International Journal of Mathematical Archive-3(8), 2012, 2978-2982
IJMA Available online through www.ijma.info ISSN 2229 – 5046

Polynomials over Euclidean Domain in Noetherian Regular Delta Near Ring Some Problems related to Near Fields of Mappings (PED-NR-Delta-NR & SPR-NFM)

N. V. Nagendram[1*]
Assistant Professor (Mathematics), Department of Science & Hmanities Lakireddy Balireddy College of Engineering Krishna District, Mylavaram 521 230, Andhra Pradesh, INDIA

B. Ramesh[2]
Assistant Professor (Mathematics), Department of Science & Humanities Lakireddy Balireddy College of Engineering Krishna District, Mylavaram 521 230, Andhra Pradesh, INDIA

(Received on: 05-08-12; Accepted on: 22-08-12)

ABSTRACT

In this paper we discuss some preliminaries and results obtained on Polynomials over Euclidean Domain in Noetherian Regular Delta Near-Rings and some problems related to Near-fields as an extension. Also, three areas of research relative to Near Fields of mappings and mentioned several questions.

Subject Classification Code: 2000 16D10; *16Y30; 20K30.*

Key words: *Polynomial, Euclidean Domain, Near Ring, Near-Field, Delta-Near Ring, Regular delta near-ring, Homogeneous functions, Forcing Linearity Numbers, rays, Near-Fields of Mappings.*

SECTION 1 INTRODUCTION:

This paper is an expanded version on Near-Fields of homogeneous functions and this paper may be thought of as a continuation of the paper, "Near – Fields of Homogeneous functions P^3 on Near Fields and K-Loops.

We discuss three areas of research related to near fields of mappings. The first , forcing linearity numbers, had been going numerous investigations. The second area rays, had its origins. The third area of research sub-fields of the zero-symmetric Near-Fields of functions on an abelian group.

Albrecht and Hausen studied near-ring of mappings, subrings of the zero-symmetric near rings of functions on abelian groups. Most likely they have undergone several iterations before reaching definitive direction.

We fix some notation for the remainder of the paper. Let N be a Near Field, always with identity. An N-module V will always mean a Unital N-Module and we denote the collection of (Left) N-modules by N-Mod. A function f: $P \rightarrow Q$ where P, Q $\in$ N – Mod is homogeneous if f(nm) = n. f(m) for all n $\in$ N, m $\in$ P. The additive group of homogeneous functions from $P \rightarrow Q$ is denoted by M_N(P, Q) and the near-Field of homogeneous functions on P is denoted by M_N (P). As usual Hom_N (P, Q) will denote the abelian group of N-Homomorphism from P to Q and End N (P), the field of Endomorphism on P.

SECTION 2 PRELIMINARIES AND FORCING LINEARITY NUMBERS:

In this section we give the preliminary definitions and examples and the required literature to this paper.

Definition 2.1: A Near – Ring is a set N together with two binary operations "+" and "." Such that

(i) (N, +) is a Group not necessarily abelian
(ii) (N, .) is a semi Group and
(iii) for all n_1, n_2, n_3 $\in$ N, $(n_1 + n_2). n_3 = (n_1 . n_3 + n_2 . n_3)$ i.e. right distributive law.

Examples 2.2: Let M_{2x2} ={ {(aij) / Z ; Z is treated as a near-ring}. M_{2x2} under the operation of matrix addition '+' and matrix multiplication'.'.

Corresponding author: *N V Nagendram*, Assistant Professor, Department of Mathematics (S&H) Lakireddy Balireddy College of Engineering, L B Reddy Nagar, Mylavaram 521 230, Krishna District Andhra Pradesh, India*

Example 2.3: Z be the set of positive and negative integers with 0. (Z, +) is a group. Define '.' on Z by a . b = a for all a, b $\in$ Z. Clearly (Z, +, .) is a near-ring.

Example 2.4: Let Z_{12} = {0, 1, 2,…, 11}. (Z12, +) is a group under '+' modulo 12.Define '.' on Z_{12} by a. b = a for all a $\in$ Z12. Clearly (Z_{12}, +, .) is a near-ring.

Definition 2.5: A near-ring N is Regular Near-Ring if each element a Є N then there

exists an element x in N such that a = axa.

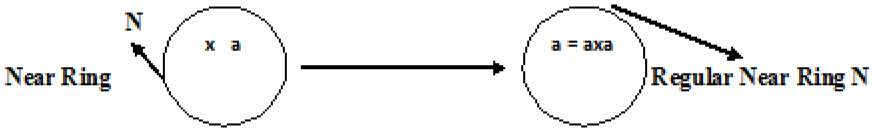


Fig. 1

Definition 2.6 : A Commutative ring N with identity is a Noetherian Regular δ-Near Ring if it is Semi Prime in which every non-unit is a zero divisor and the Zero ideal is Product of a finite number of principle ideals generated by semi prime elements and N is left simple which has N_0 = N, N_e = N.

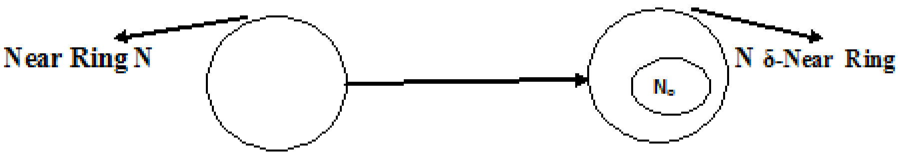


Fig. 2

Definition 2.7: A Noetherian Regular delta Near Ring (is commutative ring) N with identity, the zero-divisor graph of N, denoted Γ(N), is the graph whose vertices are the non-zero zero-divisors of N with two distinct vertices joined by an edge when the product of the vertices is zero.

Note 2.8: We will generalize this notion by replacing elements whose product is zero with elements whose product lies in some ideal I of N. Also, we determine (up to isomorphism) all Noetherian Regular delta near rings N_i of N such that Γ(N) is the graph on five vertices.

Definition 2.9: A near-ring N is called a δ-Near – Ring if it is left simple and N_0 is the smallest non-zero ideal of N and a δ-Near – Ring is a non-constant near ring.

Definition 2.10: A δ-Near-Ring N is isomorphic to δ-Near-Ring and is called a Regular δ-Near-Ring if every δ-Near-Ring N can be expressed as sub-direct product of near-rings {Ni}, Ni is a non-constant near-ring or a δ-Near-Ring N is sub-directly irreducible δ-Near-Rings Ni.

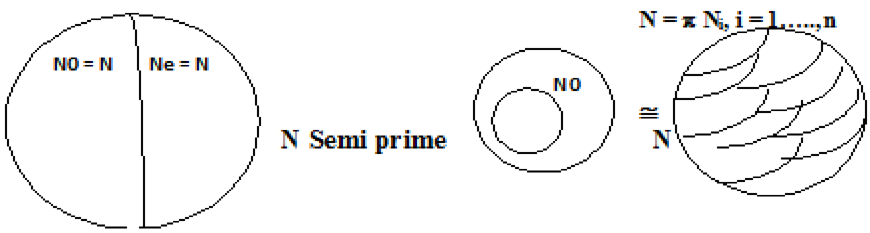


Fig. 3

Definition 2.11: Let N be a Commutative Ring. Let N be a Noetherian Regular δ-Near-Ring if each P $\in$ A (N_N) is strongly prime i.e., P is a δ-Near – Ring of N.

Example: 2.12: Let N = $\begin{bmatrix} F & F \\ 0 & F \end{bmatrix}$ where F is a field. Then P(N) = $\begin{bmatrix} 0 & F \\ 0 & 0 \end{bmatrix}$

Let, σ: N → N be defined by, $\sigma\left(\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}\right)=\begin{bmatrix} a & 0 \\ 0 & c \end{bmatrix}$ It can be seen that a σ endomorphism of N and N is a σ(*)-Ring or Noetherian Regular δ-Near– Ring.

Definition 2.13: Let (N, +, •) be a near-ring. A subset L of N is called a ideal of N provided that 1. (N, +) is a normal subgroup of (N, +), and 2. m.(n + i) — m.n ∈ L for all i ∈L and m, n ∈ N.

For all P ∈ N-Mod we have M_N (P) ⊇ End $_N$ (P) for some pairs M_N (P) = End $_N$ (P) and when M_N(P) ⊇ and ≠ End $_N$ (P) we would like to some type of measure to indicate how close (or how far away)one is to equality. The concept of forcing linearity numbers was introduced by giving such a measure.

Definition 2.14: Let K = { Q_α }, α ∈ $\mathcal{A}$ collection of proper N-sub-modules of N-module P. we say K forces linearity on P if for f ∈ M_N (P) whenever f ∈ Hom_N (Q_α , P) for each α∈ $\mathcal{A}$ then f ∈ End_N (P).

Definition 2.15: For each P ∈ N-Mod we assign number is called a forcing linearity number of P and is denoted by fln (P) defined as below:

(i) fln(P) = 0 if End $_N$ (P) = M_N (P) (ii) if fln(P) ≠ 0 and there is a finite collection K of proper sub-modules for which forces linearity then fln (P) = inf { |K| / K forces linearity on P } and (iii) fln (P) = ∞ otherwise.

Forcing linearity numbers for several pairs of (N,P) has been determined. We can mentioned here some of the references of fln:

(a) All Z- modules i.e., abelian groups
(b) Projective modules over Commutative Noetherian Regular delta near-rings
(c) finitely generated commutative Noetherian regular delta near rings
(d) modules over Artenian regular delta near rings
(e) divisible over principal ideal domains
(f) semi simple modules over integral domains, Euclidean domains
(g) Modules over complete Matrix Noetherian regular delta near rings.

SECTION 3 MAIN RESULTS:

3.1 Some Fundamental concepts on Euclidean space E over Noetherian Regular-δ Near Ring (NR-δ-NR) of a Near Field:

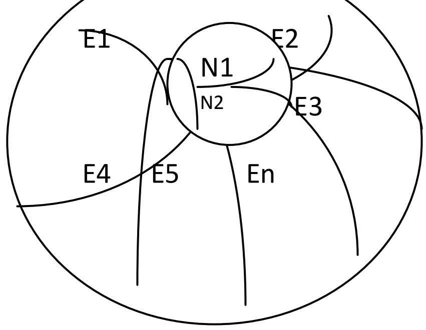
Definition 3.1.1: Let N be a Noetherian Regular -δ Near Ring. Let x be an indeterminant or variable over N. Let f(x) be the polynomial expressions in x with co-efficients in N i.e., xa_0, a_1, a_2,. . . .,a_n for all a_i in N and n ∈ Z^+ over Noetherian Regular delta Near Ring is called a polynomial.

Definition 3.1.2: Let N be a Noetherian regular delta Near Ring Let E be commutative integral domain (with or without unity) is called Euclidean space if there is a mapping ρ : N* → Z^+ such that for every a, b ∈ E, a/b → ρ(a) ≤ ρ(b) or equivalently ρ(x) ≤ ρ(xy) and (ii) for every a,b ∈ E, b ∈ E^* there exists q, r ∈ E depends on a and b such that a = qb + r with either r =0 or else ρ (r) < ρ (b) .

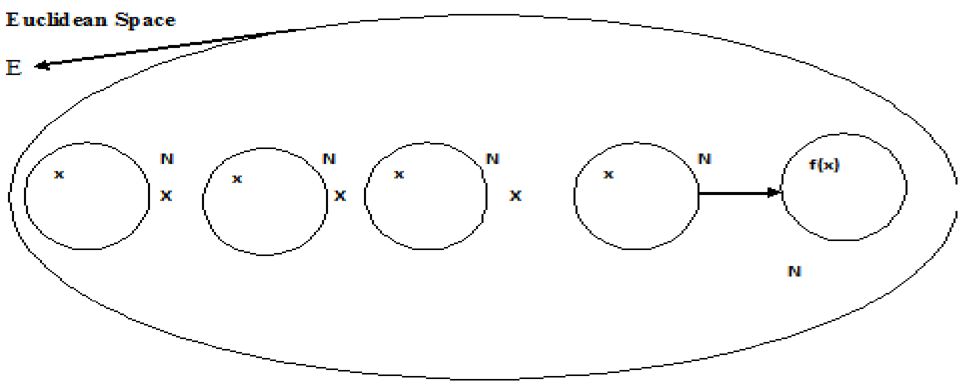
Example 3.1.3: Any field F is Euclidean Space

Example 3.1.4: Any Near Ring N = F[x] , A field F is Euclidean Space where N is the Polynomial Near Ring.

x = square root of ∑ $[xi]^2$ for all i=1,2,…,n and xi ∈ Ei



Definition 3.1.5: Let f: N x N x N x….xN → N Over Noetherian regular delta Near Ring Euclidean space E is called polynomial if (f1,f2,….,fn)(n) ∈ N ⊆ E such that f = square root of $\sum$ **[fi]^2 ∈ E.**

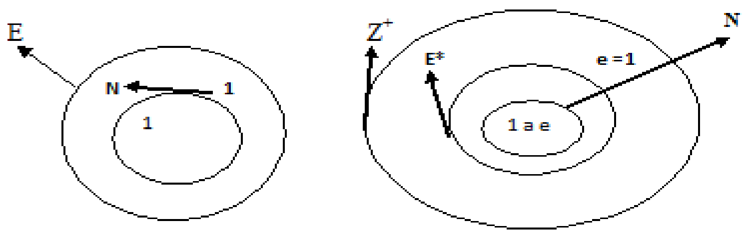


Definition 3.1.6: Polynomial in Noetherian Regular delta Near-Ring N over Euclidean Space E. Let f(x) be a Polynomial in Noetherian Regular delta Near-Ring N defined as f : N X N → N such that for every x ∈ N, f(x) ∈ N then $f_1 \in N1 \subseteq E_1$, $F_2 \in N_2 \subseteq E_2$,……. ,$f_n \in N_n \subseteq E_n$ so that (f1,f2,……,fn)(x) ∈ N ⊆ E is called a "Polynomial over Noetherian Regular Delta Near Ring of an Euclidean Space E".

3.2 Main Result on Euclidean space E over Noetherian regular Delta Near ring N:

Proposition 3.2.1: An Euclidean Space E has unity and whose group of unit is given by U (E) = {a∈E* / ρ (a) = ρ (1)} where ρ is the distance function defined from E* → Z^+ (or on E*).

Proof: [Refer C Musli Prop.4.22] by our definition of an integral domain E * ≠ ϕ and hence a least element ϕ (E*) ⊆ Z^+, ρ (E*) is non-empty subset of Z^+. by the well order principle Let m∈ρ(E*) and m = ρ (e*) and units are a ={ (a,0,0….,0), (0,a,0,…,0), (0,0,a,…,0), (0,0,0,a,…0),………., (0,0,0,…,a) } ∈ E* , ρ (1*) = ρ (e*) = m = 1.



Let ρ (1*) = ρ (e) = m = 1. And ρ (e) ≤ ρ (a + 1) ≤ ρ (a) + ρ (1) ≤ ρ (a)+0

⇒ ρ (e) ≤ ρ (a) where a ∈ { (a,0,0….,0), (0,a,0,…,0), (0,0,a,…,0), (0,0,0,a,…0),……….., (0,0,0,…,a) } ∈ E*.hence proved the proposition.

Proposition 2.2: Let f[x] be a polynomial over Euclidean domain in Noetherian Regular Delta Near rings N_i in a near-field N with [f:z(f)] is finite. Then (a) f[x] is a be a polynomial over Euclidean domain in Noetherian Regular Delta Near ring N_i in a Dickson near-field N and there exists a commutative field k such that f(x) = $k(x)^\lambda$ for some coupling map x on k. and (b) Z{f) = $f_{ix}(\Delta x) \subseteq Ux \cup \{o\}$.

Proof:

(a) Is obvious by Feigner [2].

(b) By ([3, III.5.7]) z(f(x)) ⊆ $f_{ix}(\Delta x)$. On the other hand $f_{ix}(\Delta z) \subseteq$ z(f(x)) since $f_{ix}(\Delta x)$s k{f(x)) and k(f(x)) = z(f(x)). Moreover z(f(x))\{o} ⊆ Ux by ([3, III.5.5.(b)]). For a field k, a subfield l(k) and l_1...,ln ∈ K let L(l_l,...,ln) denote the subfield generated by L ∪ {l_l , . . . , l , }. If G is a group and g ∈ G then, <g> shall denote the subgroup generated by g.

In [1] J. Ax studied a class of near fields with similar properties as finite fields called pseudo-finite fields. One can prove that pseudo-finite fields are precisely the infinite models of the first-order theory of finite fields. Similarly a near-field N is called pseudo-finite if N is an infinite model of the first-order theory of finite near-fields. The structure theory of these near-fields has been initiated by U. Feigner in [3] in near future purpose.

REFERENCES

[1] J. Ax, The elementary theory of finite fields, *Ann. of Math.* 88 (1968), 239-271.

[2] C Musli, Rings and Modules, ISBN 81-7319-037-2, pp 93.

[3] U. FELGNER, *Pseudo-finite near-fields* (Proc. Conf. Tubingen, North-Holland, Amsterdam, 1987).

[4] H. WAHLING, Theorie der Fastkorper (Thales Verlag, Essen, 1987).

[5] Proceedings *of the Edinburgh Mathematical Society* (1989) 32, 371-375 I “On Pseudo – finite near fields which have finite dimension over the centre by Peter Fuchs”.

Source of support: Nil, Conflict of interest: None Declared