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$(1, 2)^*\psi\hat{g}$ - CLOSED FUNCTIONS IN BITOPOLOGICAL SPACES

N. Ramya* & A. Parvathi

Department of Mathematics, Avinashilingam University For Women, Coimbatore-641043, India

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ABSTRACT

In this paper, we introduce $(1, 2)^*-\psi\hat{g}$-closed functions from a bitopological space X to a bitopological space Y as the image of every $\tau_{1,2}$-closed set is $(1,2)^*-\psi\hat{g}$ closed. Also we discuss about Almost $(1, 2)^*-\psi\hat{g}$-closed functions. Properties, characterizations and applications are studied.

Keywords: *$(1, 2)^*-\psi\hat{g}$-closed, $(1, 2)^*-\psi\hat{g}$-open, $(1, 2)^*-\psi\hat{g}$-closed function $(1, 2)^*-\psi\hat{g}$-open function, Almost $(1, 2)^*-\psi\hat{g}$-closed function.*

1. Preliminaries

Levine [8] introduced the concepts of generalized closed sets in topological spaces and studied their properties. Levine [8], Mashhour et al.[10] and Najas- tad[11] have introduced the concepts of semi-open sets, preopen sets and α-open sets respectively. Bhattacharya and Lahiri[5], Arya and Nour[4], Maki et al[9] introduced semi-generalized closed sets, generalized semi-closed sets and α- generalized sets, Veerakumar[16] defined $\hat{g}$-closed sets in topological spaces. Thivagar et.al[11]have introduced the concepts of $(1, 2)^*$-semi open sets,$(1, 2)^*-$ α-open sets,$(1, 2)^*$-generalized closed sets,$(1, 2)^*$-semi generalized closed sets and $(1, 2)^*-$α-generalized closed sets. In this paper, a new class of function called $(1, 2)^*-\psi\hat{g}$-closed functions, Almost $(1, 2)^*-\psi\hat{g}$-closed functions have been introduced and studied their various results. We prove that the composition of two $(1, 2)^*-\psi\hat{g}$-closed functions need not be $(1, 2)^*-\psi\hat{g}$-closed function. Also we obtain some important results in bitopological settings.

Throughout the present paper $(X, \tau_1, \tau_2), (Y, \sigma_1, \sigma_2), (Z, \eta_1, \eta_2)$ briefly X, Y, Z be bitopological space..

Definition 1.1: A subset S of a bitopological spacee (X, τ_1, τ_2) is said to be $\tau_{1,2}$-open if $S = A \cup B$ where $A \in \tau_1$ and $B \in \tau_2$. A subset S of X is said to be

[1] $\tau_{1,2}$ closed if the complement of S is $\tau_{1,2}$ –open.
[2] $\tau_{1,2}$-clopen if S is both $\tau_{1,2}$-open and $\tau_{1,2}$-closed.

Definition 1.2: Let S be a subset of the bitopological space (X, τ_1, τ_2). Then
[1] The $\tau_{1,2}$ –interior of S, denoted by $\tau_{1,2}$-int(S) is defined by $\cup G{:}G \subseteq S$ and G is $\tau_{1,2}$–open.
[2] The $\tau_{1,2}$ –closure of s, denoted by $\tau_{1,2}$–cl(s) is defined by $\cap F{:}S \subseteq F$ and F is $\tau_{1,2}$–closed.

Corresponding author: N. Ramya*
Department of Mathematics, Avinashilingam University For Women, Coimbatore-641043, India

Definition 1.3: A subset A of a bitopological space (X, τ_1, τ_2) is called

[1] $(1, 2)^*$-regular open if A $= \tau_{1,2}$-int$(\tau_{1,2} - $cl(A)).

[2] $(1, 2)^* - \alpha$-open if A $\subseteq \tau_{1,2}$-int$(\tau_{1,2} -$ cl$(\tau_{1,2} -$ int(A))).

[3] $(1, 2)^*$- generalized closed (briefly (1. 2)*- g-closed) if $\tau_{1,2} -$ cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_{1,2}$-open in X.

The complement of the sets mentioned from (i) and (ii) are called their respective closed sets and the complement of the sets mentioned above (iii) is called the respective open set..

Definition 1.4: The finite union of $(1, 2)^*$-regular open sets is said to be $\tau_{1,2} - \psi$ - open. The complement of $\tau_{1,2} - \psi$ - open is said to be $\tau_{1,2} - \psi$ -closed.

Definition 1.5: A subset A of a bitopological space(X, τ_1, τ_2) is called

[1] $(1, 2)^*$-g- closed if f(U) is $(1, 2)^*$-g-closed set in Y for every $\tau_{1,2}$ -closed set U in X.

[2] $(1, 2)^* - \psi$-closed if $(1, 2)^* - \psi$cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $(1, 2)^*$- sg-open in X.

[3] $(1, 2)^* - \psi$ -generalized closed ($(1, 2)^*\psi$-g-closed) if $(1, 2)^* - \psi$cl(A) $\subseteq$ A whenever A $\subseteq$ U and U is $\tau_{1,2}$-open in X.

[4] $(1, 2)^* - \hat{g}$-closed set if $(1, 2)^* -$ cl(A) $\subseteq$ A whenever A $\subseteq$ U and U is$(1, 2)^*$-semi-open in X.

[5]$(1, 2)^* - \psi \hat{g}$-closed if $(1, 2)^* - \psi$cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $(1, 2)^* - \hat{g}$-open in X.

Definition 1.6: A function $f : X \to Y$ is called

[1] $(1, 2)^*$- continuous if $f^{-1}(V)$ is $(1, 2)^*$ - closed in X for every $\sigma_{1,2}$-closed set V in Y.

[2] $(1, 2)^* - \psi$ -continuous if $f^{-1}(V)$ is $(1, 2)^* - \psi$ -closed in X for every $\sigma_{1,2}$-closed V in Y.

[3] $(1, 2)^* - \psi$ g-continuous if $f^{-1}(V)$ is $(1, 2)^* - \psi$ g-closed in X for every $\sigma_{1,2}$-closed V in Y.

[4] $(1, 2)^* - \psi \hat{g}$ - continuous if $f^{-1}(V)$ is $(1, 2)^* - \hat{g}$ -closed in X for every $\sigma_{1,2}$-closed V in Y.

[5] $(1, 2)^* - \psi \hat{g}$ -continuous if $f^{-1}(V)$ is $(1, 2)^* - \psi \hat{g}$ -closed in X for every $\sigma_{1,2}$-closed V in Y.

[6] $(1, 2)^* - \psi \hat{g}$ -irresolute if $f^{-1}(V)$ is $(1, 2)^* - \psi \hat{g}$ -closed in X, for every $(1, 2)^* - \psi \hat{g}$ -closed set V of Y.

[7] $(1, 2)^* - \psi \hat{g}$ -continuous if $f^{-1}(V)$ is $(1, 2)^* - \psi \hat{g}$ -closed in X for every $\sigma_{1,2}$-closed.

Definition 1.7: A space (X, τ_1, τ_2) is called $(1, 2)^* - \psi \hat{g}$ -$T_{1/2}$-space if every $(1, 2)^* - \psi \hat{g}$ -closed set is $(1, 2)^* - \psi$ -closed.

Definition 1.8: A function $f : X \to Y$ is said to be $M - (1, 2)^* - \psi \hat{g}$ -closed function if the image f(A) is $(1, 2)^* - \psi \hat{g}$ -closed in Y for every $(1, 2)^* - \psi \hat{g}$ -closed set A in X.

The complement of the $M - (1, 2)^* - \psi \hat{g}$ -closed function is said to be $M - (1, 2)^* - \psi \hat{g}$ -open function.

Definition 1.9: A space (X, τ) is called α-normal, if for every pair of disjoint closed subsets H,K there exist disjoint α-open sets U,V suchthat H$\subset \alpha$ int U, K$\subset \alpha$intV and αintU $\cap$ αintV=φ.

Definition 1.10: A function $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be almost continuous if $f^{-1}(V)$ is closed in X for every regular closed in Y.

Definition 1.11: A function $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be almost $\psi \hat{g}$ -continuous if $f^{-1}(V)$ is $\psi \hat{g}$ - closed in X for every regular closed in Y.

2. $(1,2)^* - \psi \hat{g}$ - CLOSED FUNCTIONS

Definition 2.1: A function f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be $(1, 2)^* - \psi \hat{g}$-closed if for every $\tau_{1,2}$ -closed F of X, f(F) is $(1, 2)^* - \psi \hat{g}$-closed in Y.

Theorem 2.2: Every $\tau_{1,2}$ -closed function is $(1, 2)^* \ \psi \hat{g}$-closed.

Proof: Since every $\tau_{1,2}$ -closed set is $(1, 2)^* - \psi \hat{g}$- closed. We get the result.

The converse need not be true as seen from the following example.

Example 2.3: Let X={a, b, c},τ_1 ={φ, X, {a},{b},{a,b}}, τ_2 ={φ, X, {b},{a,b}},σ_1 ={φ, X, {a},{a,c}}, σ_2 ={φ,X, {a,b}} f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the identity function then f is $(1, 2)^* - \psi \hat{g}$- closed but not $\tau_{1,2}$ -closed function, not $(1, 2)^*$- $\hat{g}$ - closed function.

Proposition 2.4: If a function ping f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(1, 2)^* - \psi \hat{g}$- closed then for every subset A of X,$(1, 2)^* - \psi \hat{g}$- cl(f(A)) $\subset$ f($\sigma_{1,2}$ − cl(A)).

Proof: Let A⊂X. Since f is (1,2)*− $\psi \hat{g}$-closed f($\sigma_{1,2}$ − cl(A)) is $(1, 2)^* - \psi \hat{g}$- closed in Y. Now f (A) ⊂ f($\sigma_{1,2}$ −cl(A)).Also f(A) ⊂ $(1, 2)^* - \psi \hat{g}$-cl(f(A)). By definition, we have $(1, 2)^* - \psi \hat{g}$- cl(f(A)) ⊂ f($\sigma_{1,2}$ − cl(A)).

Converse need not be trure as seen in the following example.

Example 2.5: Let X={a, b, c, d},τ1={φ, X, {b, c, d}}, τ2 ={φ, X},σ1={φ, X, {b}}, σ2 ={φ, X,{a}, {a, b}}, f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the idemtity function. For every subset A of X ,$(1, 2)^* - \psi \hat{g}$ -cl(f(A)) ⊂ f($\sigma_{1,2}$−cl(A)), but f is not $(1, 2)^* - \psi \hat{g}$ -closed function

Proposition 2.6: If for every subset A of X,$\tau_{1,2}$ −cl($\tau_{1,2}$ −int($\tau_{1,2}$ −cl(A))) ⊂ f($\sigma_{1,2}$ − cl(A)) then a function f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(1, 2)^* - \psi \hat{g}$ -closed.

Proof: Let A be closed in X. Since $\tau_{1,2}$ − cl($\tau_{1,2}$ − int($\tau_{1,2}$ − cl(A))) ⊂ f ($\sigma_{1,2}$ − cl(A)) ⊂ f(A).f(A)is $(1, 2)^* - \psi$ closed and hence $(1, 2)^* - \psi \hat{g}$ - closed .

Converse of the above need not be true as seen in the following example.

Example 2.7: Let X={a, b, c},τ1 ={φ, X,{a}},τ2={φ, X, {a, c}}, σ1 ={φ, X, {b}}, σ2 ={φ, X }, f:$(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the identity function then f is $(1, 2)^* - \psi \hat{g}$ -closed function, but $\tau_{1,2}$ − cl($\tau_{1,2}$ − int($\tau_{1,2}$ − cl(A))) ⊂ f($\sigma_{1,2}$ − cl(A)) is not true.

Proposition 2.7: If a function f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(1, 2)^* - \psi \hat{g}$ -closed and Y is $(1, 2)^* - \psi \hat{g}$ -$T_{1/2}$ space then $\tau_{1,2}$ −cl($\tau_{1,2}$ −int($\tau_{1,2}$ −cl(A))) ⊂ f($\sigma_{1,2}$ − cl(A)).

Proof: Let A⊂ X.Then $\tau_{1,2}$ − cl(A) is closed in X. Since f is $(1, 2)^* - \psi \hat{g}$ -closed, f($\sigma_{1,2}$ − cl(A)) is $(1, 2)^*$ −

$\psi\hat{g}$ - closed in Y and so $(1,2)^* - \alpha cl(f(\tau_{1,2} - cl(A))) \subset f(\sigma_{1,2} - cl(A))$.Hence $\tau_{1,2} - cl(\tau_{1,2} - int(\tau_{1,2} - cl(A))) \subset \tau_{1,2} - cl(\tau_{1,2} - int(\tau_{1,2} - cl(f(\tau_{1,2} - cl(A)))) \subset f(\sigma_{1,2} - cl(A))$.

Theorem 2.8: A surjection f: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2$ is $(1,2)^* - \psi\hat{g}$ - closed iff for each subset S of Y and each $\tau_{1,2}$-open set U containing $f^{-1}(S)$ there exist a $(1,2)^* - \psi\hat{g}$ -open set V of Y such that $S \subset V$ and $f^{-1}(V) \subset U$.

Proof:

Necessity: Suppose that f is $(1,2)^* - \psi\hat{g}$ -closed. Let S be a subset of Y and Y be an $\tau_{1,2}$-open subset of X containing $f^{-1}(S)$.If V=Y-f(X-U),then V is a $(1,2)^* - \psi\hat{g}$ - open set of Y, suchthat $S \subset V$ and $f^{-1}(V) \subset U$.

Sufficiency: Let F be any $\tau_{1,2}$-closed set of X. Then $f^{-1}(Y - f(F)) \subset X - F$ and X-F is $\tau_{1,2}$ -open in X. There exists $(1,2)^* - \psi\hat{g}$ - open set V of Y such that $Y - f(F) \subset V$ and $f^{-1}(V) \subset X - F$.Therefore, $Y - V \subset f(F) \subset f(X - (1,2)^* - \psi\hat{g}$ -closed functions $f^{-1}(V)) \subset Y - V$.Hence we obtain $f(F) = Y - V$ and f(F) is $(1,2)^* - \psi\hat{g}$ - closed in Y which shows that f is $(1,2)^* - \psi\hat{g}$ -closed.

Theorem 2.9: If f: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(1,2)^* - \psi\hat{g}$ - closed and A is $\tau_{1,2}$ -closed subset of X then $f|A : (A) \rightarrow (Y)$ is $(1,2)^* - \psi\hat{g}$ - closed.

Proof: Let B⊂A be $\tau_{1,2}$ -closed in A. Then B is $\tau_{1,2}$ -closed in X. Since f is $(1,2)^* - \psi\hat{g}$ - closed, f(B) is $(1,2)^* - \psi\hat{g}$ - closed in Y.But $f(B) = (f|A)(B)$. So f|A is $(1,2)^* - \psi\hat{g}$ - closed.

Remark 2.10: Composition of two $(1,2)^* - \psi\hat{g}$ - closed functions need not be $(1,2)^* - \psi\hat{g}$ - closed function.

Example 2.11: Let X=Y=Z={a, b, c}, τ_1={φ, X, {b, c}}, τ_2={φ, X, {a}}, σ_1 ={φ, X, {a}}, σ_2 ={φ, X}, η_1 ={φ, X, {b}}, η_2={φ,X,{a},{a, b}}, f: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and g : $(Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be the identity functions then f and g are $(1,2)^* - \psi\hat{g}$ -closed functions, but gof : $(X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ is not $(1,2)^* - \psi\hat{g}$ -closed function.

Proposition 2.12: If f: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $\tau_{1,2}$ -closed and g : $(Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be the $(1,2)^* - \psi\hat{g}$ - closed then gof : $(X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ is $(1,2)^* - \psi\hat{g}$ -closed.

Proof: Let A be $\tau_{1,2}$ -closed in X then f(A) is $\sigma_{1,2}$ -closed in Y. Since g is $(1,2)^* - \psi\hat{g}$ -closed, g(f(A)) is $(1,2)^* - \psi\hat{g}$ -closed in Z. Hence gof is $(1,2)^* - \psi\hat{g}$ -closed.

Theorem 2.13: Let f: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and g : $(Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be two functionpings and let gof : $(X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$) be $(1,2)^* - \psi\hat{g}$ -closed. Then

[1] If f is $(1,2)^*$-continuous and surjection then g is $(1,2)^* - \psi\hat{g}$ -closed.

[2] If g is $(1,2)^* - \psi\hat{g}$ -irresolute and injective then f is $(1,2)^* - \psi\hat{g}$ -closed.

Proof:

(i) Let A be $\sigma_{1,2}$ -closed in Y. Since f is $(1, 2)^*$-continuous, f^{-1} (A) is $\tau_{1,2}$ -closed in X. Since gof is $(1, 2)^* - \psi \hat{g}$ -closed.gof $(f^{-1}(A)) = g(f(f^{-1}(A))) = g(A)$ is $(1, 2)^* - \psi \hat{g}$ -closed.

ii) Let A be $\tau_{1,2}$ -closed in X. Since gof is $(1, 2)^* - \psi \hat{g}$ -closed, (gof)(A) is $(1, 2)^* - \psi \hat{g}$ -closed in Z. Since g is $(1, 2)^* - \psi \hat{g}$ -continuous, $g^{-1}((gof)(A)) = f(A)$ is $(1, 2)^* - \psi \hat{g}$ -closed in Y. Hence f is $(1, 2)^* - \psi \hat{g}$ -closed.

Proposition 2.14: For any bijection f: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ the following statements are equivalent.

[1] f is a $(1, 2)^* - \psi \hat{g}$ -open function.

[2] f is a $(1, 2)^* - \psi \hat{g}$ -closed function.

[3] f^{-1}: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(1, 2)^* - \psi \hat{g}$ -continuous.

Proof: (i) $\Rightarrow$ (ii)Let f be a $(1, 2)^* - \psi \hat{g}$ -open function. Let U be $\tau_{1,2}$ -closed in X. Then X-U is $\tau_{1,2}$ -open in X. By assumption,$f(X - U)$ is a $(1, 2)^* - \psi \hat{g}$ -open function and it implies Y-f(U) is $(1, 2)^* - \psi \hat{g}$ -open function and hence f(U) is $(1, 2)^* - \psi \hat{g}$ -closed.

(ii) $\Rightarrow$ (iii) Let V be $\tau_{1,2}$ -closed in X. By (ii) $f(V) = (f^{-1})^{-1}(V)$ is $(1, 2)^* - \psi \hat{g}$ -closed in Y.

(iii) $\Rightarrow$ (i) Let V be $\tau_{1,2}$ -open in X. By (iii) $(f^{-1})^{-1}(V) = f(V)$ is $(1, 2)^* - \psi \hat{g}$ -open in Y.

Definition 2.15: A function $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be M $- (1, 2)^* - \psi \hat{g}$ -closed function if the image f(A) is $(1, 2)^* - \psi \hat{g}$ -closed in Y for every $(1, 2)^* - \psi \hat{g}$ - closed set A in X.

Remark 2.16: Every M $- (1, 2)^* - \psi \hat{g}$ -closed function is $(1, 2)^* - \psi \hat{g}$ -closed function, but the converse need not be true as seen in the following example.

Example 2.17: Let X= {a, b, c} $\tau_1 = \{\varphi, X, \{a\}\}$, $\tau_2 = \{\varphi, X\}$,$\sigma_1 = \{\varphi, X, \{b\}\}$, $\sigma_2 = \{\varphi, X, \{a\}, \{a, b\}\}$. f: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity function then f is $(1, 2)^* - \psi \hat{g}$ -closed function, but not M $- (1, 2)^* - \psi \hat{g}$ -closed function

Definition 2.18: A function f: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $(1, 2)^* - \hat{g}$ - Continuous if f (V) is $\tau_{1,2} - \hat{g}$ -closed in X for every $\sigma_{1,2}$-closed set V in Y.

Theorem 2.19: If f: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is a $(1, 2)^* - \hat{g}$ -continuous and M $- (1, 2)^* - \psi \hat{g}$ -closed function in X then f(A) is $(1, 2)^* - \psi \hat{g}$ -closed in Y for every $(1, 2)^* - \psi \hat{g}$ -closed set A of X.

Proof: Let A be any $(1, 2)^* - \psi \hat{g}$ -closed set of X and V be any $\sigma_{1,2} - \hat{g}$ -open set of Y containing f(A).Since f is $(1, 2)^* - \hat{g}$ -continuous, $f^{-1}(V)$ is $\tau_{1,2} - \hat{g}$ open in X and $A \subset f^{-1}(V)$. Therefore $(1, 2)^* - \psi\,cl(A) \subset f^{-1}(V)$ and hence $f(1, 2)^* - \psi\,cl(A)) \subset V$ .Since f is M $- (1, 2)^* - \psi \hat{g}$ -closed,$(1, 2)^* - \psi\,cl(A))$ is$(1, 2)^* - \psi \hat{g}$ -closed in Y ans hence we obtain $(1, 2)^* - \psi\,cl(f(A)) \subset (1, 2)^* - \psi\,cl(f((1, 2)^* - \psi\,cl(A))) \subset V$ .Hence f(A) is $(1, 2)^* - \psi \hat{g}$ -closed in Y.

Proposition 2.20: For any bijection f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ the following statements are equivalent.

[1] f^{-1}: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a $(1,2)^* - \psi\hat{g}$-irresolute.

[2] f is a $M - (1,2)^* - \psi\hat{g}$-open function.

[3] f is a $M - (1,2)^* - \psi\hat{g}$-closed function.

Proof: (i) $\Rightarrow$ (ii)Let U be a $(1,2)^* - \psi\hat{g}$-open in X. By (i) $(f^{-1})^{-1}(U) = f(U)$ is $(1,2)^* - \psi\hat{g}$-open in Y. Hence (ii) holds.

(ii) $\Rightarrow$ (iii) Let V be $(1,2)^* - \psi\hat{g}$-closed in X. By (ii) $f(X - V = Y - f(V)$ is $(1,2)^* - \psi\hat{g}$-open in Y. That is f(V) is $(1,2)^* - \psi\hat{g}$-closed in Y and so f is a $M - (1,2)^* - \psi\hat{g}$-closed function.

(iii) $\Rightarrow$ (i) Let V be $(1,2)^* - \psi\hat{g}$-closed in X. By (iii) $f(V) = (f^{-1})^{-1}(V)$ is $(1,2)^* - \psi\hat{g}$-closed n Y. Hence (i) holds.

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