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ON INTUITIONISTIC Q-FUZZY IMPLICATIVE Q-IDEALS IN Q-ALGEBRA

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ABSTRACT

In this paper, we introduce the concept of an Intuitionistic Q-fuzzy implicative Q-ideal in Q-algebras and some related properties are investigated. We also define the level subsets of an intuitionistic Q-fuzzy implicative Q-ideals in Q-algebras and discussed some of its properties.

Keywords: *Intuitionistic fuzzy set , intuitionistic Q-fuzzy set, intuitionistic fuzzy implicative Q-ideals, intuitionistic Q-fuzzy implicative Q-ideals , intuitionistic Q-fuzzy closed implicative Q-ideals.*

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1. INTRODUCTION

The concept of fuzzy set was introduced by Zadeh [10] in 1965.Several researchers explored on the generalization of the notion of fuzzy sets. The concept of intuitionistic fuzzy set was introduced by K.T. Atanassov [1,2] ,as a generalization of the notion of fuzzy set in 1986.Since then the literature on these concepts has been growing rapidly.

Y. Imai and K. Iseki introduced two classes of abstract algebras: BCK-algebra and BCI-algebras. It is known that the class of BCK-algebras is proper subclass of the class of BCI- algebras. J. Neggers, S.S. Ahn and H.S. Kim [4] introduced a notion, called Q –algebras. In this paper, we introduce the notion of Intuitionistic Q-fuzzy implicative Q-ideals in Q-algebra and investigate some properties.

2. PRELIMINARIES

In this section, we site the fundamental definitions that will be used in the sequel.

2.1 Definition: A Q-algebra is a nonempty set X with a constant 0 and a binary operation * satisfying the following axioms

i. $x * x = 0$
ii. $x * 0 = x$
iii. $(x * y) * z = (x * z) * y$ for all $x, y, z \in X$,

In X, we can define a binary relation $\le$ by $x \le y$ if and only if $x * y = 0$.

2.1 Example Let X = {0, 1, 2, 3} be a set with the following table

*	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	0	0	0
3	3	3	3	0

(X,*, 0) is a Q-algebra.

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2.2 Definition: Let (X,*, 0) be a Q-algebra. A non empty set I of X is called an ideal of X if it satisfies
i. $0 \in I$
ii. $x * y \in I$ and $y \in I$ imply $x \in I$, for all x, y $\in X$.

2.3 Definition: An ideal A of a Q-algebra X is said to be closed if $0 * x \in A$ for all $x \in A$.

2.4 Definition: A fuzzy set μ in a nonempty set X we mean a function $\mu: X \rightarrow [0, 1]$ and the complement of μ denoted by μ^c, is the fuzzy set in X given by $\mu^c(x) = 1- \mu(x)$ for all $x \in X$.

2.5 Definition: A fuzzy set μ in a Q-algebra X is called a fuzzy subalgebra of X if
$\mu(x*y) \geq \min\{\mu(x),\mu(y)\}$, for all x, y $\in X$

2.6 Definition: A fuzzy set μ in a Q-algebra X is called a fuzzy ideal of X if
i. $\mu(0) \geq \mu(x)$
ii. $\mu(x) \geq \min\{\mu(x*y), \mu(y)\}$ for all x, y $\in X$

2.7 Definition: An intuitionistic fuzzy subset (IFS) A in a set X is defined as an object of the form $A = \{\langle X, \mu(x),\lambda(x)\rangle / x\in X\}$, where $\mu : X \rightarrow [0,1]$ and $\lambda : X \rightarrow [0,1]$ define the degree of membership and the degree of non-membership of each element $x\in X$ to the set A respectively and for every $x\in X$, $0 \leq \mu(x) + \lambda(x) \leq 1$. For the sake of simplicity we use the symbol $A= (X,\mu,\lambda)$ for the intuitionistic fuzzy set $A =\{\langle X,\mu(x),\lambda(x)\rangle / x\in X\}$.

2.8 Definition: An intuitionistic fuzzy set $A = (X, \mu, \lambda)$ is called an intuitionistic fuzzy subalgebra of X if it satisfies
i. $\mu(x*y) \geq \min\{\mu(x), \mu(y)\}$
ii. $\lambda(x*y) \leq \max\{\lambda(x), \lambda(y)\}$, for all x, y $\in X$

2.1 Theorem: Every intuitionistic fuzzy subalgebra $A = (X, \mu, \lambda)$ of X satisfies the inequalities $\mu(0) \geq \mu(x)$ and $\lambda(0) \leq \lambda(x)$.

Proof: For all x, y $\in X$.
$\mu(0) = \mu(x*x) \geq \min\{\mu(x),\mu(x)\}= \mu(x)$ and $\lambda(0) = \lambda(x*x) \leq \max\{\lambda(x), \lambda(x)\} = \lambda(x)$,

2.9 Definition: A non empty set I of a Q-algebra X is called an implicative Q-ideal of X if
i. $0 \in I$
ii. $(x* y)* z \in I$ and $y* z \in I$ then $x* z \in I$, for all x, y, z $\in X$.

2.10 Definition: A fuzzy set μ in a Q-algebra X is called a fuzzy implicative Q- ideal of X if
i. $\mu(0) \geq \mu(x)$
ii. $\mu(x*z) \geq \min\{\mu((x*y)*z), \mu(y*z)\}$ for all x, y, z $\in X$

2.11 Definition: An intuitionistic fuzzy set $A = (X,\mu,\lambda)$ in a Q-algebra X is called an intuitionistic fuzzy implicative Q-ideal of X if
i. $\mu(0) \geq \mu(x)$ and $\lambda(0) \leq \lambda(x)$
ii. $\mu(x*z) \geq \min\{\mu((x*y)*z),\mu(y*z)\}$
iii. $\lambda(x*z) \leq \max\{\lambda((x*y)*z, \lambda(y*z)\}$ for all x, y, z $\in X$

2.12 Definition: An intuitionistic fuzzy set $A = (X,\mu,\lambda)$ in a Q-algebra X is called an intuitionistic fuzzy closed implicative Q- ideal of X if it satisfies
i. $\mu(0*x) \geq \mu(x)$ and $\lambda(0*x) \leq \lambda(x)$
ii. $\mu(x*z) \geq \min\{\mu((x*y)*z),\mu(y*z)\}$
iii. $\lambda(x*z) \leq \max\{\lambda((x*y)*z, \lambda(y*z)\}$ for all x, y, z $\in X$

2.13 Definition: Let $A=(X,\mu,\lambda)$ be an intuitionistic fuzzy set in a Q- algebra X the set $U(\mu;s) =\{x\in X / \mu(x) \geq s\}$ is called an upper s-level of μ and the set $L(\lambda; t) =\{x\in X / \lambda(x) \leq t\}$ is called lower t - level of λ.

3. Intuitionistic Q- fuzzy implicative Q-ideal

In this section Q-fuzzy implicative Q-ideal, Q-fuzzy closed implicative Q-ideal, Intuitionistic Q-fuzzy implicative Q-ideal, Intuitionistic Q-fuzzy closed implicative Q-ideal, homomorphism of Q-algebra are defined and some related properties are discussed.

3.1 Definition: A Q-fuzzy subset $\mu:XxQ \to [0,1]$ in a Q- algebra X is called a Q- fuzzy implicative Q- ideal of X if
i. $\mu(0, q) \geq \mu(x,q)$
ii. $\mu((x*z), q) \geq \min\{\mu((x*y)*z,q),\mu(y*z,q)\}$ for all x, y, z $\in$X, q$\in$Q.

3.2 Definition: An intuitionistic Q-fuzzy set A in an non empty set X is an object having the form $A=\{(X,\mu(x, q),\lambda(x, q))/x\in X,\ q\in Q\}$,where the function μ: XxQ $\to$ [0,1] and λ : XxQ $\to$ [0,1] denote the degree of membership (namely $\mu(x, q)$) and degree of non-membership (namely $\lambda(x, q)$) and $0\leq \mu(x, q)+ \lambda(x, q) \leq 1$, for all x$\in$X and q$\in$Q .

For the sake of simplicity, we use the symbol $A=(X,\mu,\lambda)$ for the intuitionistic Q- fuzzy set $A=\{(X, \mu(x, q),\lambda(x, q))/x \in X, q\in Q\}$.

3.3 Definition: An intuitionistic Q- fuzzy set A of a Q-algebra X is a subalgebra if
i. $\mu(x*y, q) \geq \min\{\mu(x, q), \mu(y, q)\}$
ii. $\lambda(x*y, q) \leq \max\{\lambda(x, q), \lambda(y, q)\}$ for all x, y, z $\in$ X and q $\in$ Q.

3.4 Definition: An intuitionistic Q- fuzzy set $A=(X,\mu,\lambda)$ in a Q- algebra is called an intuitionistic Q- fuzzy implicative Q- ideal of X, if it satisfies the following axioms:
i. $\mu(0,q) \geq\mu(x,q)$ and $\lambda(0,q) \leq \lambda(x, q)$
ii. $\mu(x*z, q) \geq \min\{\mu((x*y)*z,q), \mu(y*z,q)\}$
iii. $\lambda(x*z, q) \leq \max\{\lambda(x*y)*z, q), \lambda(y*z, q)\}$ for all x, y, z $\in$X and q $\in$ Q.

3.1 Theorem: Let $A=(X,\mu,\lambda)$ be an intuitionistic Q- fuzzy implicative Q-ideal of a Q- algebra X. If $x \leq y$ in X then,

$$\mu(x, q) \geq \mu(y, q) \text{ and } \lambda(x, q) \leq \lambda(y, q).$$

Proof : Let $x \leq y$ implies $x*y=0$

$$\begin{aligned}\mu(x, q) = \mu(x*0, q) &\geq \min\{\mu((x*y)*0,q), \mu(y*0,q)\}\\ &= \min\{\mu(0*0,q), \mu(y*0,q)\}\\ &= \min\{\mu(0,q), \mu(y, q)\}\\ &= \mu(y, q) \text{ (Since } \mu(0) \geq \mu(y))\end{aligned}$$

Therefore, $\mu(x, q) \geq \mu(y, q)$

Similarly, $\lambda(x, q) \leq \lambda(y, q)$.

3.5 Definition: Let $A=(X,\mu,\lambda)$ be an intuitionistic Q- fuzzy set in X. then
i. $\square A=(X, \mu, \mu^c)$ and
ii. $\Diamond A=(X, \lambda^c, \lambda)$

3.2 Theorem: If $A=(X,\mu,\lambda)$ is an intuitionistic Q- fuzzy implicative Q- ideal of a Q- algebra X, then $\square A=(X, \mu,\mu^c)$ is also an intuitionistic Q- fuzzy implicative Q- ideal of X.

Proof: Let A is an intuitionistic Q- fuzzy implicative Q-ideal of X. We have $\mu(0, q) \geq \mu(x, q)$
$1-\mu^c(0, q) \geq 1-\mu^c(x, q)$

$\mu^c(0,q) \leq \mu^c(x, q)$ For any x $\in$ X and q $\in$ Q.

and $\mu(x*z, q) \geq \min\{\mu((x*y)*z, q), \mu(y*z, q)\}$

$1- \mu^c(x*z, q) \geq \min\{1-\mu^c((x*y)*z, q) ,1- \mu^c(y*z, q)\}$

$$\begin{aligned}\mu^c(x*z, q) &\leq 1-\min\{1-\mu^c((x*y)*z, q) ,1- \mu^c(y*z,q)\}\\ &\leq \max\{\mu^c((x*y)*z,q), \mu^c(y*z,q)\}\end{aligned}$$

Hence $\square A=(X, \mu, \mu^c)$ is an intuitionistic Q- fuzzy implicative Q- ideal of X.

3.3 Theorem: Let $A=(X, \mu, \lambda)$ be an intuitionistic Q- fuzzy implicative Q-ideal of a Q-algebra X then $\Diamond A=(X, \lambda^c, \lambda)$ is also an intuitionistic Q- fuzzy implicative Q-ideal of X.

Proof: Let A is an intuitionistic Q- fuzzy implicative Q-ideal of X. We have
$\lambda(0, q) \leq \lambda(x, q)$

$1-\lambda^c(0, q) \leq 1-\lambda^c(x, q)$

$\lambda^c(0,q) \geq \lambda^c(x, q)$ for any $x \in X$ and $q \in Q$.

Now for any x, y, z $\in$X and q $\in$ Q.

$\lambda(x*z, q) \leq \max\{\lambda((x*y)*z,q), \lambda(y*z,q)\}$

$1-\lambda^c(x*z, q) \leq \max\{1-\lambda^c((x*y)*z,q), 1-\lambda^c(y*z,q)\}$

$\lambda^c(x*z, q) \geq 1-\max\{1-\lambda^c((x*y)*z,q), 1-\lambda^c(y*z,q)\}$
$= \min\{\lambda^c((x*y)*z,q), \lambda^c(y*z,q)\}$

$\lambda^c(x*z, q) \geq \min\{\lambda^c((x*y)*z,q), \lambda^c(y*z,q)\}$

Hence ◊A=(X, λ^c, λ) is an intuitionistic Q-fuzzy implicative Q-ideal of X.

3.4 Theorem: Let A=(X,μ,λ) be an intuitionistic Q-fuzzy implicative Q-ideal of a Q-algebra if and only if □A, ◊A are intuitionistic Q-fuzzy implicative Q-ideal of a Q-algebra X.

Proof: It is clear.

3.6 Definition: An intuitionistic Q-fuzzy set A=(X,μ,λ) in a Q-algebra X is called an intuitionistic Q-fuzzy closed implicative Q-ideal of X, if it satisfies the following condition
i. $\mu(0*x, q) \geq \mu(x, q)$ and $\lambda(0*x, q) \leq \lambda(x, q)\}$
ii. $\mu(x*z,q) \geq \min\{\mu((x*y)*z,q), \mu(y*z,q)\}$
iii.$\lambda(x*z,q) \leq \max\{\lambda((x*y)*z,q), \lambda(y*z,q)\}$ for all x ,y, z $\in$X and q $\in$ Q.

3.5 Theorem: If A=(X,μ,λ) be an intuitionistic Q-fuzzy closed implicative Q-ideal of a Q-algebra X, then
□A= (X, μ, μ^c) is also an intuitionistic Q-fuzzy closed implicative Q-ideal of X.

Proof: For any x $\in$ X and q $\in$ Q,
$\mu(0*x, q) \geq \mu(x, q)$

$1-\mu^c(0*x, q) \geq 1-\mu^c(x, q)$

$\mu^c(0*x, q) \leq \mu^c(x, q)$

Hence □A= (X, μ,μ^c) is an intuitionistic Q- fuzzy closed implicative Q-ideal of X.

3.6 Theorem: If A=(X,μ,λ) be an intuitionistic Q-fuzzy closed implicative Q-ideal of a Q-algebra X, then
◊A=(X, λ^c, λ) is also an intuitionistic Q-fuzzy closed implicative Q-ideal.

Proof: For any x $\in$ X and q $\in$ Q,
$\lambda(0*x, q) \leq \lambda(x, q)$

$1-\lambda^c(0*x, q) \leq 1-\lambda^c(x, q)$

$\lambda^c(0*x, q) \geq \lambda^c(x, q)$.

Hence ◊A=(X, λ^c, λ) is an intuitionistic Q-fuzzy closed implicative Q-ideal of X.

3.7 Theorem: A=(X,μ,λ) be an intuitionistic Q-fuzzy closed implicative Q-ideal of a Q-algebra X if and only if □A , ◊A are intuitionistic Q-fuzzy closed implicative Q-ideal of a Q-algebra X.

Proof: It is clear.

3.8 Theorem: Let A=(X,μ,λ) be an intuitionistic Q-fuzzy implicative Q-ideal of a Q-algebra X if and only if the non – empty upper s-level cut U(μ;s) and the non-empty lower t-level cut L(λ;t) are implicative Q-ideals of X, for any
s, t $\in$[0,1].

Proof: Suppose A=(X, μ, λ) be an intuitionistic Q-fuzzy implicative Q-ideal of a Q-algebra X. For any s, t $\in$ [0, 1] define the sets

$U(\mu:s) = \{(x, q) \in XxQ \,/\, \mu(x, q) \geq s\}$ and $L(\lambda;t) = \{(x, q) \in XxQ \,/\, \lambda(x, q) \leq t\}$

Since $U(\mu:s) \neq \phi$, for $(x, q) \in U(\mu:s)$

$\mu(x, q) \geq s$

$\mu(0, q) \geq \mu(x,q) \geq s$

$\mu(0, q) \geq s$ implies $(0, q) \in U(\mu:s)$

Let $(x*y)*z, q) \in U(\mu:s)$ and $(y*z, q) \in U(\mu:s)$

$\mu((x*y)*z, q) \geq s$ and $\mu(y*z, q) \geq s$

Since $\mu((x*z ,q) \geq \min \{ \mu((x*y)*z ,q), \mu(y*z,q) \}$
$\geq \min \{s, s\} = s$

$\mu((x*z, q) \geq s$ implies $(x*z, q) \in U(\mu: s)$

Hence U(μ:s) is an implicative Q-ideal of X.

Similarly we can prove that L(λ;t) is an implicative Q-ideal of X

Conversely,

Suppose that for any s, t $\in$[0, 1], U(μ:s) and L(λ;t) are implicative Q-ideals of X.

If possible, assume $x_0, y_0 \in X$ and $q_0 \in Q$ such that $\mu(0,q_0) < \mu(x_0,q_0)$ and $\lambda(0,q_0) > \lambda(y_0,q_0)$

take $s_0 = (1/2)\,[\mu(0,q_0)+ \mu(x_0,q_0)]$

That is $s_0 < \mu(x_0,q_0)$ and $0 \leq \mu(0,q_0) < s_0 < 1$, $(x_0,q_0) \in U(\mu:s_0)$ and $(0,q_0) \notin U(\mu:s_0)$

Since $U(\mu:s_0)$ is an implicative Q-ideal of X we have $(0, q_0) \in U(\mu:s_0)$ and $\mu(0,q_0) \geq s_0$

Therefore Our assumption is wrong.

Hence $\mu(0,q) \geq \mu(x,q)$ for all $x \in X$ and $q \in Q$.

Similarly by taking $t_0 = (1/2)\,[\lambda(0,q_0)+ \lambda(y_0,q_0)]$

We can show that $\lambda(0, q) \leq \lambda(y, q)$ for all $y \in X$ and $q \in Q$.

ii. If possible, assume that $x_0, y_0, z_0 \in X$ and $q_0 \in Q$ and such that

$\mu(x_0*z_0, q_0) < \min \{\mu((x_0* y_0)* z_0, q_0), \mu(y_0* z_0, q_0)\}$

Take $s_0 = (1/2)\,[\mu(x_0*z_0,q_0)+\min \{ \mu((x_0* y_0)* z_0,q_0), \mu((y_0* z_0,q_0)\}]$

and $s_0 < \min \{ \mu((x_0* y_0)* z_0,q_0), \mu((y_0* z_0,q_0)\}]$ and $s_0 > \mu(x_0*z_0, q_0)$

Therefore $s_0 < \mu((x_0* y_0)* z_0,q_0)$, $s_0 < \mu(y_0* z_0,q_0)$ and $s_0 > \mu(x_0*z_0, q_0)$ $(x_0*z_0, q_0) \notin U(\mu:s_0)$

Since $U(\mu:s_0)$ is an implicative Q-ideal of X.

$((x_0* y_0)* z_0,q_0) \in U(\mu:s_0)$, $(y_0* z_0,q_0) \in U(\mu:s_0)$ imply that $(x_0*z_0, q_0) \in U(\mu:s_0)$

Therefore Our assumption is wrong.

Hence $\mu(x*z, q) \geq \min \{\mu((x*y)*z, q), \mu(y*z, q)\}$ for any x, y, z $\in$X and q $\in$Q.

Similarly we can prove that

$\lambda (x*z, q) \leq \max \{\lambda ((x*y)*z, q), \lambda (y*z, q)\}$ for any x, y, z $\in$X and q $\in$Q.

Hence A is an intuitionistic Q- fuzzy implicative Q-ideal of a Q-algebra X.

3.9 Theorem: $A=(X,\mu,\lambda)$ be an intuitionistic Q-fuzzy closed implicative Q-ideal of a Q-algebra X if and only if the non empty upper s-level cut $U(\mu;s)$ and the non-empty lower t-level cut $L(\lambda;t)$ are closed implicative Q-ideals of X, for any s, t $\in$[0,1].

Proof: Suppose $A=(X,\mu,\lambda)$ be an intuitionistic Q- fuzzy closed implicative Q-ideal of a Q-algebra X.$\mu(0*x, q) \geq \mu(x, q)$ and $\lambda(0*x, q) \leq \lambda (x, q)$ for any $x \in X$ and $q \in Q$.

For any $(x, q) \in U (\mu; s) \Rightarrow \mu(x, q) \geq s \Rightarrow \mu(0*x, q) \geq s \Rightarrow (0*x, q) \in U(\mu;s)$ and

$(x, q) \in L(\lambda;t) \Rightarrow \lambda (x, q) \leq t \Rightarrow \lambda (0*x, q) \leq t \Rightarrow (0*x, q) \in L(\lambda;t)$

Hence $U(\mu;s)$ and $L(\lambda;t)$ are closed Q-ideals of X for s, t $\in$[0,1].

Conversely,

$U(\mu;s)$ and $L(\lambda;t)$ are closed Q-ideals of X for s, t $\in$[0,1].

To show $A=(X,\mu,\lambda)$ is an intuitionistic Q- fuzzy closed Q-ideal of X it is enough to show that $\mu(0*x, q) \geq \mu(x, q)$ and

$\lambda (0*x, q) \leq \lambda(x, q)$ for any $x\in X$ and $q\in Q$.

If possible assume that $x_0 \in X$ and $q_0 \in Q$ such that $\mu(0* x_{0,} q_0) < \mu(x_{0,} q_0)$

take $s_0 = (1/2) [\mu(0* x_{0,} q_0) + \mu(x_{0,} q_0)]$

$\Rightarrow \mu(0* x_{0,} q_0) < s_0 < \mu(x_{0,} q_0)$

$\Rightarrow (x_{0,} q_0) \in U(\mu;s_0)$ but $((0* x_{0,} q_0) \notin U(\mu;s_0)$

which is a contradiction to the definition of closed Q-ideal.

Hence $\mu(0*x, q) \geq \mu(x, q)$ for any $x \in X$ and $q \in Q$.

Similarly we can prove that $\lambda(0*x, q) \leq \lambda(x, q)$ for any $x \in X$ and $q \in Q$.

3.7 Definition: Let f be a mapping on a set XxQ and $A=(X,\mu,\lambda)$ an intuitionistic Q- fuzzy set in X. Then the fuzzy sets u and v on f (XxQ) defined by

$$u (y, q) = \sup_{x, q) \in f^{-1}(y, q)} \mu(x, q) \text{ and}$$

$$v(y, q) = \inf_{(x, q) \in f^{-1}(y, q)} \lambda (x, q)$$

for all $(y, q) \in f (XxQ)$ is called the image of A under f. If u, v are fuzzy sets in f (XxQ). Then the fuzzy setsμ = u o f and λ = v o f are called the pre – images of u and v respectively under f.

3.8 Definition: A function $f : XxQ \rightarrow YxQ$ is said to be a homomorphism of Q-algebras if
$f [(x,q)*(y,q)] = f (x,q) * f (y,q) = f (x*y,q)$

3.10 Theorem: Let f: $XxQ \rightarrow YxQ$ be an onto homomorphism of Q-algebras. If B=(Y, u, v) is an intuitionistic Q-fuzzy implicative Q-ideal of Y then the pre-image of B under f is an intuitionistic Q- fuzzy implicative Q-ideal of X.

Proof: Let A=(X,μ,λ) where $\mu = u \circ f$ and $\lambda = v \circ f$ is the pre-image of B=(Y, u, v) under f.

Since B=(Y, u, v) is an intuitionistic Q- fuzzy implicative Q-ideal of Y we have
$u(0^1,q) \geq u(f(x, q)) = u^\circ f(x, q) = \mu(x, q)$ and $v(0^1,q) \leq v(f(x, q)) = v^\circ f(x, q) = \lambda(x, q)$

on the other hand,

$u(0^1,q) = u(f(0,q)) = u^\circ f(0,q) = \mu(0,q)$ and $v(0^1,q) = v(f(0,q)) = v^\circ f(0,q) = \lambda(0,q)$

$\mu(0,q) = u(0^1,q) \geq \mu(x, q)$ implies $\mu(0,q)) \geq \mu(x, q)$

$\lambda(0,q)= v(0^1,q) \leq \lambda(x, q)$ implies $\lambda(0,q) \leq \lambda(x, q)$. for all x, y ∈ X and q∈Q.

ii. Now show that $\mu(x*z, q) \geq \min\{\mu((x*y)*z, q), \mu(y*z, q)\}$ and

$\lambda(x*z, q) \leq \max\{\lambda((x*y)*z, q), \lambda(y*z, q)\}$, for all x, y, z ∈ X q∈Q.

We have,
$$\begin{aligned}\mu(x*z,q) &= u^\circ f(x*z, q) = u(f(x*z, q)) \\ &= u(f(x,q)*f(z,q)) \\ &\geq \min\{u(f(x*y),q)*f(z,q)), u(f(y*z,q))\} \\ &= \min\{u(f((x*y)*z,q), u(f(y*z,q))\} \\ &= \min\{u^\circ f((x*y)*z, q), u^\circ f(y*z, q)\} \\ &= \min\{\mu((x*y)*z, q), \mu(y*z, q)\end{aligned}$$

Hence $\mu(x*z, q) \geq \min\{\mu((x*y)*z, q), \mu(y*z, q)$ is true for all x, y∈X and q∈Q.

Similarly, we can prove that $\lambda(x*z, q) \leq \max\{\lambda((x*y)*z, q), \lambda(y*z, q)\}$ for all x, y∈ X and q∈Q.

Hence the pre-image A=(X,μ,λ) of B is an intuitionistic Q- fuzzy implicative Q-ideal of X.

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