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QUALITY CONTROL FOR PROBABILISTIC SINGLE-ITEM EOQ MODEL WITH ZERO LEAD TIME UNDER TWO RESTRICTIONS: A GEOMETRIC PROGRAMMING APPROACH

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ABSTRACT

In this paper, we provided a simple method to determine statistical quality control for probabilistic EOQ model, that has varying order cost and zero lead time. The model was restricted to the expected holding cost and the expected available limited storage space. The problem was then solved using a modified geometric programming method.

Keywords: *Quality control, inventory, order cost, holding cost, storage area, lead time.*

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1. INTRODUCTION:

The simple EOQ model is the most fundamental of all inventory models. It is assumed that the expected order cost and demand rate are constants. Fabrycky and Banks [4] studied some probabilistic models of the case where both demand and procurement lead time were identically and independently rodmen variables distributed. Unconstrained probabilistic inventory problem with constant cost units has been treated by Taha [6]. Also, Abou-El-Ata and Kotb [1] developed a crisp inventory model under two restrictions. Teng and Yang [7] studied deterministic inventory lot-size models with time-varying demand and cost under generalized holding costs. Other related studies were presented by Cheng [2] and Mandal et al. [5].

In this paper, we have proposed statistical quality control (SQC) for constrained probabilistic single-item EOQ model with varying order cost and zero lead time. The varying order cost was continually increasing function of number of periods per inventory cycle. The constraints were proposed to be the expected holding cost and the expected available limited storage space. The optimal number of periods and the optimal maximum inventory level were obtained using a modified geometric programming method. Finally, the average of the subgroup ranges approach was used to confirm that the production process is in control.

2. FUNDAMENTAL ASSUMPTIONS AND NOTATION:

The following assumptions were made for developing the model:

1. Demand rate is random variable having a known probability distribution.
2. Lead time is zero.
3. Shortages are not allowed.
4. Review of the stock level is made every N period.
5. Ordering cost $C_O(N) = \alpha + \beta N$, $\alpha > 0$, $\beta \geq 0$ is continuous increasing function of the number of periods. Where α and β are real constants selected to provide the best fit of the estimated cost function.
6. Quality control (QC) is the objective.

In addition, the following notation was adopted for developing the model:

C_h = Holding cost.
C_p = Purchase cost.
C_O = Ordering cost.
$C_O(N)$ = Varying order cost per period.
CL = Control limit.
$\overline{D}$ = Expected annual demand rate.
K_1 = Limitation on the expected holding cost.
K_2 = Limitation on the storage area.
N = Number of periods.
N^* = Optimal number of periods.
LCL = Lower control limit.
Q_m = Maximum inventory level.
Q_m^* = Optimal maximum inventory level.
R = The subgroup ranges.
$\overline{R}$ = The average of the subgroup ranges (CL).
S = Available storage area.
S_R^2 = The variance of the subgroup ranges.
$\overline{TC}$ = Expected total cost.
UCL = Upper control limit.

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3. MATHEMATICAL FORMULATION:

The annual expected total cost was composed of three components (expected purchase cost, expected ordering cost and expected holding cost) according to the basic assumptions and notation of the EOQ model provided by Fabrycky and Banks [4]:

$$\overline{TC} = C_p\,\overline{D} + \frac{C_O(N)}{N} + \frac{C_h\,\overline{D}\,N(2k-1)}{2} \tag{1}$$

The restrictions on the expected holding cost and the expected storage area are the following two conditions:

$$\frac{C_h\,\overline{D}\,N}{2} \le K_1 \qquad \text{and} \qquad S\,\overline{D}\,N \le K_2 \tag{2}$$

In order to solve this primal function which was a convex programming problem, it can be rewritten in the following form:

$$\min \overline{TC} = C_p\,\overline{D} + \frac{\alpha}{N} + \beta + \frac{C_h\,\overline{D}\,N\,k_3}{2} \quad , \quad k_3 = 2k-1 \tag{3}$$

Subject to:

$$\frac{C_h\,\overline{D}\,N}{2K_1} \le 1 \qquad \text{and} \qquad \frac{S\,\overline{D}\,N}{K_2} \le 1 \tag{4}$$

The term $C_p\,\overline{D} + \beta$ is constant and hence can be ignored.

Applying Duffin et al. [3] results of geometric programming technique on (3) and (4), the enlarged predual function can be written in the form:

$$G(\underline{W}) = \left(\frac{\alpha}{N\,W_1}\right)^{W_1} \left(\frac{C_h\,\overline{D}\,N\,k_3}{2\,W_2}\right)^{W_2} \left(\frac{C_h\,\overline{D}\,N}{2\,K_1\,W_3}\right)^{W_3} \left(\frac{S\,\overline{D}\,N}{K_2\,W_4}\right)^{W_4}$$

$$= \left(\frac{\alpha}{W_1}\right)^{W_1} \left(\frac{C_h\,\overline{D}\,k_3}{2\,W_2}\right)^{W_2} \left(\frac{C_h\,\overline{D}}{2\,K_1\,W_3}\right)^{W_3} \left(\frac{S\,\overline{D}}{K_2\,W_4}\right)^{W_4} \times N^{-W_1+W_2+W_3+W_4} \tag{5}$$

Where $\underline{W} = W_j\ ,\ j = 1,2,3,4\ \left(0 < W_j < 1\right)$ are the weights, and could be easily deduced from equation through the use of the following normal and orthogonal conditions:

$$\left.\begin{aligned} W_1 + W_2 &= 1 \\ \text{and} \qquad -W_1 + W_2 + W_3 + W_4 &= 0 \end{aligned}\right\} \tag{6}$$

These are two linear equations in four unknowns having an infinite number of solutions. However, the problem is to select the optimal solution of the weights $W_j^*\ ,\ 0 < W_j^* < 1\ ,\ j = 1,2,3,4$.

By solving equations (6), we had:

$$\left.\begin{aligned} W_1 &= \frac{1 + W_3 + W_4}{2} \\ \text{and} \qquad W_2 &= \frac{1 - W_3 - W_4}{2} \end{aligned}\right\} \tag{7}$$

Substituting W_1 and W_2 in equation (5), then the dual function is:

$$g(W_3\,, W_4) = \left(\frac{2\alpha}{1+W_3+W_4}\right)^{\frac{1+W_3+W_4}{2}} \left(\frac{C_h\,\overline{D}\,k_3}{1-W_3-W_4}\right)^{\frac{1-W_3-W_4}{2}} \left(\frac{C_h\,\overline{D}}{2\,K_1\,W_3}\right)^{W_3} \left(\frac{S\,\overline{D}}{K_2\,W_4}\right)^{W_4} \tag{8}$$

In order to find the optimal W_3 and W_4 which maximize $g(W_3, W_4)$, the logarithm of both side of equation (8), and the partial derivatives were taken with respect to W_3 and W_4, respectively. Setting each of them to equal zero and simplifying, we got:

$$\left(\frac{2\,\alpha}{C_h\,\overline{D}\,k_3}\right)\left(\frac{1-W_3-W_4}{1+W_3+W_4}\right)\left(\frac{C_h\,\overline{D}}{2\,e\,K_1\,W_3}\right)^2 = 1 \quad (9)$$

and

$$\left(\frac{2\,\alpha}{C_h\,\overline{D}\,k_3}\right)\left(\frac{1-W_3-W_4}{1+W_3+W_4}\right)\left(\frac{S\,\overline{D}}{e\,K_2\,W_4}\right)^2 = 1 \quad (10)$$

Multiplying relation (9) by the inverse of relation (10), we had:

$$\frac{W_3}{W_4} = \frac{C_h\,K_2}{2\,S\,K_1} \quad (11)$$

Substituting W_3 and W_4 into relations (9) and (10), respectively, we had:

$$f_i(W_j) = W_j^3 + C_i\,W_j^2 + B_i\,W_j - B_i\,C_i = 0 \quad , \quad j = 3\,,4 \quad , \quad i = 1\,,2 \quad (12)$$

where

$$B_1 = \frac{\alpha\,C_h\,\overline{D}\,k_3}{2e^2\,K_1^2} \quad , \quad B_2 = \frac{2\,\alpha\,S^2\,\overline{D}}{e^2\,C_h\,K_2^2} \quad , \quad C_1 = \frac{C_h\,K_2}{C_h\,K_2 + 2\,S\,K_1} \quad , \quad C_2 = \frac{2\,S\,K_1}{C_h\,K_2 + 2\,S\,K_1}$$

It is clear that $f_i(0) < 0$ *and* $f_i(1) > 0$, $i = 1\,,2$, which means that there exists a root $W_j \in (0\,,1)$, $j = 3\,,4$. The trial and error approach can be used to find these roots. However, we shall first verify any root W_j^*, $j = 3\,,4$ calculated from equations (12) to maximize $f_i(W_j)$, $i = 1{,}2$, $j = 3\,,4$, respectively. This was confirmed by the second derivative to $ln g(W_3\,,W_4)$ with respect to W_3 *and* W_4, respectively, which was always negative.

Thus, the roots W_3^* *and* W_4^* calculated from equations (12) maximize the dual function $g(W_3\,,W_4)$. Hence, the optimal solutions were W_3^* *and* W_4^* of equations (12), respectively. W_1^* *and* W_2^* were evaluated by substituting the value of W_3^* *and* W_4^* in expression (7).

To find the optimal number of periods N^* and the optimal maximum inventory level Q_m^*, we applied the results of Duffin et al. [3] for geometric programming as indicated blow:

$$\frac{\alpha}{N^*} = W_1^*\,g(W_3^*\,,W_4^*)$$

and

$$\frac{C_h\,\overline{D}\,N^*\,k_3}{2} = W_2^*\,g(W_3^*\,,W_4^*)$$

By solving these relations, the optimal number of periods is given by:

$$N^* = \sqrt{\frac{2\alpha W_2^*}{C_h\overline{D}\,k_3\,W_1^*}} = \sqrt{\frac{2\alpha\left(1-W_3^*-W_4^*\right)}{C_h\overline{D}\,k_3\left(1+W_3^*+W_4^*\right)}} \quad (13)$$

and the optimal maximum inventory level Q_m^* is:

$$Q_m^* = \overline{D}\,N^*\,g(N^*) = k\,\sqrt{\frac{2\,\alpha\,\overline{D}\,\left(1-W_3^*-W_4^*\right)}{C_h\,k_3\left(1+W_3^*+W_4^*\right)}} \quad (14)$$

4. STATISTICAL QUALITY CONTROL:

The decision variables N^* and $Q_{m\,i}^*$, $i = 1\,,2\,,3$ should be computed to confirm that the production process is in control. Assume the parameters of the inventory model as:

$C_{oi} = \$11\,,12.86\,,12.86$ *per procurement*, $i = 1\,,2\,,3$,
$C_{hi} = \$0.05\,.0.0965\,,0.0965$ *per unit per period* , $i = 1\,,2\,,3$,
$C_{pi} = \$25\,,1780.87\,,1780.87$ *per unit* , $i = 1\,,2\,,3$,
$\overline{D}_i = 2\,,25\,,25$ *units per period* , $i = 1\,,2\,,3$, $k = 4$,
$S = 50$ *cubic units per item*,

$K_{1i} = \$1000\,, 188.98\,, 188.98\; per\ unit\,, i = 1, 2\,, 3\,,$

$K_{2i} = 200\,, 0.27\,, 0.27\; cubic\ units\ of\ space\,, i = 1\,, 2\,, 3.$

The optimal results of N^* and Q^*_{mi}, $i = 1, 2, 3$ for different values of α are shown in table: 1

α	1	2	5	8	10	15	30	50
N^*_1	2.634	3.209	3.961	4.306	4.454	4.692	5.001	5.154
N^*_2	4.953	6.520	10.106	13.317	15.398	20.515	35.645	55.701
N^*_3	0.0395	0.0452	0.0510	0.0531	0.0539	0.0551	0.0564	0.0570
Q^*_{m1}	21.1072	25.672	32.688	34.448	35.632	37.536	40.008	41.232
Q^*_{m2}	495.3	652.0	1010.6	1331.7	1539.8	2051.5	3564.5	5570.1
Q^*_{m3}	3.95	4.52	5.10	5.31	5.39	5.51	5.64	5.70
R	491.35	647.48	1005.5	1326.39	1534.41	2045.99	3558.86	5564.4

Table 1

In order to study the statistical quality control of this model, apply control limits (CL) method when σ is unknown as:
The average of the subgroup ranges (CL) is:

$$\overline{R} = \frac{\sum_{j=1}^{n} R_j}{n} = 2021.800$$

the standard deviation of the subgroup ranges is:

$$S_R = \sqrt{\frac{\sum_{j=1}^{n} \left(R_j - \overline{R}\right)^2}{n}} = 1727.847$$

The lower control limit is $LCL = \overline{R} - 3S_R = 0$, the average of the subgroup ranges is $\overline{R} = 2021.800$ and the upper control limit is $UCL = \overline{R} + 3S_R = 7205.341$. It is clear that $LCL < \overline{R} < UCL$. Therefore the production process is in control.

5. CONCLUSION:

This work investigated how ordering cost function, two constraints and geometric programming approach affect the probabilistic inventory model. Ordering cost function was assumed to depend on number of periods. In addition, the constraints were expected holding cost and expected available limited storage space. A geometric programming approach was devised to determine the optimal solution for probabilistic inventory model, number of periods and maximum inventory level instead of the traditional Lagrangian method. Finally, Statistical Quality Control of the EOQ model is confirmed.

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