2(y_n,Tz,t), M_2(y_n,TSy_n,t), M_1(z,STz,t)\}$$
$$= \lim_{n\to\infty} \min\{M_1(z,x_n,t), M_2(y_n,Tz,t), M_2(y_n,y_{n+1},t), M_1(z,STz,t)\}$$
$$= \min\{M_1(z,z,t), M_2(w,Tz,t), M_2(w,w,t), M_1(z,STz,t)\}$$
$$= \min\{1, M_2(w,Tz,t), 1, M_1(z,STz,t)\}$$
$$\geq M_1(z,STz,t)$$

Also

$$M_1(z, STz, qt) = \lim_{n\to\infty} M_1(x_n, STz, qt)$$
$$= \lim_{n\to\infty} M_1(Sy_n, STz, qt)$$
$$\geq \lim_{n\to\infty} \min\{M_2(y_n,Tz,t), M_1(z, Sy_n), M_1(z, STz,t), M_2(Tz,TSy_n,t)\}$$
$$= \lim_{n\to\infty} \min\{M_2(y_n,Tz,t), M_1(z, x_n,t), M_1(z,STz,t), M_2(Tz,y_{n+1},t)\}$$
$$= \min\{M_2(w,Tz,t), M_1(z,z,t), M_1(z,STz,t), M_2(Tz,w,t)\}$$
$$= \min\{M_2(w,Tz,t), 1, M_1(z,STz,t), M_2(Tz,w,t)\}$$
$$\geq M_2(Tz,w,t)$$

(i.e), $M_1(z,STz,t) \geq M_2(Tz,w,t/q)$

hence

$M_2(Tz,w,qt) \geq M_2(Tz,w,t/q)$, which is a contradiction. (Since $q<1$)

Thus $Tz = w$.

Now we prove $Sw = z$.

Suppose $Sw \neq z$.

Then we have

$$M_1(Sw,z,qt) = \lim_{n\to\infty} M_1(Sw,x_{n+1},qt)$$
$$= \lim_{n\to\infty} M_1(Sw,STx_n,qt)$$
$$\geq \lim_{n\to\infty} \min\{M_2(w,Tx_n,t), M_1(x_n,Sw,t), M_1(x_n,STx_n,t), M_2(Tx_n,TSw,t)\}$$
$$= \lim_{n\to\infty} \min\{M_2(w,y_{n+1},t), M_1(x_n,Sw,t), M_1(x_n,x_{n+1},t), M_2(y_{n+1},TSw,t)\}$$
$$= \min\{ M_2(w,w,t), M_1(z,Sw,t), M_1(z,z,t), M_2(w,TSw,t)\}$$
$$= \min\{1, M_1(z,Sw,t), 1, M_2(w,TSw,t)\}$$
$$\geq M_2(w,TSw,t)$$

$$M_2(w,TSw,qt) = \lim_{n\to\infty} M_2(y_{n+1},TSw,qt)$$
$$= \lim_{n\to\infty} M_2(Tx_n,TSw,qt)$$
$$\geq \lim_{n\to\infty} \min\{M_1(x_n,Sw,t), M_2(w,Tx_n,t), M_2(w,TSw,t), M_1(x_n,STx_n,t)\}$$
$$= \min\{ M_1(z,Sw,t), 1, M_2(w,TSw,t), 1\}$$
$$\geq M_1(z,Sw,t)$$

(i.e), $M_2(w,TSw,t) \geq M_1(Sw,z,t/q)$

Hence

$M_1(Sw,z,qt) \geq M_2(w,TSw,t) \geq M_1(Sw,z,t/q)$

Which is a contradiction. (since $q<1$)

© 2012, IJMA. All Rights Reserved

Thus Sw = z.

Therefore we have STz = Sw = z and TSw = Tz = w. Thus the point z is a fixed point of ST and the point w is a fixed point of TS.

Uniqueness: Let z′ be the another fixed point of ST such that $z \neq z'$.

Now

$$\begin{aligned} M_1(z,z',qt) &= \lim_{n\to\infty} M_1(Sw,STz',qt) \\ &\geq \lim_{n\to\infty} \min\{M_2(w,Tz',t), M_1(z',Sw,t), M_1(z',STz',t), M_2(Tz',w,t)\} \\ &= \min\{M_2(w,Tz',t), M_1(z',z,t), 1, M_2(Tz',w,t)\} \\ &\geq M_2(Tz',w,t) \end{aligned}$$

Also

$$\begin{aligned} M_2(Tz',w,qt) &= \lim_{n\to\infty} M_2(Tz',y_{n+1},qt) \\ &= \lim_{n\to\infty} M_2(Tz',TSy_n,qt) \\ &\geq \lim_{n\to\infty} \min\{M_1(z',Sy_n,t), M_2(y_n,Tz',t), M_2(y_n,TSy_n,t), M_1(z',STz',t)\} \\ &= \min\{M_1(z',z,t), M_2(w,Tz',t), 1, 1\} \\ &\geq M_1(z',z,t) \end{aligned}$$

(i.e), $M_2(Tz',w,t) \geq M_1(z',z,t/q)$

Therefore we have

$$M_1(z, z',qt) \geq M_2(Tz',w,t) \geq M_1(z,z',t)$$

Which is a contradiction. (since $q<1$)

Thus $z = z'$.

So the point z is the unique fixed point of ST. Similarly, we prove the point w is also a unique fixed point of TS. .

This completes the proof.

Remark 2.8: If $(X,M_1,*)$ and $(Y,M_2,*)$ are the same fuzzy metric spaces, then by the above theorem 2.7, we get the following theorem, as corollary.

Corollary 2.9: Let $(X,M,*)$ be a complete fuzzy metric space. If S and T are mappings from X into itself satisfying the following conditions:

$$M(Tx,TSy,qt) \geq \min\{M(x,Sy,t), M(y,Tx,t), M(y,TSy,t), M(x,STx,t)\}$$
$$M(Sy,STx,qt) \geq \min\{M(y,Tx,t), M(x,Sy,t), M(x,STx,t), M(Tx,TSy,t)\}$$

for all x, y in X where $q < 1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y. Further Tz = w and Sw = z.

Theorem 2.10: Let $(X,M_1,*)$ and $(Y,M_2,*)$ be two complete fuzzy metric spaces. If T is a mapping from X into Y and S is a mapping from Y into X , satisfying following conditions:

$$M_2(Tx, TSy, qt) \geq \min\{M_1(x, Sy, t), M_2(y, Tx, t), M_2(y, TSy, t), M_1(x, STx, t), M_1(Sy, STx, t)\} \quad (1)$$
$$M_1(Sy, STx, qt) \geq \min\{M_2(y, Tx, t), M_1(x, Sy, t), M_1(x, STx, t), M_2(Tx, TSy, t), M_2(y, TSy, t)\} \quad (2)$$

for all x in X and y in Y where $q < 1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y. Further Tz = w and Sw = z.

Proof: Let x_0 be an arbitrary point in X. Define a sequence $\{x_n\}$in X and $\{y_n\}$ in Y, as follows:

$$x_n = (ST)^n x_0, \; y_n = T(x_{n-1}) \text{ for } n = 1,2,\dots .$$

we have

$$\begin{aligned} M_1(x_n, x_{n+1}, qt) &= M_1((ST)^n x_0, (ST)^{n+1} x_0, qt) \\ &= M_1(S(T(ST)^{n-1} x_0, ST(ST)^n x_0, qt) \\ &= M_1(ST(x_{n-1}), STx_n, qt) \\ &= M_1(Sy_n, STx_n, qt) \\ &\geq \min\{M_2(y_n,Tx_n,t), M_1(x_n,Sy_n,t), M_1(x_n,STx_n,t), M_2(Tx_n,TSy_n), M_2(y_n,TSy_n,t)\} \\ &= \min\{M_2(y_n,y_{n+1},t), M_1(x_n,x_n,t), M_1(x_n,x_{n+1},t), M_2(y_{n+1},y_{n+1},t), M_2(y_n,y_{n+1},t)\} \\ &= \min\{M_2(y_n,y_{n+1},t), 1, M_1(x_n,x_{n+1},t), 1, M_2(y_n,y_{n+1},t)\} \\ &\geq M_2(y_n, y_{n+1},t) \end{aligned}$$

Also we have

$$\begin{aligned} M_2(y_n,y_{n+1},qt) &= M_2(Tx_{n-1},Tx_n,qt) \\ &= M_2(Tx_{n-1}, TSy_n,qt) \\ &\geq \min\{M_1(x_{n-1},Sy_n,t), M_2(y_n,Tx_{n-1},t), M_2(y_n,TSy_n,t), M_1(x_{n-1},STx_{n-1},t), M_1(Sy_n,STx_{n-1},t)\} \\ &= \min\{M_1(x_{n-1},x_n,t), M_2(y_n,y_n,t), M_2(y_n,y_{n+1},t), M_1(x_{n-1},x_n,t), M_1(x_{n-1},x_n,t\}\} \\ &= \min\{M_1(x_{n-1},x_n,t), 1, M_2(y_n,y_{n+1},t), M_1(x_{n-1},x_n,t), M_1(x_{n-1},x_n,t\}\} \\ &\geq M_1(x_{n-1},x_n,t) \end{aligned}$$

Now

$$\begin{aligned} M_1(x_n, x_{n+1},qt) &\geq M_2(y_n,y_{n+1},t) \\ &\geq M(x_{n-1}, x_n,t/q) \\ &\vdots \\ &\geq M_1(x_0,x_1,t/q^{2n-1}) \to 1 \text{ as } n\to\infty \text{ (since } q<1) \end{aligned}$$

Thus $\{x_n\}$ is a Cauchy sequence in X. Since $(X,M_1, *)$ is complete, it converges to a point z in X. Similarly, we can prove that the sequence $\{y_n\}$ is also a Cauchy sequence in Y and it converges to a point w in Y.

Now we prove Tz = w

We have

$$\begin{aligned} M_2(Tz,w,qt) &= \lim_{n\to\infty} M_2(Tz,y_{n+1},qt) \\ &= \lim_{n\to\infty} M_2(Tz, TSy_n,qt) \\ &\geq \lim_{n\to\infty} \min\{M_1(z,Sy_n,t), M_2(y_n,Tz,t), M_2(y_n,TSy_n,t), M_1(z,STz,t), M_1(Sy_n,STz,t)\} \\ &= \lim_{n\to\infty} \min\{M_1(z, x_n,t), M_2(y_n, Tz,t), M_2(y_n, y_{n+1},t), M_1(z,STz,t), M_1(x_n,STz,t)\} \\ &= \min\{1, M_2(w, Tz,t), 1, M_1(z,STz,t), M_1(z,STz,t)\} \\ &\geq M_1(z,STz,t) \end{aligned}$$

$$\begin{aligned} M_1(z,STz,qt) &= \lim_{n\to\infty} M_1(x_n, STz, qt) \\ &= \lim_{n\to\infty} M_1(Sy_n, STz, qt) \\ &\geq \lim_{n\to\infty} \min\{M_2(y_n, Tz, t), M_1(z, Sy_n, t), M_1(z, STz, t), M_2(Tz, TSy_n, t), M_2(y_n, TSy_n, t)\} \\ &= \lim_{n\to\infty} \min\{M_2(y_n, Tz, t), M_1(z, x_n, t), M_1(z, STz, t), M_2(Tz,y_{n+1},t), M_2(y_n, y_{n+1},t)\} \\ &= \min\{M_2(w, Tz, t), 1, M_1(z, STz, t), M_2(Tz, w, t), 1\} \\ &\geq M_2(Tz,w,t) \end{aligned}$$

Hence

$$M_2(Tz, w, qt) \geq M_2(Tz, w, t/q)$$

Thus Tz = w.

Now we prove Sw = z.

Suppose $Sw \neq z$.

$$\begin{aligned} M_1(Sw, z, qt) &= \lim_{n\to\infty} M_1(Sw, x_{n+1}, qt) \\ &= \lim_{n\to\infty} M_1(Sw, STx_n, qt) \end{aligned}$$

$\geq \lim_{n\to\infty} \min\{M_2(w,Tx_n,t), M_1(x_n,Sw,t), M_1(x_n,STx_n,t), M_2(Tx_n,TSw,t), M_2(y_n,TSx_n,t)\}$
$= \lim_{n\to\infty} \min\{M_2(w,y_{n+1},t), M_1(x_n,Sw,t), M_1(x_n,x_{n+1},t), M_1(y_{n+1},TSw,t), M_2(y_n,y_{n+1}.t)\}$
$= \min\{1, M_1(z,Sw,t), 1 , M_1(w,TSw,t), 1\}$
$\geq M_2(w,TSw,t)$

Now

$M_2(w,TSw,qt) = \lim_{n\to\infty} M_2(y_{n+1},TSw,qt)$
$= \lim_{n\to\infty} M_2(Tx_n,TSw,qt)$
$\geq \lim_{n\to\infty} \min\{M_1(x_n,Sw,t), M_2(w,Tx_n,t), M_2(w,TSw,t), M_1(x_n,STx_n,t), M_1(Sw,STx_n,t)\}$
$= \lim_{n\to\infty} \min\{M_1(x_n,Sw,t), M_2(w,y_{n+1},t), M_2(w,TSw,t), M_1(x_n,x_{n+1},t), M_1(Sw,x_{n+1},t)\}$
$= \min\{M_1(z,Sw,t), 1 , M_2(w,TSw,t), 1 , M_1(Sw,z,t)\}$
$\geq M_1(Sw,z,t)$

Hence

$M_1(Sw,z,qt) \geq M_2(w,TSw,t)$
$\geq M_1(Sw,z,t/q)$

Which is a contradiction. (since $q< 1$)

Thus $Sw = z$.

Therefore we have $STz = Sw = z$ and $TSw = Tz = w$. Thus the point z is a fixed point of ST and the point w is a fixed point of TS.

Uniqueness: Let z' be the another fixed point of ST such that $z \neq z'$.

Now

$M_1(z,z',qt) = M_1(Sw,STz',qt)$
$\geq \min\{M_2(w,Tz',t), M_1(z',Sw,t), M_1(z',STz',t), M_2(Tz',TSw,t), M_2(w,TSw,t)\}$
$= \min\{M_2(w,Tz',t), M_1(z',z,t), 1 , M_2(Tz',w,t), 1 \}$
$\geq M_2(Tz',w,t)$

$M_2(Tz',w,qt) = M_2(Tz',TSw,t)$
$\geq \min\{M_1(z',Sw,t), M_2(w,Tz',t), M_1(z',TSz',t), M_1(z',z,t), M_1(z,STz',t)\}$
$= \min\{M_1(z',z,t), M_2(w,Tz',t), 1, M_1(z',z,t), M_1(z, z',t)\}$
$\geq M_1(z, z',t)$

Hence

$M_1(z,z',qt) \geq M_2(Tz',w,t)$
$\geq M_1(z,z',t/q)$

Which is a contradiction. (since $q<1$)

Thus $z = z'$.

So the point z is the unique fixed point of ST. Similarly, we prove the point w is also a unique fixed point of TS.

This completes the proof.

Remark 2.11: If $(X,M_1, *)$ and $(Y,M_2, *)$ are the same fuzzy metric spaces, then by the above theorem 2.10, we get the following theorem, as corollary.

Corollary 2.12: Let $(X, M, *)$ be a complete fuzzy metric space. If S and T are mappings from X into itself satisfying the following conditions:

$M(Tx, TSy, qt) \geq \min\{M(x, Sy, t), M(y, Tx, t), M(y, TSy, t), M(x, STx, t),M(Sy, STx, t)\}$

$M(Sy, STx, qt) \geq \min\{M(y, Tx, t), M(x, Sy, t), M(x, STx, t), M(Tx, TSy, t), M(y, TSy, t)\}$

for all x, y in X where q < 1, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y. Further Tz = w and Sw = z.

Theorem 2.13: Let $(X,M_1, *)$ and $(Y,M_2, *)$ be two complete fuzzy metric spaces. If T is a mapping from X into Y and S is a mapping from Y into X , satisfying the following conditions:

$$M_2(Tx, TSy, t) \geq \min\{M_1(x, Sy, t), M_1(Sy, STx, t), M_2(y, Tx, t) * M_2(y, TSy, t), M_1(x, STx, t)\} \quad (1)$$

$$M_1(Sy, STx, t) \geq \min\{M_1(x, Sy, t) * M_1(x, STx, t), M_2(y, TSy, t), M_2(y, Tx, t), M_2(Tx, TSy, t)\} \quad (2)$$

for all x in X and y in Y where q < 1, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y.

Further Tz = w and Sw = z.

Proof. Let x_0 be an arbitrary point in X. Define a sequence $\{x_n\}$in X and $\{y_n\}$ in Y, as follows:

$$x_n = (ST)^n x_0, \quad y_n = T(x_{n-1}) \text{ for } n = 1,2,\dots .$$

We have

$$\begin{aligned} M_1(x_n, x_{n+1}, qt) &= M_1((ST)^n x_0, (ST)^{n+1} x_0, qt) \\ &= M_1(S(T(ST)^{n-1} x_0, ST(ST)^n x_0, qt) \\ &= \mathbf{M_1(ST(x_{n-1}), STx_n, qt)} \\ &= M_1(Sy_n, STx_n, qt) \\ &\geq \min\{M_1(x_n,Sy_n,t) * M(x_n,STx_n,t), M_2(y_n,TSy_n,t), M_2(y_n,Tx_n,t), M_2(Tx_n,TSy_n,t)\} \\ &= \min\{M_1(x_n,x_n,t) * M_1(x_n,x_{n+1},t), M_2(y_n,y_{n+1},t), M_2(y_n,y_{n+1},t), M_2(y_{n+1},y_{n+1},t)\} \\ &= \min\{1 * M_1(x_n,x_{n+1},t), M_2(y_n,y_{n+1},t), M_2(y_n,y_{n+1},t), 1\} \\ &\geq M_2(y_n,y_{n+1},t) \end{aligned}$$

Now

$$\begin{aligned} M_2(y_n,y_{n+1},qt) &= M_2(Tx_{n-1},Tx_n,qt) \\ &= M_2(Tx_{n-1},TSy_n,qt) \\ &\geq \min\{M_1(x_{n-1},Sy_n,t),M_1(Sy_n,STx_{n-1},t),M_2(y_n,Tx_{n-1},t) * M_2(y_n,TSy_n,t), M_1(x_{n-1},STx_{n-1},t)\} \\ &= \min\{M_1(x_{n-1},x_n,t), 1, 1*M_2(y_n,y_{n+1},t), M_1(x_{n-1},x_n,t)\} \\ &\geq M_1(x_{n-1},x_n,t) \end{aligned}$$

Hence

$$\begin{aligned} M_1(x_n,x_{n+1},qt) &\geq M_2(y_n,y_{n+1},t) \\ &\geq M_1(x_{n-1},x_n,t/q) \\ &\vdots \\ &\geq M_1(x_0,x_1,t/q^{2n-1}) \to 1 \text{ as } n\to\infty \quad (\text{since } q < 1) \end{aligned}$$

Thus $\{x_n\}$ is a Cauchy sequence in $(X,M_1, *)$. Since $(X,M_1, *)$ is complete, it converges to a point z in X. Similarly, we can prove that the sequence $\{y_n\}$ is also a Cauchy sequence in $(Y,M_2, *)$. Since $(Y\ M_2, *)$ is complete, it converges to a point w in Y.

Now we prove Tz = w .

Suppose $Tz \neq w$.

We have

$$\begin{aligned} M_2(Tz,w,qt) &= \lim_{n\to\infty} M_2(Tz,y_{n+1},qt) \\ &= \lim_{n\to\infty} M_2(Tz,TSy_n,qt) \\ &\geq \lim_{n\to\infty} \min\{M_1(z,Sy_n,t),M_1(Sy_n,STz,t), M_2(y_n,Tz,t)*M_2(y_n,TSy_n,t), M_1(z,STz,t)\} \\ &= \lim_{n\to\infty} \min\{M_1(z,x_n,t), M_1(x_n,STz,t), M_2(y_n,Tz,t)*M_2(y_n,y_{n+1},t), M_1(z,STz,t)\} \\ &\geq M_1(z,STz,t) \end{aligned}$$

Now

$$
\begin{aligned}
M_1(z,STz,qt) &= \lim_{n\to\infty} M_1(x_n,STz,qt) \\
&= \lim_{n\to\infty} M_1(Sy_n,STz,qt) \\
&\geq \lim_{n\to\infty} \min\{M_1(z,Sy_n,t)*M_1(z,STz,t),M_2(y_n,,TSy_n,t),M_2(y_n,Tz,t), M_2(Tz,TSy_n,t)\} \\
&= \lim_{n\to\infty} \min\{M_1(z,x_n,t)*M_1(z,STz,t), M_2(y_n,y_{n+1},t), M_2(y_n,Tz,t), M_2(Tz, y_{n+1},t)\} \\
&= \min\{1*M_1(z,STz,t), 1, M_2(w,Tz,t), M_2(Tz, w,t)\} \\
&\geq M_2(Tz,w,t)
\end{aligned}
$$

Hence

$M_2(Tz,w,qt) \geq M_1(z,STz,t) \geq M_2(Tz,w,t/q)$ (since $q<1$) which is a contradiction.

Thus $Tz = w$.

Now we prove $Sw = z$.

Suppose $Sw \neq z$.

We have

$$
\begin{aligned}
M_1(Sw,z,qt) &= \lim_{n\to\infty} M_1(Sw,x_{n+1},qt) \\
&= \lim_{n\to\infty} M_1(Sw,STx_n,qt) \\
&\geq \lim_{n\to\infty} \min\{M_1(x_n,Sw,t)*M_1(x_n,STx_n,t), M_2(w,TSw,t), M_2(w,Tx_n,t), M_2(Tx_n,TSw,t)\} \\
&= \lim_{n\to\infty} \min\{M_1(x_n,Sw,t)*M_1(x_n,x_{n+1},t), M_2(w,TSw,t), M_2(w,y_{n+1},t), M_2(y_{n+1},w,t)\} \\
&\geq M_2(w,TSw,t)
\end{aligned}
$$

Now

$$
\begin{aligned}
M_2(w,TSw,qt) &= \lim_{n\to\infty} M_2(y_{n+1},TSw,qt) \\
&= \lim_{n\to\infty} M_2(Tx_n, TSw,qt) \\
&\geq \lim_{n\to\infty} \min\{M_1(x_n,Sw,t), M_1(Sw,STx_n,t), M_2(w,Tx_n,t)*M_2(w,TSw), M_1(x_n,Tx_n,t)\} \\
&= \lim_{n\to\infty} \max\{M_1(x_n,Sw,t),M_1(Sw,x_{n+1},t),M_2(w,y_{n+1},t)*M_2(w,TSw,t),M_1(x_n,y_{n+1},t)\} \\
&\geq M_1(z,Sw,t)
\end{aligned}
$$

Hence

$M_1(Sw,z,qt) \geq M_2(w,TSw,t) \geq M_1(z,Sw,t/q)$ (since $q<1$) which is a contradiction.

Thus $Sw = z$.

We have $STz = Sw = z$ and $TSw = Tz = w$. Thus the point z is a fixed point of ST in X and the point w is a fixed point of TS in Y.

Uniqueness: Let $z' \neq z$ be the another fixed point of ST in X.

We have

$$
\begin{aligned}
M_1(z, z',qt) &= M_1(Sw, STz',qt) \\
&\geq \min\{M_1(z',Sw,t)*M_1(z',STz',t), M_2(w,TSw,t), M_2(w,Tz',t), M_2(Tz',TSw,t)\} \\
&= \min\{M_1(z',z,t)*1, 1, M_2(w,Tz',t), M_2(Tz',w,t)\} \\
&\geq M_2(Tz',w,t)
\end{aligned}
$$

Now

$$\begin{aligned} M_2(Tz',w,qt) &= M_2(Tz',TSw,qt) \\ &\geq \min\{M(z',Sw,t), M_1(Sw,STz',t), M_2(w,Tz',t)*M_2(w,TSw,t), M_1(z',STz',t)\} \\ &= \min\{M_1(z',z,t), M_1(z,z',t), M_2(w,Tz',t)*1, 1\} \\ &\geq M_1(z,z',t) \end{aligned}$$

Hence

$$M_1(z, z',qt) \geq M_2(Tz',w,t) \geq M_1(z,z',t/q) \text{ which is a contradiction.}$$

Thus $z = z'$.

So the point z is a unique fixed point of ST. Similarly, we prove the point w is also a unique point of TS.

This completes the proof.

Remark 2.14: If $(X, M_1, *)$ and $(Y, M_2, *)$ are the same fuzzy metric spaces, then by the above theorem 2.13, we get the following theorem, as corollary.

Corollary 2.15: Let $(X, M, *)$ be a complete fuzzy metric space. If S and T are mappings from X into itself satisfying the following conditions:

$$M(Tx,TSy,t) \geq \min\{M(x,Sy,t), M(Sy,STx,t), M(y,Tx,t)*M(y,TSy,t), M(x,STx,t)\}$$

$$M(Sy,STx,t) \geq \min\{M(x,Sy,t)*M(x,STx,t), M(y,TSy,t), M(y,Tx,t), M(Tx,TSy,t)\}$$

for all x, y in X where $q < 1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y. Further $Tz = w$ and $Sw = z$.

REFERENCES

[1] Cho Y.J., Fixed points in fuzzy metric spaces, J. Math. Vol.5, No.4, (1997), 949-962.

[2] Deng Z., Fuzzy pseudo metric space, J. Math. Anal. Appl., 86 (1982), 74-95.
.
[3] Erceg M.A, Metric space in fuzzy set theory, J. Math. Anal. Appl., (1979),205 – 230.

[4] Fang J.X., On fixed point theorems in fuzzy metric spaces, Fuzzy sets and systems, 46 (1992), 107-113.

[5] George A., Veeramani P., On some results in fuzzy metric spaces, fuzzy sets, and systems, 64 (1994), 395-399.

[6] Grabiec M., Fixed point in fuzzy metric space, Fuzzy sets and systems, 27(1988), 385-389.

[7] Kramosil I., Michalek J., Fuzzy metric and statistical metric spaces, Kybernetica, 11(1975), 326-334.

[8] Veerapandi T., Thiripura Sundari T., Paulraj Joseph J., Some fixed point theorems two metric spaces, International journal of Mathematical Archive-3(3) (2012), 826-837.

[9] Zadeh L.A., Fuzzy sets, Inform. and Control, 8 (1965), 338- 353.

[10] Zaheer K. Ansari, Rajesh Shrivastava, Gunjan Ansari, Arun Garg, Some fixed point theorems in fuzzy 2-metric and fuzzy 3-metric spaces, Int. J.Contemp. Math, Sciences, Vol.6, 2011, no. 46, 2291- 2301.

Source of support: Nil, Conflict of interest: None Declared