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ON THE SOLUTION OF MULTIPLE OBJECTIVE INTEGER LINEAR FRACTIONAL PROGRAMMING PROBLEMS WITH UNCERTAIN DATA

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ABSTRACT

An algorithm for solving a multiobjective integer linear fractional programming problem with random parameters in the objective functions (SMOILFP) is investigated. These random parameters are considered to be statistically independent random variables. The basic idea in the developed solution algorithm in treating the (SMOILFP) is to convert the probabilistic nature of this problem into a deterministic version and thus the problem is reduced to a single objective. A numerical example has been given as a problem test of (SMOILFP) and LINGO software packages has been used in solving the numerical example.

Keywords: *Multi-objective Programming, Fractional Programming, Stochastic Programming, Integer Programming.*

MSC 2000: *90C10; 90C15; 90C 29; 90C32.*

1. INTRODUCTION

One of the common problems in the practical application of mathematical programming is the difficulty for determining the proper values of model parameters. The values of these parameters are often influenced by random events that are impossible to predict i.e., some or all of the model parameters may be random variables. What is needed is a way to formulating the problem so that the optimization will directly consider the uncertainty. One such approach for mathematical programming under uncertainty is Stochastic Programming (SP).

The SP is an optimization technique in which the constraints and/or the objective function of an optimization problem contains certain random variables. In the stochastic linear programming literature (Infanger [1], Kall and Wallace [2]), several researchers suggested various models. A bibliography has been presented by Stancu-Minasian and Wets [27]. Model coefficients of most of these models are assumed to follow independent normal distribution because deriving the deterministic equivalent of the objective function and/or constraints of the model is well known (Kall and Wallace [2]) in this case.

In recent years methods of stochastic optimization have become increasingly important in scientifically based decision-making involved in practical problems arising in economic, industry, healthcare, transportation, agriculture, military purposes and technology.

Fractional programming has been widely reviewed by many authors (Schaible [3], Nagih and Plateau [4]) and there are entire books and chapters devoted to this subject (Craven [5], Stancu-Minasian [6], Horst et al. [7] and Frenk and Schaible [8]). Schaible [3] has published a comprehensive review of the work in fractional programming, outlining some of its major developments. Stancu-Minasian's textbook [6] contains the state-of-the-art theory and practice of fractional programming, allowing the reader to quickly become acquainted with what has been done in the field.

Different approaches have been proposed in the literature to solve both continuous linear fractional programming (LFP) and integer linear fractional programming (ILFP) problems. These can be divided in studies that have developed solution methods (e.g., [6, 9-13]) and those which concentrated on applications (e.g. [6, 8]).

The multiple objective linear fractional programming (MOLFP) problem is one of the most popular models used in multiple criteria decision making. Numerous studies and applications have been reported in the literature in hundreds of books, monographs, articles, and books' chapters. For an overview of these studies and applications, see, for instance, [6, 14-21], and references therein.

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In the present paper we develop a solution procedure for solving stochastic multiobjective integer linear fractional programming problem with random parameters in the objective functions.

This paper is organized as follows: The problem formulation and the solution concept are introduced in section 2. In section 3, a solution procedure to solve the formulated problem is described. An illustrative example as a test problem is given in section 4 to clarify the solution algorithm and finally, the paper is concluded in section 5 with some points for further research.

2. PROBLEM FORMULATION AND THE SOLUTION CONCEPT

In what follows, we consider the multiobjective integer linear fractional programming problems with random parameters in the objective functions:

$$\begin{gathered}(\text{SMOILFP}): F(x)=\{F_1(x),F_2(x),\ldots,F_k(x)\}\\ \text{subject to}\\ x\in \mathrm{M}\\ F_i(x)=\frac{N_i(x)+\alpha_i}{D_i(x)+\beta_i}=\frac{\sum_{j=1}^{q}c_{ij}x_j+\alpha_i}{\sum_{j=1}^{q}d_{ij}x_j+\beta_i},(i=1,2,\ldots,k)\end{gathered} \tag{1}$$

In the above problem, $k \geq 2$, α_i , β_i are scalars for each $i = (1,2,\ldots,k)$, $N_i(x)$ and $D_i(x)$ are random variables. For the sake of simplicity, we assume that $N_i(x) \sim N\left(\sum_{j=1}^{n}u_{cij}x_j, \sum_{j=1}^{n}s_{cij}^2x_j^2\right)$ and $D_i(x) \sim N\left(\sum_{j=1}^{n}u_{dij}x_j, \sum_{j=1}^{n}s_{dij}^2x_j^2\right)$ where u_{cij} , u_{dij} are means and s_{cij}^2 , s_{dij}^2 are variances.

The set M is defined as the feasible region and might be, for example, of the form:

$$M = \{x \in R^n \,|Ax \leq b,\ x \geq 0 \text{ and integer}\} \tag{2}$$

where A is an $m \times n$ real matrix, x is an n-vector of integer decision variables, b is an m-vector of the constraints right-hand sides, R^n is the n-dimensional Euclidean space and T denotes the transpose.

Definition 1 [24]. A point $x^* \in M$ is said to be an efficient solution for the problem (SMOILFP) if and only if there does not exist another $x \in M$ such that $F(x) \geq F(x^*)$ and $F(x) \neq F(x^*)$.

Definition 2 [24]. A point $x^* \in M$ is said to be a weakly efficient solution for the problem (SMOILFP) if and only if f there does not exist another $x \in M$ such that $F(x) \geq F(x^*)$.

In what follows, an equivalent stochastic multiobjective linear fractional programming problem (SMOLFP) associated with problem (SMOILFP) can be stated with the help of cutting-plane technique [22] together with Balinski algorithm [23]. This equivalent (CHMOLFP) can be written in the following form:

$$\begin{gathered}(\text{SMOLFP}): F(x)=\{F_1(x),F_2(x),\ldots,F_k(x)\}\\ \text{subject to}\\ x\in [\mathrm{M}]\\ F_i(x)=\frac{N_i(x)+\alpha_i}{D_i(x)+\beta_i}=\frac{\sum_{j=1}^{q}c_{ij}x_j+\alpha_i}{\sum_{j=1}^{q}d_{ij}x_j+\beta_i},(i=1,2,\ldots,k)\end{gathered} \tag{4}$$

where $[\mathrm{M}]$ is the convex hull of the feasible region M defined by (2) earlier.

This convex hull is defined by

$$[M]=M_R^{(s)}=\left\{x\in R^n \,\middle|\, A^{(s)}x\le b,\ x\ge 0\right\} \tag{5}$$

and in addition,

$$A^{(s)}=\begin{bmatrix} A\\ \dots\\ a_1\\ \vdots\\ a_s\end{bmatrix} \text{ and } b^{(s)}=\begin{bmatrix} b\\ \dots\\ b_1\\ \vdots\\ b_s\end{bmatrix} \tag{6}$$

are the original constraint matrix A and the right-hand side vector b, respectively, with s-additional constraints to an efficient non-redundant cut in the form $a_i x \le b_i$.

The basic idea in treating problem (SMOLFP) is to convert the probabilistic nature of this problem into a deterministic version. Here, the idea of employing deterministic version will be illustrated by using the interesting technique which is indicated in [24, 25]. In this case, the objective functions can be rewritten in the deterministic form as:

Case 1: when $\alpha_i>0,(i=1,2,\dots,k)$

$$\lambda_i\left[\sum_{j=1}^{n}u_{dij}x_j+\beta_i\right]-\sum_{j=1}^{n}u_{cij}x_j+\phi^{-1}\left(q_i^{(2)}\right)\sqrt{\sum_{j=1}^{n}\left(\lambda_i^2 s_{dij}^2+s_{cij}^2\right)x_j^2}\le\alpha_i \tag{7}$$

where λ_i $(i=1,2,\dots,k)$ are unknown parameters, it is less than or equal to $F_i(x)$. That is $F_i(x)\ge\lambda_i$ i.e. $\dfrac{N_i(x)+\alpha_i}{D_i(x)+\beta_i}\ge\lambda_i \Rightarrow 0\le N_i(x)+\alpha_i-\lambda_i\left[D_i(x)+\beta_i\right]$

Case 2: when $\alpha_i\le 0,(i=1,2,\dots,k)$

$$\sum_{j=1}^{n}u_{cij}x_j-\lambda_i\left[\sum_{j=1}^{n}u_{dij}x_j+\beta_i\right]+\phi^{-1}\left(p_i^{(2)}\right)\sqrt{\sum_{j=1}^{n}\left(\lambda_i^2 s_{dij}^2+s_{cij}^2\right)x_j^2}\ge\alpha_i \tag{8}$$

As indicated in [24], a classic method to generate dominated solutions is to use the weighted sums of the objective functions. Define a new unction namely linear stochastic scalarizing function as follows

$$\max_{\lambda\in M'}\sum_{i=1}^{k}\xi_i\lambda_i \tag{9}$$

where $\xi_i\in\xi\subset(0,1),(i=1,2,\dots,k)$ is related stochastic weight in the objective function λ_i and M' may be the defined solution space. Assume that the set ξ is finite and hence compact as well $\sum_{i=1}^{k}\xi_i=1$.

Case 1: when $\alpha_i>0,(i=1,2,\dots,k)$

$$\max_{\lambda\in M'}\sum_{i=1}^{k}P_i^{(2)}\lambda_i \tag{10}$$

$$0<P_1^{(2)}<P_2^{(2)}<\dots<P_k^{(2)}<1 \tag{11}$$

$$\sum_{i=1}^{k}P_i^{(2)}=1 \tag{12}$$

$\sum_{i=1}^{k}q_i^{(2)}$ may not be equal to one.

where M' is non-empty, convex and compact set.

Definition 3[24]. In equation (10), $\lambda_i^* \in \lambda$ is efficient if and only if there does not exist another $\lambda_i \in \lambda$ such that $\sum_{i=1}^{k} P_i^{(2)}\lambda_i \geq \sum_{i=1}^{k} P_i^{(2)}\lambda_i^*$ and $\sum_{i=1}^{k} P_i^{(2)}\lambda_i \neq \sum_{i=1}^{k} P_i^{(2)}\lambda_i^*$ along with the conditions equations (11) and (12) where $\lambda \in R^k$ is a feasible set.

Case 2: when $\alpha_i \leq 0, (i = 1, 2, \ldots, k)$

$$\max_{\lambda \in M'} \sum_{i=1}^{k} q_i^{(2)}\lambda_i \tag{13}$$

$$0 < q_1^{(2)} < q_2^{(2)} < \ldots < q_k^{(2)} < 1 \tag{14}$$

$$\sum_{i=1}^{k} q_i^{(2)} = 1 \tag{15}$$

$\sum_{i=1}^{k} p_i^{(2)}$ may not be equal to one.

where M' is non-empty, convex and compact set.

Definition 4[24]. In equation (13), $\lambda_i^* \in \lambda$ is efficient if and only if there does not exist another $\lambda_i \in \lambda$ such that $\sum_{i=1}^{k} q_i^{(2)}\lambda_i \geq \sum_{i=1}^{k} q_i^{(2)}\lambda_i^*$ and $\sum_{i=1}^{k} q_i^{(2)}\lambda_i \neq \sum_{i=1}^{k} q_i^{(2)}\lambda_i^*$ along with the conditions equations (14) and (15) where $\lambda \in R^k$ is a feasible set.

3. A PROPOSED SOLUTION ALGORITHM FOR SOLVING PROBLEM (SMOILFP)

In this section, a solution algorithm to solve multiobjective integer linear fractional programming problem with random parameter in the objective functions (SMOILFP) is described in a series of steps. The suggested algorithm can be summarized in the follow manner:

Step1. Characterize the set $[M] = M_R^{(s)}$.

Step2. Formulate the deterministic multiobjective integer linear fractional programming problem (MILFP) corresponding to the problem (SMOILFP).

Step3. Formulate the integer linear fractional programming problem with a single-objective function.

Step4. Solve the integer linear fractional programming problem with a single-objective function by using LINGO [26] software to obtain the efficient integer solution x^* with the corresponding optimal parameters $\lambda^* \in R^k$.

4. TEST PROBLEM

In this section, an illustrative example is given to clarify the proposed solution algorithm This example is adapted from one appearing in Charles, V. and Dutta, D. [9] and the LINGO [11] software package is used in the computational process.

The problem to be solved here is the following multiobjective integer linear fractional programming problem involving random parameters in the objective functions:

$(SMOILFPP)$

$$\max\ F(x) = \left\{ \frac{c_{11}x_1 + c_{12}x_2 + \alpha_1}{d_{11}x_1 + d_{12}x_2 + \beta_1}, \frac{c_{21}x_1 + c_{22}x_2 + \alpha_2}{d_{21}x_1 + d_{22}x_2 + \beta_2}, \frac{c_{31}x_1 + c_{32}x_2 + \alpha_3}{d_{31}x_1 + d_{32}x_2 + \beta_3} \right\}$$

$$\begin{aligned} \text{subject to} \quad & -x_1 + 4x_2 \le 0 \\ & 2x_1 - x_2 \le 8 \\ & x_1, x_2 \ge 0 \text{ and integer} \end{aligned}$$

where $\alpha_1 = -1,\ \alpha_2 = 0,\ \alpha_3 = 0,\ \beta_1 = \beta_2 = \beta_3 = 1$

The convex hull $M_R^{(s)} = [M]$ is given by

$$[M] = \left\{ x \in R^2 \middle| -x_1 + 4x_2 \le 0,\ 2x_1 - x_2 \le 8,\ x_1 \le 4,\ x_1, x_2 \ge 0 \right\}$$

where $s = 1$ an efficient Gomory cut: $x_1 \le 4$ and then the problem (SMOILFPP) can be formulated as:

$$\max\ F(x) = \left\{ \frac{c_{11}x_1 + c_{12}x_2 + \alpha_1}{d_{11}x_1 + d_{12}x_2 + \beta_1}, \frac{c_{21}x_1 + c_{22}x_2 + \alpha_2}{d_{21}x_1 + d_{22}x_2 + \beta_2}, \frac{c_{31}x_1 + c_{32}x_2 + \alpha_3}{d_{31}x_1 + d_{32}x_2 + \beta_3} \right\}$$

$$\text{subject to} \quad x \in [M]$$

The means and variances of the random variables are given in the following table

r.v	c_{11}	c_{12}	c_{21}	c_{22}	c_{31}	c_{32}	d_{11}	d_{12}	d_{21}	d_{22}	d_{31}	d_{32}
mean	6	3	10	2	5	3	7	4	8	2	2	8
variance	4	1	2	1	3	1	4	2	5	1	1	5

Let us take $P_1^{(2)} = 0.10,\ P_2^{(2)} = 0.30,\ P_3^{(2)} = 0.60$. The deterministic equivalent with single-objective function of the problem is given as

$$\begin{aligned} & \max\ 0.10\lambda_1 + 0.30\lambda_2 + 0.60\lambda_3 \\ & \text{subject to} \\ & 6x_1 + 3x_2 - \lambda_1(7x_1 + 4x_2 + 1) - 1.285\sqrt{(4\lambda_1^2 + 4)x_1^2 + (2\lambda_1^2 + 1)x_2^2} \ge -1 \\ & 10x_1 + 2x_2 - \lambda_2(8x_1 + 2x_2 + 1) - 0.525\sqrt{(5\lambda_2^2 + 2)x_1^2 + (\lambda_1^2 + 1)x_2^2} \ge 0 \\ & 5x_1 + 3x_2 - \lambda_3(2x_1 + 8x_2 + 1) + 0.255\sqrt{(\lambda_3^2 + 3)x_1^2 + (5\lambda_3^2 + 1)x_2^2} \ge 0 \\ & -x_1 + 4x_2 \le 0 \\ & 2x_1 - x_2 \le 8 \\ & x_1 \le 4 \\ & x_1, x_2 \ge 0 \text{ and integer} \\ & \lambda_1, \lambda_2, \lambda_3 \ge 0 \end{aligned}$$

The optimal integer solution is obtained as follows:

$$x_1 = 4,\ x_2 = 0,\ \lambda_1 = 0.4703350,\ \lambda_2 = 1.039035,\ \lambda_3 = 2.573822.$$

and the corresponding objective function value of the problem (SMOILFPP) is found $[0.7931,\ 1.2121,\ 2.2222]$

5. CONCLUSION

In this paper, we have suggested a procedure for solving multiobjective integer linear fractional programming problem with random parameters in the objective functions. An illustrative numerical example has been given to clarify the suggested solution procedure.

There are however several open points for future research in the area of stochastic multiobjective integer linear fractional programming to be studied. Some of these points of interest are stated in the following:

1. A study is needed to investigate multiobjective integer linear fractional programming problem having random parameters in the objective functions and in the constraints.
2. A solution method is required for solving chance-constrained large-scale multiobjective integer linear fractional programming problem.
3. An algorithm is should be suggested to solve multiobjective bi-level integer linear and nonlinear fractional programming problems under fuzziness and randomness.

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