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On Semi – modules of Artinian Regular Delta Near rings (SM-AR-δ-NR)

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ABSTRACT

Let N be a right Artinian Regular Delta Near ring with identity and $M_n(N)$ be the Regular Delta Near Ring of n X n matrices over N is the sense of Meldrum and Vander walt. The purpose of this paper is to define the notions of semi modules and equi prime semi modules in Artinian Regular Delta Near Rings. We show that (1) – there is a one to one correspondence between the set of semi modules of N and the set of full semi modules of Matrix Artinian Regular Delta Near Ring $M_n(N)$. (2) – There is a one to one correspondence between the set of Equiprime semi modules of N and the set of Equiprime semi modules of Matrix Artinian Regular Delta Near Ring $M_n(N)$.

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SECTION 1 INTRODUCTION:

In this paper I generalize the concept of a module of a right Artinian regular delta near ring N and call it a semi-module of N, and I give some examples of semi – modules which are not modules , also I shown that there is a one to one correspondence between the set of semi modules of N and the set of full semi modules of Matrix Artinian Regualar delta near ring M_n (N) as it is I introduced in section 3 the concept of equiprime semi module of a Artinian regular delta near ring N and I show that there is one to one correspondence between the set of equiprime semi modules of N and the set of equiprime semi modules of Matrix Artinian Regular delta Near Ring M_n (N) as in section 4. I note if N is a right zero symmetric Regular delta near ring then every semi module of N is a two sided subgroup of N. Furthery we define the equiprime two sided subgroup of a Artinian regular delta near ring N and we show that there is one to one correspondence between the set of equiprime two sided subgroups of N and the set of equiprime two sided subgroups of Matrix Artinian Regular delta near ring M_n (N) as in section 5.

SECTION 2 PRELIMINARIES:

Definition 2.1: A Near Ring N that satisfies the minimum condition for right modules, i.e. a Near Ring N in which any non-empty set M of right modules that is partially ordered by inclusion has a minimal element [1] — a right module from M that does not strictly contain right modules from M . In other words, an Artinian Near Ring is a Near Ring which is a right Artinian module over itself. A Near Ring N is an Artinian Near Ring if and only if it satisfies the decreasing chain condition for right modules, i.e. for any decreasing sequence of right modules $B_1 \supseteq B_2 \supseteq \ldots$ there exists a natural number m such that $B_m = B_{m+1} = \ldots$ The definition of a left Artinian Near Ring is similar.

Definition 2.2: Artinian Module: A Module M of an Artinian Regular Delta Near ring N that satisfies the decreasing chain condition for sub-modules. The class of Artinian modules is closed with respect to passing to sub-modules, quotient modules, finite direct sums and extensions. Extension in this context means that if the modules B and A/B are Artinian, then so is A. Each Artinian module can be decomposed into a direct sum of sub-modules which are no longer decomposable into a direct sum. A module has a composition series if and only if it is both Artinian and Noetherian. See also Noetherian Near Ring N(say)

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Definition 2.3: Artinian module: A module for which every sub-module has a finite system of generators. Equivalent conditions are: Every strictly ascending chain of sub-modules breaks off after finitely many terms; every non-empty set of sub-modules ordered by inclusion contains a maximal element. Sub-modules and quotient modules of a Noetherian module are Noetherian. If, in an exact sequence O → M' → M → M" → O, M' and M'' are Artinian, then so is M A module over a Artinian if and only if it is finitely generated.

Definition 2.4: Artinian Near Ring: A Near Ring N satisfying one of the following equivalent conditions:
1) N is a left (or right) Artinian module over itself;
2) every left (or right) modulel in N has a finite generating set;
3) every strictly descending chain of left (or right) modules in N breaks off after finitely many terms.

Example 2.5: An example of a Artinian Near Ring is any principal module ring, i.e. a Near Ring in which every module has one generator.

A right Artinia Near Ring need not be left Artinian and vice versa. For example, let N be the ring of Matrices of the form $\begin{Vmatrix} \alpha & \alpha \\ 0 & \beta \end{Vmatrix}$ where α is a rational integer and α and β are rational numbers, with the usual addition and multiplication. Then N is right, but not left, Artinian , since the left module of elements of the form $\begin{Vmatrix} 0 & \alpha \\ 0 & 0 \end{Vmatrix}$ does not have a finite generating set.

Quotient Near Rings and finite direct sums of Artinian Near Rings are again Artinian, but a sub-ring of a Artinian Near ring need not be Artinian Near. For example, a polynomial ring in infinitely many variables over a field is not Artinian Near Ring, although it is contained in its field of fractions, which is Artinian Near Ring.

If N is a left Artinian Near Ring, then so is the polynomial Near Ring N[X]. The corresponding property holds for the Near Ring of formal power series over a Artinian Near Ring. In particular, polynomial Near Rings of the form $K[X_1, X_2, \ldots, X_n]$ or $Z[X_1, X_2, \ldots, X_n]$, where **K** is a field and **Z** the ring of integers, and also quotient Near Rings of them, are Artinian. Every Artinian Near Ring is Noetherian. The localization of a commutative Artinian Near Ring **N** relative to some multiplicative system say **S** is again Artinian Near Ring. If in a commutative Artinian Near Ring **N**, $\mathfrak{m}$ is a module such that no element of the form $1+M$. where $m \in m$ is a divisor of zero, then $\cap_{k=1}^{\infty} m^K = 0$. This means that any such module $\mathfrak{m}$ defines on **N** a separable $\mathfrak{m}$-adic topology. In a commutative Artinian Near Ring every module has a representation as an in contractible intersection of finitely many primary modules. Although such a representation is not unique, the number of modules and the set of prime modules associated with the given primary modules are uniquely determined.

Note 2.6: Noetherian rings are named after E. Noether, who made a systematic study of such rings and carried over to them a number of results known earlier only under more stringent restrictions (for example, Lasker's theory of primary decompositions).

Artinian Near Rings are named after Artin, who made a systematic study of such rings and carried over to them a number of results known earlier only under more stringent restrictions.

Definition 2.7 Jacobson radical of a Near Ring N: The module J(N) of an associative Near ring J(N) $\subseteq$ ASS.(N) where Associate Algebras and N which satisfies the following two requirements:

(1) **J(N)** is the largest quasi-regular module in **N**(a Near Ring N is called quasi-regular if the equation a + x + ax =0 is solvable for any of its elements a);

(2) The quotient ring N = N / J(N) contains no non-zero quasi-regular modules. The radical was introduced and studied in detail in 1945 by N. Jacobson [3].

The Jacobson radical always exists and may be characterized in very many ways: J(N) is the intersection of the kernels of all irreducible representations of the Near Ring **N**; it is the intersection of all modular maximal right modules it is the intersection of all modular maximal left modules; it contains all quasi-regular one-sided modules; it contains all one-sided nil modules; etc. If M is a module of N, then J(I) = I $\cap$ J(N). If N is the Artininan Near Ring of all matrices of order n over N, then $J(A_n) = (J(A))_n$.

If the following $\circ$-composition is introduced on the associative Near Ring **N**:

a° b = a + b + ab then the radical J(N) in the semi-Near Ring < N, ° > will be a sub-near Ring with respect to the composition ○.

There are no non-zero irreducible finitely-generated modules over a quasi-regular associative near ring (i.e. an associative near ring coinciding with its own Jacobson radical), but there exist simple associative quasi-regular near rings. The Jacobson radical of the associative near ring N is zero if and only if N is a sub-direct sum of primitive Near Rings.

Definition 2.8 Semi-simple Artinian Near **Ring:** A Artinian near ring N with zero radical. More precisely, if r is some radical then the Artinian Near Ring N is called r-semi-simple Near Ring if r(N) = 0 . Frequently, by an associative semi-simple near ring one understands a classical semi-simple Artinian Near Ring.

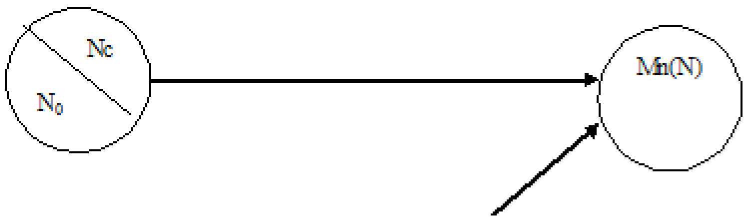
Definition 2.9 Balanced Artinian Near Ring *on the left (right)*: Artinian Near Ring N over which all left (right) modules are balanced. Atinian Near ring N is balanced on the left if and only if all its quotient rings are QF-1-Near Rings, that is, if all the exact left modules over it are balanced. In particular, a Near Ring N is balanced if all its quotient near rings are quasi-Frobenius. Every balanced near ring can be split into a direct sum of a uni-serial near ring and near rings of matrices over local near rings of a special type. Every balanced Artinian Near Ring is semi-perfect. Artinian balanced Near Ring is an Artinian ring.

Let N be a Atinian Regular delta Near ring (not necessarily commutative) and let M be a left (or right) N-module. Then M Artinian means that every simple descending chain of sub-modules M_1 M_2 M_3 stabilizes, that is, for some r and all n 0, $M_r = M_{r+n}$. Equivalently, every non-empty family of sub-modules of M contains members that are minimal in that family. We say that N is left (or right) Artinian if it is Artinian as a left (or right) module over itself.

To some extent, arguments with Artinian modules and semi-modules are very similar to arguments with Noetherian modules, semi-modules that is, semi-modules and modules in which every simple ascending chain stabilizes.

SECTION 3 SEMI MODULES OF ARTINIAN REGULAR DELTA NEAR RINGS:

In this section N is a right Artinian Regular delta near ring with identity. I introduce the concept of a semi module of N and show that there is one to one correspondence between the set of semi modules of N and the set of full of semi modules of Matrix Artinian Regular delta near rings M_n (N).



Regular Delta Near Ring Matrix Artinian Regular delta Near Ring Mn(N)

Fig 1

Definition 3.1: A non empty set M of a Artinian regular delta near ring N is called a right module of N if there exists m ∈ M and n ∈ N such that mn ∈ M for every element of M in N.

Definition 3.2: A non empty set M of a Artinian regular delta near ring N is called a semi-module of N if it satisfies the following

(a) (M, +) is a sub group of Artinian regular delta near ring (N , +) and K + M = M+ K for any sub group K of (N,+) ,

(b) MN ⊆ M and

(c) n (a + b) – n. b ∈ M for every n , b ∈ N, for every a ∈ M.

Example 3.3: Let K = P x Q be the direct product of two groups P and Q where P = < p, q / 9 p = 3q=0, q + p – q = 4p > and Q = < r > , |r | = 9 Let us define multiplication on K by lm =0 for all l , m ∈ K. K is a Artinian Regular delta near ring which is called the zero symmetric Artinian Regular delta Near Ring on K.

Let M = < q > x Q it is clear that M is a sub group of K which permutes with each subgroup of K but M = < q > x Q is not a normal in K So M is a semi module of K which is not a module.

Example 3.4: In above example 3.3, if we define a multiplication operation on K by lm = l for all l, m ∈ K. Then K is a Artinian regular delta near ring which is called the trivial constant Artininan Regular delta near ring on K. Then < q > x Q is a semi module of K which is not module.

Example 3.5: In above example 3.3., If we define multiplication on K by

lm = {0 if l =0
{m if h ≠ 0 for all l, m are belongs to K.

Then K is Artinian regular delta near ring which is called the trivial zero-symmetric Artinian regular delta near ring on K. Then < q > x Q is a semi module of K which is not a module.

Proposition 3.6: If L a semi module of Artinian Regular Delta Near Ring N, then let us define $L^* = \{ P \in M_n(N) / P\alpha \in L^n$ for all $\alpha \in N^n\}$ is a semi module of a Matrix Artinina Regular delta near ring $M_n(N)$.

Proof: Let us have $(L^n, +)$ is a subgroup of $(N^n, +)$ since (L, +) is a subgroup of (N, +) so $(L^*, +)$ is a subgroup of (Mn(N) , +). Now we show that $L^*+K= K+L^*$ for a subgroup K of (Mn(N), +). Let K be a subgroup of (Mn(N), +), $u\in N^n$ then $Ku = \{Au / A \in K\}$ is a subgroup of $(N^n, +)$ but L permute with all subgroups of (N, +) so $(L^n, +)$ permute with all subgroups of $(N^n, +)$.

We show that $L^* u + Ku = K u + L^* u , \forall u \in N^n$

Let $u \in N^n$ take $A u \in K u$ and $D u \in L^* u$ we have $A u + D u \in K u + L^n = L^n + K u$ so that there exists $B u \in Ku$, $z \in L^n$ such that $Au + Du = Z + Bu \Rightarrow Au + Du - Bu = Z \in L^n$.

$\Rightarrow (A + D – B)u = Z \in L^n$ for every $u \in N^n$.

Suppose, $C = A + D – B \in L^* \Rightarrow A + D = C + B$ i.e., $L^* u + K u = K u + L^* u , \forall u \in N^n$--------(1)

By (1), to show that the required $(L^* + K) u = L^*u + K u = Ku + L^*u = (K + L^*)u , \forall u \in N^n$

Therefore, $L^* + K = K + L^*$. Hence proved.

If $D \in L^*$, $B \in M_n(N)$ $(DB)u = D(Cu) \in L^n \Rightarrow L^* Mn(N) \subseteq L^*$.

We show that for any $A, C \in Mn(N)$ and $D \in L^*$: $A(C + D) - AC \in L^*$

i.e., $(A(C + D) – AC) u = A (Cu + Du) – (AC) u \in L^n, \forall u \in N^n$ where N is an Artinina Regular delta near ring.

Moreover, we show that for any $A \in Mn(N)$, $\alpha \in N^n$, $\beta \in L^n$, $A(\alpha + \beta) - A\alpha \in L^n$

i.e., there is $\gamma \in L^n$ such that $A(\alpha + \beta) - A\alpha = \gamma$ for all $\alpha \in N^n$, $\beta \in L^{n.}$

Let $\alpha = (\alpha_1, \alpha_2 , \alpha_3 ,…..\alpha_n) \in N^n$, $\beta = (b_1, b_2, b_3,….., b_n) \in L^{n.}$

By induction w(A). If w(A) = 1 i.e., $A = g^r_{ij}$ then

$$\begin{aligned} g^r_{ij}(\alpha + \beta) - g^r_{ij}(\alpha) &= g^r_{ij}(\alpha_1 +b1, \alpha_2 +b2 , \alpha_3 +b3,…..\alpha_n+b_n) - g^r_{ij}(\alpha_1, \alpha_2 , \alpha_3 ,…..\alpha_n) \\ &= (0,0….,r(\alpha_j+b_j),…..,0) – (0,0,…..r\alpha_j,…..,0) \\ &= (0, 0…., r(\alpha_j+b_j)- r\alpha_j …..,0) \end{aligned}$$

Put L is left semi module of N, then $r(\alpha_j+b_j)- r\alpha_j \in L$ and so we have
$g^r_{ij}(\alpha + \beta) - g^r_{ij}(\alpha) = (0,0….,r(\alpha_j+b_j)- r\alpha_j …..,0) \in L^n$.

Result holds good for any Matrix Artinian regular delta near ring A, w(A) < m; m ≥ 2. Let w(A) = m then A = L + M, where w(L), w(M) < w(A). by induction hypothesis,

(i) $$\begin{aligned} A(\alpha + \beta) = (L+M)(\alpha + \beta) = L (\alpha + \beta) + M(\alpha + \beta) &= \delta + L(\alpha) + \eta + M\alpha , \ \delta, \eta \in L^n \\ &= \delta + \theta + L\alpha + M\alpha, \theta \in L^n \\ &= \delta + \theta + (L + M)\alpha \\ &= \delta + \theta + A(\alpha) \end{aligned}$$

(ii) $$\begin{aligned} A(\alpha + \beta) = (LM)(\alpha + \beta) = L (M(\alpha + \beta)) &= L(\delta + M(\alpha), \ \delta \in L^n \\ &= \eta + L(M\alpha), \eta \in L^n \\ &= \eta + L(M\alpha) \\ &= \eta + A(\alpha) \text{ therefore } L^* \text{ is a sei module of } Mn(N). \end{aligned}$$

So, SM (N), SM (Mn(N)) be the set of semi modules in N, Mn(N) respectively. Hence Proved.

Definition 3.7: A semi module of M_n (N) is called full if it of the form L^* for some semi module of Artinian regular delta near ring N. For instance {0} and M_n (N) are full semi modules of M_n (N).

Theorem 3.8: There is a bijection between the set of semi modules of N and the set of full semi modules of M_n (N) given by L → L^* and its inverse by Q → Q_* such that $(L^*)_* = L$ and $(Q_*)^* = Q$ for a semi module L of N and a full semi module Q of M_n (N), where M_n (N) Is a Matrix Artinian Regular delta near rings of N.

Proof: Let SM(N) (resp. SM(M_n (N))) bet the set of semi modules of N (resp. M_n (N)). Let us define a mapping ϕ: SM(N) → SM(M_n (N)) by ϕ (L) = L^* Now clearly ϕ is injection mapping. Further if Q ∈ SM (M_n (N)) then Q_* ∈ SM(N) and if Q is full semi module of M_n (N) the b Q = $(Q_*)^*$ Finally, Q = $(Q_*)^*$ = ϕ (Q_*) . Hence ϕ is onto. Hence Proved the theorem.

SECTION 4: RESULTS RELATED TO SEMI MODULES OF ARTINIAN REGULAR DELTA NEAR RINGS AND EQUIPRIME SEMI MODULES OF MATRIX ARTINIAN REGULAR DELTA NEAR RING:

Definition 4.1: If P is a semi module of N, then P is called an equiprime semi module of N if and only if for all p ∈ N – P and r, s ∈ N, axr – axy ∈ P for all x ∈ N implies r – s ∈ P. Artinian Regular delta near ring N is called an Equiprime Artinian regular delta near ring if the zero symmetric semi module is an Equiprime Artinian Regular delta near ring.

Lemma 4.2: If 0 → M^1 → M → M^{11} → 0 is a short exact sequence of N-modules, then M is Artinian Regular Delta Near Ring [resp. Noetherian Regular Delta Near Ring] if and only if both M^1 ; M^{11} are Artinian Regular Delta Near Ring [resp. Noetherian Regular Delta Near Ring].

Proof: For the `if' direction in both cases, consider a chain of sub-modules M_m of M. Projecting to M^{11} , the image chain stabilizes, say from the image of M_r. From this point, the kernels of M_m → M ^{11} ; m r; form a chain of sub-modules of M^1 that stabilizes. The 5-lemma then implies the M_m stabilize as well.

If M satisfies a chain condition then obviously so does M^1. But also, given a chain in M^{11}, the inverse images in M form a chain. Since M → M^{11} is surjective, the fact that the chain in M stabilizes implies the original chain in M^{11} stabilizes.

Lemma 4.3: Consider a finite chain of sub-modules of M, say (0) = M_0 M_1 $M_{2\ldots}$ M_s = M. Then M is Artinian Regular Delta Near Ring [resp. Noetherian Regular Delta Near Ring] if and only if each quotient module M_{j+1}=M_j is Artinian Regular Delta Near Ring [resp. Noetherian Regular Delta Near Ring].

Proof: Apply Lemma 1 inductively for j=0 to the sequences 0 → M_j → M_{j+1} → M_{j+1}=M_j → 0:

Example 4.4: Modules over any ring that are also finite dimensional vector spaces over a field, and for which all sub-modules are vector subspaces, are clearly both Artinian and Noetherian. In fact, for vector spaces, finite dimensionality is equivalent to either chain condition separately.

Example 4.5: Non-commutative examples of type Ex. 3.3 include matrix rings over a field or a division algebra and group rings of finite groups over a field. Commutative examples include commutative rings B that are finitely generated (f.g.) over a field K and in which every element is algebraic over K. Zero divisors and nilpotent elements are allowed here. The conditions are just equivalent to saying K, B is a finitely generated (f.g.) integral extension, so B is finitely generated (f.g.) as K-module.

Example 4.6: Simple modules N, that is modules with no sub-modules other than (0) and N, are both Artinian and Noetherian. By Lemma 2, any module M that admits a = ax for some x ∈ N, hence (1 x) a = 0. But (1 x) is a unit, since it is contained in no maximal ideals, so we have a contradiction.

Lemma 4.7:. A commutative Artinian Regular Delta Near Ring is Noetherian Regular Delta Near Ring.

Proof: Since N^k = (0), we have (not necessarily distinct) maximal ideals m_1; m_2; ; m_r with (0) = $m_1 m_2$ m_r. Consider the following iteration

A ⊃ m_1 ⊃ $m_1 m_2$ ⊃ $m_1 m_2 \ldots\ldots m_r$ = (0):

Since A is Artinian, the j^{th} quotient in this series is Artinian, but is also a vector space over the field A=m_j . Thus these vector spaces are finite dimensional, hence these quotients are Noetherian A-modules, from which we conclude from Lemma 2 that A is Noetherian.

Theorem 4.8: If A is a commutative Artinian Regular delta Near Ring with $N^k = (0)$, where N = Q P_j is the nilradical, then by the Chinese Remainder Theorem Y A ' A=P_j^k; which is seen to be a finite product of local Noetherian rings, each with a nilpotent maximal ideal.

Proof: Is obvious

Lemma 4.9: In a commutative Artinian ring every prime ideal is maximal. Also, there are only nitely many prime ideals.

Proof: Consider a prime P A. Consider x 62P. The power ideals (x^m) decrease, so we get (x^n) = (x^{n+1}) for some n. Then $x^n = ax^{n+1}$, so x^n(1 ax) = 0. But x^n 62P, hence 1 ax 2 P, which implies A = P +Ax. Thus P is maximal.

For the second statement, consider an ideal J minimal among all finite products Q Q of distinct primes, J = P_j. Given any prime P, we have P JP = J = P_j , the middle relation because of the minimality of J. Thus P contains some P_j , and since all primes are maximal P = P_j Q T

Lemma 4.10: In a commutative Artinian regular delta Near Ring, the nilradical N = P_j = P_j is a nilpotent ideal. That is, N^k = (0), some k =1.

Proof: Here, the P_j denote the nitely many distinct primes of A. The powers of N decrease, so we can nd a smallest k so that $NN^k = N^k$. Claim that N^k = (0). If not, consider a minimal non-zero ideal J so that JN^k 6= (0). (N is one such ideal, so the family is non-empty.) Clearly J = (a) must be principal. Then $aNN^k = aN^k$ 6= (0), so by minimality (a) = aN. But then in this formula, one can obviously replace the common exponent k for the various maximal ideals by exponents k_j , the smallest integers so that $P_j^{kj} = P_j^{kj+1}$.

A finite composition series with simple quotients M_{j+1}=M_j is both Artinian regular delta near ring and Noetherian regular delta near ring.

In fact, the converse is true. Starting with such an M, choose a minimal (and therefore simple) proper sub-module (0) M_1. Then choose M_2 minimal among sub-modules containing M_1 properly. So M_2=M_1 is simple. Continuing, the process must terminate with M after finitely many steps because M is Artinian (or Noetherian) regular delta near ring. [One can also start with the Noethrian hypohesis and choose a maximal proper submodule of M and work down. The Artinian hypothesis guarantees the process stops at (0) after nitely many steps.]

Note 4.11: Finite direct products of Artinian modules or Regulr delta near rings are Artinian reguar delta near rings.

Note 4.12: Suppose A is a commutative local Noetherian ring, with nilpotent maximal ideal m. Then A is Artinian regular delta near ring. Namely, apply Lemma 2 to the lteration A m m^2 m^k = 0. Each quotient mj =m^{j+1} is a finite dimensional vector space over the field A=m, since all the m^j are finitely generated modules in A. But also, the A-sub-modules of each such quotient are the same thing as A=m sub-vector spaces, hence these quotients are Artinian regular delta near ring A-modules.

Note 4.13: If A is any Artinian (or Noetherian) commutative ring and m A is a maximal module, then A=m^k is Artinian regular delta near ring. This just more or less repeats the paragraph above, after noting that A=m^k is indeed local. Any element of A not in m is invertible modulo m^k.

Note 4.14: There is a good structure theory for both non-commutative and commutative Artinian Regular Delta Near Rings. Here I will just deal with the commutative case. The theorem is that any commutative Artinian Regular delta near ring is a finite direct product of regular delta near rings.

SECTION 5 EQUIPRIME TWO SIDED SUBGROUPS OF ARTINIAN REGULAR DELTA NEAR RINGS:

It is clear that in any Artinian Regular delta near ring N every module M is a semi module of N also if N is a zero symmetric right Artinian regular delta near ring then every semi module of N is a two sided subgroup of N.

In this section N is a right zero symmetric Artinian regular delta near ring also we define the Equiprime two sided subgroup of an Artinian Regular delta near ring N and we show that there is a one to one correspondence between the set of Equiprime two sided subgroups of N and the set of Equiprime two sided subgroups of Matrix Artinian Regular delta near rings $M_n(N)$.

Definition 5.1: If P is a two sided subgroup of Artinina regular delta near ring N , then P is called an Equiprime two sided subgroup of N if and only if for all $x \in N - P$ and r , s $\in$ N , xkr – xky $\in$ P for all k $\in$ N implies r – s $\in$ P.

Proposition 5.2: Let P be an equiprime two sided subgroup of N. Then P* is an equoiprime semi module of Mn(N).

Proof: Proof is obvious and refers [3]

Note 5.3: Let P be an equi prime two sided subgroup of Mn(N) then P = $(P_*)^*$

Theorem 5.4: The mapping P→P* defines a 1-1 correspondence between the sets of Equiprime two sided subgroup of N and Mn(N)

Proof: Let E (N) and E (Mn(N)) are the sets of Equiprime two sided subgroups of N and Mn(N).

Consider the function, ϕ : E(N) → E(Mn(N)) defined by ϕ (P) = P* for all P belongs toE(N).It is easy t show that ϕ is injection. Suppose that P belong to E(Mn(N)), then by P_* is in E(N) . this follows that P = $(P_*)^*$ = ϕ (P_*). Hence the ϕ is onto Hence proved the theorem.

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