

ON R^* - CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper, a new class of sets called R^* -closed sets in topological spaces is introduced and their properties are discussed.

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Keywords: R^* - closed sets, R^* - open sets, R^* -continuous, R^* -irresoluteness.

1. INTRODUCTION

Way back in 1970, Levine [11] introduced the concept of generalized closed sets. This study has been furthered by general topologists. Stone [19], Cameron [5] Gnanamabal [9], N Palaniappan and Rao [15] introduced regular open sets, regular semi open sets ,generalized pre regular closed sets, regular generalized closed sets respectively. N.

Nagaveni [14], Sundaram and SheikJohn[18], S.S. Benchalli and R.S. Wali [4], Sharmishta Battacharya [17], Sanjay Mishra[16] delved into the study of weak generalized closed sets, weakly closed sets, regular w-closed sets, regularized weak closed sets, regular generalized weakly- closed sets respectively. This paper is an attempt to study a new class of sets called R^* -closed sets.

Throughout this paper, we consider spaces on which no separation axioms are assumed unless explicitly stated. The topology of a given space X is denoted by τ and (X, τ) is replaced by X if there is no confusion. For $A \subset X$, the closure and the interior of A in X are denoted by $cl(A)$ and $int(A)$ respectively.

2. PRELIMINARIES

Definition 2.1. A subset A of a topological space (X, τ) is called

- (1) a regular open [19] if $A = int(cl(A))$ and regular closed [19] if $A = cl(int(A))$
- (2) a pre open [13] if $A \subseteq int(cl(A))$ and pre closed [13] if $cl(int(A)) \subseteq A$.
- (3) a semi open [10] if $A \subseteq cl(int(A))$ and semi closed [10] if $int(cl(A)) \subseteq A$.
- (4) a semi-preopen [1] if $A \subseteq cl(int(cl(A)))$ and semi-pre closed [1] if $int(cl(int(A))) \subseteq A$

The intersection of all regular closed(semi- closed, pre-closed, semi- pre-closed) subset of (X, τ) containing A is called the regular closure (semi-closure, pre-closure, semi-pre-closure resp.) of A and is denoted by $rcl(A)$ ($scl(A)$, $pcl(A)$, $spcl(A)$ resp.)

Definition 2.2. [6] A subset A of a space (X, τ) is called regular semi open set , if there is a regular open set U such that $U \subset A \subset cl(U)$. The family of all regular semi open sets of X is denoted by $RSO(X)$.

Lemma2.3. [5] Every regular semi open set in (X, τ) is semi open but not conversely.

Lemma2.4. [8] If A is regular semi open in (X, τ) , then $X \setminus A$ is also an regular semi open.

Lemma2.5. [8] In a space (X, τ) , the regular closed sets, regular open sets and clopen sets are regular semi open.

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Definition2.6. [8] A subset A of a space (X, τ) is said to be semi regular open if it is both semi open and semi closed.

Definition2.7. A subset of a topological space (X, τ) is called

- (1) a generalized closed (briefly g- closed) [11] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
- (2) a semi generalized closed (briefly sg-closed) [3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.
- (3) a generalized semi closed (briefly gs-closed) [2] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
- (4) a generalized semi preclosed (briefly gsp- closed) [6] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
- (5) a regular generalized (briefly rg-closed) [15] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.
- (6) a generalized pre-closed (briefly gp- closed) [12] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
- (7) a generalized pre regular closed (briefly gpr-closed) [9] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.
- (8) a weakly generalized closed (briefly wg - closed) [14] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
- (9) a weakly closed (briefly w-closed) [18] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.
- (10) a semi weakly generalized closed (briefly swg-closed) [14] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is semi open.
- (11) a regular weakly generalized closed (briefly rwg-closed) [14] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.
- (12) a regular w-closed (briefly rw-closed)[4] if $cl(A) \subset U$ whenever $A \subset U$ and U is regular semi open in X.
- (13) a regular generalized weak closed set (briefly rgw-closed) [16] if $cl(int(A)) \subset U$ whenever $A \subset U$ and U is regular semi open.

The complements of the above mention closed sets are their respective open sets.

3. R*-CLOSED SETS AND R*-OPEN SETS

Definition 3.1. A subset A of a space (X, τ) is called R*-closed if $rcl(A) \subset U$ whenever $A \subset U$ and U is regular semi open in (X, τ) .

We denote the set of all R*- closed sets in (X, τ) by $R^*C(X)$.

Example 3.2. Let $X= \{a, b, c\}$ $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}\}$

R*-closed sets are $\{X, \phi, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.

Remark 3.3. Closed sets and R*-closed sets are independent of each other.

Example 3.4. Let $X= \{a, b, c, d\}$ $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ R*-closed sets are $\{X, \phi, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}\}$ Closed sets are $\{X, \phi, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$

Theorem 3.5.

- (1).Every R*-closed sets is rg- closed.
- (2).Every R*-closed set is gpr- closed.
- (3) Every R*-closed set is rwg-closed.
- (4).Every R*- closed sets is rw-closed.
- (5).Every R*-closed set is pr-closed.
- (6).Every R*-closed sets is rgw- closed.

Proof: Straight forward.

Converse of the theorem need not be true as seen in the following example.

Example 3.6

a) Let $X= \{a, b, c, d\}$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$

Let $A=\{c\}$. A is rg-closed, gpr closed, rwg closed, but not R*-closed set.

b) Let $X= \{a, b, c, d\}$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$

Let $A = \{d\}$ is rw-closed set but not R^* -closed set.

c) Let $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$

$R^*C(X) = \{\{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X, \phi\}$

Let $A = \{c\}$. A is pr-closed set but not R^* -closed set.

d) Let $X = \{a, b, c, d\}$ $\tau = \{X, \phi, \{a\}, \{d\}, \{a, d\}, \{a, b\}, \{a, b, d\}\}$

$R^*C(X) = \{X, \phi, \{c\}, \{c, d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}\}$

$A = \{b\}$ is rgw closed set but not R^* -closed set.

Remark 3.7. g-closed, gs-closed, gp-closed, gsp-closed sets are independent with R^* -closed sets.

Example 3.8. In example 3.4. $A = \{a, b\}$ is R^* -closed set but it is not g-closed, gs-closed, gp-closed and gsp-closed.

$B = \{d\}$ is g-closed, gs-closed, gp-closed and gsp-closed but not R^* -closed set.

Remark 3.9. The following example shows that R^* -closed sets are independent of wg-closed, w-closed, sg-closed, swg-closed.

Example 3.10. Let $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$.

(1) Closed sets are $\{X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{d\}\}$

(2) R^* -closed set are $\{X, \phi, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$.

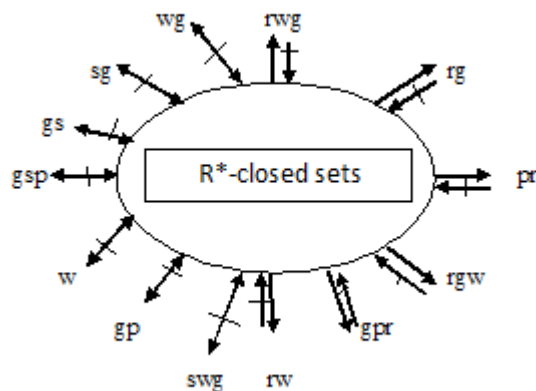
(3) wg-closed sets are $\{\{c\}, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}, X, \phi\}$.

(4) w-closed sets are $\{\{d\}, \{c, d\}, \{b, c, d\}, \{a, c, d\}, X, \phi\}$.

(5) sg-closed sets are $\{\{a\}, \{b\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}, \{a, c, d\}, X, \phi\}$.

(6) swg-closed sets are $\{\{c\}, \{d\}, \{c, d\}, \{b, c, d\}, \{a, c, d\}, X, \phi\}$.

Remark 3.11 From the above discussion we have the following diagram.



Theorem 3.12. The union of the two R^* -closed sets is an R^* -closed subset of X .

Proof: Assume that A and B are R^* -closed sets in X . Let U the regular semi open in X such that $(A \cup B) \subset U$. Then $A \subset U$ and $B \subset U$. Since A and B are R^* -closed, $\text{rcl}(A) \subset U$ and $\text{rcl}(B) \subset U$ respectively. Hence $\text{rcl}(A \cup B) \subset U$. Therefore $A \cup B$ is R^* -closed.

Remark 3.13 The intersection of two R^* -closed sets in X need not be R^* -closed in X .

Example 3.14. Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$. R^* -closed sets are $\{X, \phi, \{b\}, \{a, b\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}\}$

Let $A = \{a, d\}$ and $B = \{a, b\}$

Therefore $A \cap B = \{a\} \notin R^*$ - closed set.

Theorem 3.15. If a subset A of X is R^* -closed set in X , then $\text{rcl}(A) \setminus A$ does not contain any non-empty regular semi open set in X .

Proof: Suppose that A is R^* -closed set in X . Let U be a regular semi open set such that $\text{rcl}(A) \setminus A \supset U$ and $U \neq \emptyset$. Now $U \subset X \setminus A$ implies $A \subset X \setminus U$. Since U is regular semi open, by Lemma 2.4. $X \setminus U$ is also regular semi open in X . Since A is R^* -closed in X , by definition $\text{rcl}(A) \subset X \setminus U$. So $U \subset X \setminus \text{rcl}(A)$, hence $U \subset \text{rcl}(A) \cap X \setminus \text{rcl}(A) = \emptyset$. This shows that $U = \emptyset$, which is a contradiction.

Hence $\text{rcl}(A) \setminus A$ does not contain any non-empty regular semi open set in X .

Remark 3.16. If $\text{rcl}(A) \setminus A$ contain no non-empty regular semi open subset of X , then A need not to be R^* -closed

Example.3.17. In example 3.1 $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b, c\}\}$

Let $A = \{a\}$. $\text{rcl}(A) \setminus A = \{c, d\}$ does not contain a non empty regular semi open set, but $A = \{a\}$ is not R^* -closed.

Corollary 3.18. If a subset A of X is R^* -closed in X , then $\text{rcl}(A) \setminus A$ does not contain any non empty regular open set in X .

Proof: Follows from theorem 3.13, since every regular open set is regular semi open.

Corollary 3.19. If a subset A of X is R^* -closed set in X then $\text{rcl}(A) \setminus A$ does not contain any non empty regular closed set in X .

Proof: Follows the theorem 3.13 and the fact that every regular closed set is regular semi open.

Theorem 3.20. For any element $x \in X$. The set $X \setminus \{x\}$ is R^* -closed or regular semi open.

Proof: Suppose $X \setminus \{x\}$ is not regular semi open, then X is the only regular semi open set containing $X \setminus \{x\}$. This implies $\text{rcl}(X \setminus \{x\}) \subset X$. Hence $X \setminus \{x\}$ is R^* -closed or regular semi open set in X .

Theorem 3.21. If A is regular open and R^* -closed. Then A is regular closed and hence r -clopen.

Proof: Suppose A is regular open and R^* -closed. $A \subset \text{rcl}(A)$ and by hypothesis $\text{rcl}(A) \subset A$. Also $A \subset \text{rcl}(A)$, so $\text{rcl}(A) = A$. Therefore A is regular closed and hence r -clopen.

Theorem 3.22. If A is an R^* -closed subset of X such that $A \subset B \subset \text{rcl}(A)$, then B is an R^* -closed set in X .

Proof: Let A be an R^* -closed set of X such that $A \subset B \subset \text{rcl}(A)$. Let U be a regular semi open set of X such that $B \subset U$, then $A \subset U$. Since A is R^* -closed, we have $\text{rcl}(A) \subset U$. Now $\text{rcl}(B) \subset \text{rcl}(\text{rcl}(A)) = \text{rcl}(A) \subset U$, therefore B is an R^* -closed set in X .

Remark 3.23. The converse of the theorem 3.22 need not be true.

Example 3.24. Consider the topological space (X, τ) , where $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Let $A = \{c\}$, $B = \{c, d\}$, $\text{rcl}(A) = \{c, d\}$

Then A and B are such that $A \subset B \subset \text{rcl}(A)$ where B is R^* -closed set in (X, τ) but A is not R^* -closed set in (X, τ) .

Theorem 3.25. Let A be the R^* -closed in (X, τ) . Then A is regular closed if and only if $\text{rcl}(A) \setminus A$ is regular semi open.

Proof: Suppose A is regular closed in X . Then $\text{rcl}(A) = A$ and so $\text{rcl}(A) \setminus A = \emptyset$, which is regular semi open in X .

Conversely, suppose $\text{rcl}(A) \setminus A$ is regular semi open in X . Since A is R^* -closed by theorem 3.13 $\text{rcl}(A) \setminus A$ does not contain any non-empty regular semi open set in X . Then $\text{rcl}(A) \setminus A = \emptyset$. Hence A is regular closed in X .

Theorem 3.26. If a subset A of a topological space X is both regular semi open and R*- closed, then it is regular closed.

Proof: By hypothesis, we have $\text{rcl}(A) \subset A$. Hence A is regular closed.

Theorem 3.27. In a topological space X , if $\text{RSO}(X) = \{X, \phi\}$, then every subset of X is an R*-closed set.

Proof: Let X be a topological space and $\text{RSO}(X) = \{X, \phi\}$. Let A be any subset of X. Suppose $A = \phi$, then ϕ is an R*-closed set in X. Suppose $A \neq \phi$, then X is the only regular semi open set containing A and so $\text{rcl}(A) \subset X$. Hence A is R*- closed set in X.

Remark 3.28. The converse of theorem 3.27 need not to be true in general as seen from the following example.

Example 3.29. Let $X = \{a, b, c, d\}$ be with the topology $\tau = \{X, \phi, \{a, b\}, \{c, d\}\}$. Then every subset of X is R*-closed set in X. But $\text{RSO}(X) = \{X, \phi, \{a, b\}, \{c, d\}\}$.

Definition 3.30. A subset A in X is called R*-open if A^c is R*-closed in X.

Theorem 3.31. A subset A of X is said to be R*-open if $F \subseteq \text{rint}(A)$ whenever F is regular semi open and $F \subseteq A$.

Proof: Necessity .Let F be regular semi open such that $F \subseteq A$. $X - A \subseteq X - F$. Since X-A is R*-closed, $\text{rcl}(X-A) \subseteq X - F$. Thus $F \subseteq \text{rint}(A)$.

Sufficiency. Let U be any regular semi open set such that $X - A \subseteq U$.We have $X - U \subseteq A$ and by hypothesis $X - U \subseteq \text{rint}(A)$. That is $\text{rcl}(X-A) = X - \text{rint}(A) \subseteq U$.Therefore (X-A) is R*-closed and hence A is R*-open.

Theorem 3.32. Finite intersection of R*-open sets is R*-open.

Proof: Let A and B be R*-open sets in X . Then $A^c \cup B^c$ is R*-closed set. This implies $(A \cap B)^c$ is R*-closed set. Therefore $A \cap B$ is R*-open.

Theorem 3.33. If A is R*-closed subset of (X, τ) and F be a regular closed set in $\text{rcl}(A) \setminus A$, then R*-open set.

Proof: Let A be an R*-closed subset of (X, τ) and F be a regular closed subset such that $F \subseteq \text{rcl}(A) - A$. By corollary 3.19. , $F = \phi$ and thus $F \subseteq \text{rint}(\text{rcl}(A) - A)$.

By Theorem 3.31, $\text{rcl}(A) - A$ is R*- open.

Lemma 3.34. If the regular open and regular closed sets of X coincide, then all subset of X are R*-closed (and hence all are R*-open).

Proof: Let A be a subset of X which is regular open such that $A \subseteq U$ and U is regular open, then $\text{rcl}(A) \subseteq \text{rcl}(U) \subseteq U$.

Therefore A is R*-closed.

4. R*-CONTINUOUS AND R*-IRRESOLUTE FUNCTIONS

Definition 4.1. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called R*-continuous function if every $f^{-1}(V)$ is R*closed in (X, τ) for every closed set V in (Y, σ) .

Definition 4.2. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called R*-irresolute if $f^{-1}(V)$ is R*-closed in (X, τ) for every R*-closed set V in (Y, σ) .

Example 4.3. Let $X = \{a, b, c, d\}$ $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}, \{a, b\}, \{a, b, c\}\}$ and $Y = X$

$\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}\}$. $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity mapping, then f is R*-continuous.

Example 4.4. $X = \{a, b, c, d\}$ $\tau = \{X, \phi, \{a\}, \{d\}, \{a, d\}, \{a, b\}, \{a, b, d\}\}$ $Y = X$ and $\sigma = \{Y, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$
 Define the function $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = d, f(b) = a, f(c) = b, f(d) = c$.

The inverse image of every R^* -closed sets is R^* -closed under f . Hence f is R^* -irresolute.

Remark 4.5. The composition of two R^* -continuous function need not be R^* -continuous.

Example 4.6 . Let $X = \{a, b, c, d\} = Y = Z$ $\tau = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$
 $\sigma = \{X, \phi, \{a\}, \{a, b\}, \{a, b, c\}\}$, $\eta = \{X, \phi, \{a\}, \{d\}, \{a, d\}, \{a, b\}, \{a, b, d\}\}$.

Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = a, f(b) = d, f(c) = b, f(d) = c$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ by $g(a) = a, g(b) = d, g(c) = b, g(d) = c$. Here both f and g are R^* -continuous but $g \circ f$ is not R^* -continuous.

Remark 4.7. R^* -continuity and continuity are independent concepts.

Example 4.8. Let $X = \{a, b, c, d\}$ $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}, \{a, b\}, \{a, b, c\}\}$ and $Y = \{a, b, c, d\}$ $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}\}$ defined by the function $f: (X, \tau) \rightarrow (Y, \sigma)$, the identity mapping. f is R^* - continuous function but not continuous .

Example 4.9. Let $X = \{a, b, c, d\}$ $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ $Y = \{a, b, c, d\}$ $\sigma = \{Y, \phi, \{a, b, c\}\}$ $f: X \rightarrow Y$, the identity mapping . f is continuous but not R^* -continuous .

Remark 4.10. Every R^* -irresolute function is R^* -continuous but not conversely.

Example 4.11. Let $X = \{a, b, c, d\}$ $\tau = \{ \{a\}, \{c\}, \{a, c\}, \{a, b\}, \{a, b, c\}, X, \phi \}$ $Y = \{a, b, c, d\}$ $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}\}$. Define $f: X \rightarrow Y$, the identity mapping. f is R^* -continuous but not R^* -irresolute.

Theorem 4.12.

- (a) Every R^* -continuous mapping is rw-continuous
- (b) Every R^* -continuous mapping is rg-continuous.
- (c) Every R^* -continuous mapping is pr-continuous
- (d) Every R^* -continuous mapping is rwg-continuous
- (e) Every R^* -continuous mapping is rgw-continuous
- (f) Every R^* -continuous mapping is gpr-continuous.

Proof: Obvious

The converse of the above need not be true as seen in the following examples.

Example 4.13. Consider $X = \{a, b, c, d\}$ $\tau = \{ X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ and $Y = \{a, b, c, d\}$, $\sigma = \{Y, \phi, \{a, b, c\}\}$.
 Define $f: (X, \tau) \rightarrow (Y, \sigma)$, the identity mapping.

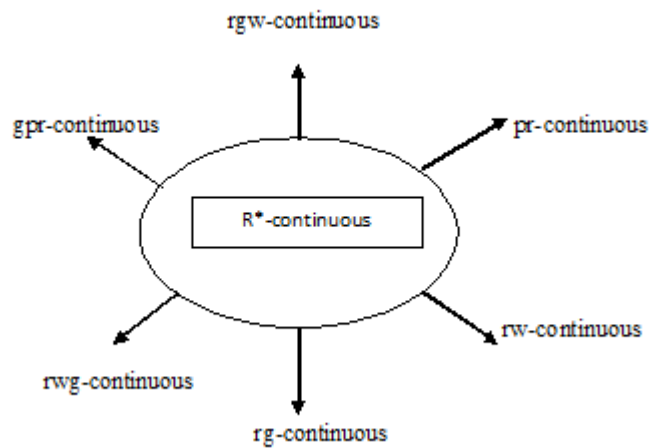
The mapping f is both rw-continuous and rg- continuous but not R^* -continuous.

Example 4.14. $X = \{a, b, c, d\}, \tau = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$ and $Y = \{a, b, c, d\}, \sigma = \{Y, \phi, \{a, b, c\}\}$

Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = a, f(b) = b, f(c) = d, f(d) = c$

The function f is pr-continuous, rwg-continuous, rgw-continuous and gpr-continuous but not R^* -continuous.

Remark 4.15. From the above theorem the following diagram is implicated.



Theorem:4.16.

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be any two function, then

- (1). $g \circ f$ is R^* -continuous if g is continuous and f is R^* -continuous.
- (2). $g \circ f$ is R^* -irresolute if g is R^* -irresolute and f is R^* -irresolute.
- (3). $g \circ f$ is R^* -continuous if g is R^* -continuous and f is R^* -irresolute.

Proof:

- (1). Let V be closed set in (Z, η) . Then $g^{-1}(V)$ is closed set in (Y, σ) , since g is continuous and R^* -continuity of f implies $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is R^* -closed in (X, τ) . That is $(g \circ f)^{-1}(V)$ is R^* -closed in (X, τ) . Hence $g \circ f$ is R^* continuous.
- (2). Let V be R^* -closed set in (Z, η) . Since g is irresolute, $g^{-1}(V)$ is R^* -closed set in (Y, σ) . As f is R^* -irresolute $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is R^* -closed in (X, τ) . Hence $g \circ f$ is R^* -irresolute.
- (3). Let V be closed in (Z, η) . Since g is R^* -continuous, $g^{-1}(V)$ is R^* -closed in (Y, σ) . As f is R^* -irresolute $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is R^* -closed in (X, τ) . Therefore $(g \circ f)$ is R^* -continuous.

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