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Strongly g-closed sets and strongly g**-closed sets

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ABSTRACT

*In this paper we introduce and investigate the concept of strongly g-closed set and strongly g**-closed set.*

Keywords: *Strongly g-closed set, strongly g**-closed set.*

1. INTRODUCTION

N. Levine **[2]** introduced generalized closed sets in 1970, M.K.R.S. Veera Kumar **[7]** introduced the genralised g*-closed sets in 2000. R. Parimelazhagan and V. Subramania Pillai**[5]** introduced the strongly g*-closed set in 2012. In this paper we introduce and study the concept of strongly g-closed sets and strongly g**-closed sets.

2. PRELIMINARIES

Throughout this paper (X, τ) represents non-empty topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A), int(A) and C((X, τ)) denote the closure of A, interior of A and the closed sets of (X, τ) respectively.

Let us recall the following definitions, which are useful in this sequel.

Definition 2.1: A subset A of a space(X, τ) is called a
1. semi-open set **[1]** if A $\subseteq$ cl(int(A)) and a semi-closed set if int(cl(A)) $\subseteq$ A.
2. regular-open set **[1]** if A = int(cl(A)) and a regular-closed set if int(cl(A)) = A
3. pre-open set **[3]** if A $\subseteq$ int(cl(A)) and pre-closed set if cl(int(A) $\subseteq$ A.

Definition 2.2: A subset A of a space(X, τ) is called a
1. regular generalized closed (briefly rg-closed) set [4] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in (X, τ).
2. generalized closed (briefly g-closed) set **[2]** if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
3. generalized g star(briefly g*-closed) set **[7]** if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in (X, τ).
4. generalized g star star(briefly g**-closed) set **[6]** if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g*-open in (X, τ).
5. strongly g star closed(briefly strongly g*-closed) set **[5]** if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in (X, τ).

3. BASIC PROPERTIES OF STRONGLY g-closed sets and strongly g**-closed sets

We introduce the following definitions.

Definition 3.1: Let (X,τ) be a topological space and A be its subset. Then A is said to be a strongly g-closed set if cl(int(A) $\subseteq$ U whenever $A \subseteq U$ and U is open in X .

Definition 3.2: Let (X,τ) be a topological space and A be its subset. Then A is said to be a strongly g**-closed set if cl(int(A) $\subseteq$ U whenever $A \subseteq U$ and U is g*-open in X.

Proposition 3.3: Every closed set is strongly g-closed.

Proof follows from the definitions.

Proposition 3.4: Every closed set is strongly g**-closed.

Proof follows from the definitions.

The converse of the above propositions need not true in general as seen in the following examples.

Example 3.5: Let $X = \{a,b,c\}$, $\tau = \{\varphi,\{a\},\{a,c\},X\}$. Then A = {a,b}is a strongly g-closed set but not a closed set of (X,τ).

Example 3.6: Let $X = \{a,b,c\}$, $\tau = \{\varphi,\{a\},X\}$. Then A = {b} is a strongly g**-closed set but not a closed set of (X,τ).

Proposition 3.7: Every strongly g*-closed set is strongly g-closed.

Proof follows from the definitions.

The converse of the above propositions need not true in general as seen in the following examples.

Example 3.8: Let X = {a, b, c, d} and $\tau = \{\varphi,\{a\},X\}$. Then A = {a, b}is strongly g-closed but not strongly g*-closed in (X,τ).

Proposition 3.9: Every strongly g**-closed set is strongly g-closed.

Proof follows from the definitions.

Proposition 3.10: Every strongly g*-closed set is strongly g**-closed but not conversely.

Proof follows from the definitions.

Example 3.11: In example (3.8), A = {a, b} is strongly g**-closed but not strongly g*-closed in (X,τ).

Proposition 3.12: Every g-closed set is strongly g-closed but not conversely.

Proof follows from the definitions.

Example 3.13: Let X = {a, b, c} and $\tau = \{\varphi,\{a\},\{a,b\},X\}$. Then A = {b} is strongly g-closed but not g-closed in (X,τ).

Proposition 3.14: Every g**-closed set is strongly g**-closed but not conversely.

Proof follows from the definitions.

Example 3.15: Let X = {a, b, c} and $\tau = \{\varphi,\{a\},\{a,b\},X\}$. Then A = {b} is strongly g-closed but not g-closed in (X,τ).

Proposition 3.16: Every g*-closed set is strongly g-closed but not conversely.

Proof follows from the definitions.

Example 3.17: In example (3.6) A = {b} is strongly g-closed but not g*-closed in (X,τ).

Proposition 3.18: Every g*-closed set is strongly g**-closed but not conversely.

Proof follows from the definitions.

Example 3.19: In example (3.6) A = {b} is strongly g**-closed but not g*-closed in (X,τ).

Remark 3.20: g**-closedness and strongly g*-closedness are independent as seen in the following examples.

Example 3.21: In example (3.5) A = {c}is strongly g*-closed but not g**-closed in (X,τ) and in example (3.8) B = {a, b}is g**-closed but not strongly g*-closed in (X,τ)

Remark 3.22: g-closedness and strongly g*-closedness are independent as seen in the following examples.

Example 3.23: In example (3.13) A = {b} is strongly g*-closed but not g-closed in (X,τ)

Example 3.24: In example (3.8) A = {a,}I s g-closed but not strongly g*-closed in (X,τ)

Remark 3.25: g-closedness and strongly g**-closedness are independent as seen in the following examples.

Example 3.26: In example (3.13) A = {b} is strongly g**-closed but not g-closed in (X,τ)

Example 3.27: In example (3.10) A = {a, b} is g-closed but not strongly g**-closed in (X,τ)

Proposition 3.28: Every g**-closed set is strongly g-closed but not conversely.

Proof follows from the definitions.

Example 3.29: In example (3.5) A = {c} is strongly g-closed but not g**-closed in (X,τ).

Theorem 3.30: If A subset of a topological space (X,τ) is both open and strongly g-closed then it is closed.

Proof: Suppose A is both open and strongly g-closed. Since A is strongly g-closed cl(int(A)$\subseteq$A.
That is cl(A) = cl(int(A) $\subseteq$ A. $\therefore$ A is closed.

Corollary 3.31: If A subset of a topological space (X,τ) is both open and strongly g-closed then it is both regular open and regular closed in X.

Corollary 3.32: If A subset of a topological space (X,τ) is both open and strongly g-closed then it is rg-closed in X.

Theorem 3.33: If A subset of a topological space (X,τ) is both open and strongly g**-closed then it is closed.

Proof is similar to theorem (3.30).

Corollary 3.34: If A subset of a topological space (X,τ) is both open and strongly g**-closed then it is both regular open and regular closed in X.

Corollary 3.35: If A subset of a topological space (X,τ) is both open and strongly g**-closed then it is rg-closed in X.

Theorem 3.36: If A subset of a topological space (X,τ) is both strongly g-closed and semi-open then it is g-closed.

Proof: Since A is strongly g-closed cl(int(A) $\subseteq$A whenever A $\subseteq$U and U is open in X.

cl(int(A) $\supseteq$A since A is semi-open. Then cl(A) $\subseteq$ cl(int(A) $\subseteq$U. Hence A is g-closed.

Corollary 3.37: If A subset of a topological space (X,τ) is both strongly g-closed then it is g-closed in X.

Proof: Since every open set is semi-open the result follows from the above theorem.

Theorem 3.38: If A subset of a topological space (X,τ) is both strongly g**-closed and semi-open then it is g**-closed.

Corollary 3.39: If A subset of a topological space (X,τ) is both strongly g**-closed then it is g**-closed in X.

Theorem 3.40: If A subset of a topological space (X,τ) is both strongly g-closed then cl(int(A)) – A contains no non empty closed set.

Proof: Suppose F is a closed set such that F ⊆ cl(int(A)) – A. Then F ⊆ cl(int(A)) ∩ A^c, which implies F ⊆ cl(int(A)) and F ⊆ A^c . F ⊆ A^c implies A ⊆ F^c where F^c is open. Therefore cl(int(A)) ⊆ F^c since A is strongly g-closed.

Then F ⊆ $(cl(int(A)))^c$. Hence F ⊆ (cl(int(A))) ∩ $(cl(int(A)))^c$ = φ.

Therefore cl(int(A)) – A contains no non empty closed set.

Theorem 3.41: If A subset of a topological space (X,τ) is both strongly g**-closed then cl(int(A)) – A contains no non empty closed set.

Proof: Since every open set is g*-open the result follows from the previous theorem.

Theorem 3.42: If A is strongly g-closed and A ⊆ B ⊆cl(int(A)), then B is strongly g-closed.

Proof: Let B ⊆ U where U is open which implies A ⊆ U where U is open.

∴ cl(int(A)) ⊆ U, since A is strongly g-closed. cl(int(B)) ⊆ cl(B) ⊆ cl(int(A)) ⊆ U. ∴ B is strongly g-closed.

Theorem 3.43: If A is strongly g**-closed and A ⊆ B ⊆cl(int(A)), then B is strongly g**-closed.

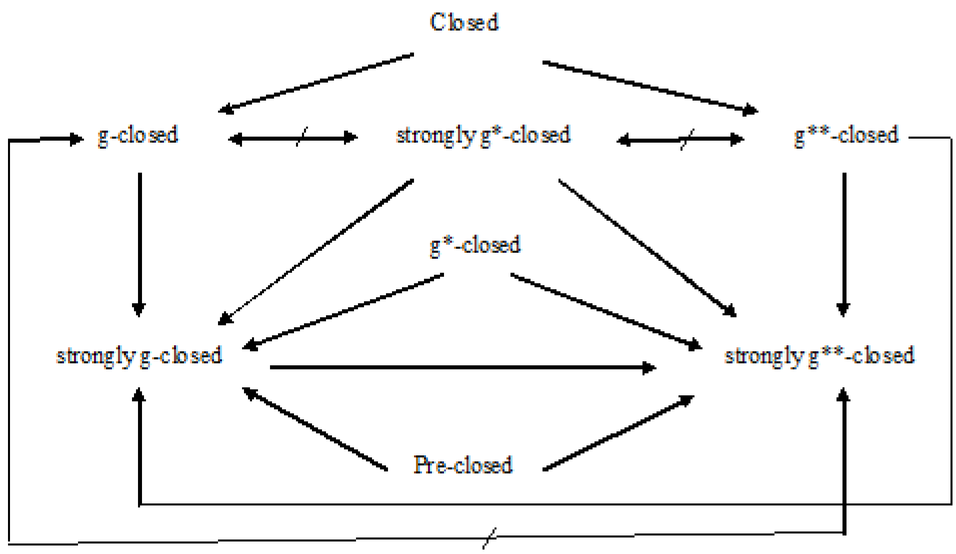
Proof is similar to the above theorem.

Proposition 3.44: Every pre-closed set is (i) strongly g-closed (ii) strongly g**-closed but not conversely.

Proof follows from the definitions.

Example 3.45: In example (3.5) A = {a, b} is strongly g-closed and strongly g**-closed but it is not pre-closed since cl(int(A)) = X ⊄ A

The above results can be represented in the following figure.



Where A ⟶ B (resp. A ⟷̸ B) represents A implies B (resp. A and B are independent).

REFENRENCES

[1] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70(1963), 36-41.

[2] N. Levine, Generalized closed sets in topology, Rend. Circ. Math. Palermo, **19**(2) (1970), 89-96.

[3] A. S. Mashhour, M.E. Abd El-Monsef and S.N. El-Deeb, On pre-continuous and weak pre-continuous mappings, Proc. Math. and Phys. Soc. Egypt, **53**(1982), 47- 53.

[4] N. Palaniappan and K.C. Rao, Regular generalized closed sets, Kyungpook Math. J., **33**(2) (1993), 211-219.

[5] R. Parimelazhagan and V. Subramania Pillai, Strongly g*-closed sets in Topological spaces, Int. Journal of Math. Analysis, Vol. 6,(30) (2012), 1481-1489.

[6] Pauline Mary Helen. M, Veronica Vijayan, Ponnuthai Selvarani, g**-closed sets in topological spaces, IJM A 3(5), (2012),1-15.

[7] M. K. R. S. Veerakumar, Between closed sets and g-closed sets, Mem. Fac. Sci. Koch. Univ. Ser. A, Math., **17**(1996), 33-42.

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