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SLIP EFFECTS ON MHD OSCILLATORY FLOW OF JEFFREY FLUID IN A CHANNEL WITH HEAT TRANSFER

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1. INTRODUCTION

The terms 'oscillatory' or 'unsteady' are generally used in the literature to describe the flows in which velocity or pressure or both depend on time. Oscillatory flow is a periodic flow that oscillates around a zero value. From technological view, oscillatory flow is always important for it has many practical applications, as for example in the aerodynamics of a helicopter rotor or in a fluttering airfoil as well as in a variety of bio-engineering problems.

The study of the flow of a viscous incompressible electrically conducting fluid in the presence of a magnetic field in a channel is motivated by several important problems of geophysical and astrophysical interest and fluid engineering. It is useful in Astrophysics because much of the universe is filled with widely spaced charged particles and permeated by magnetic fields. Geophysicists encounter MHD phenomena in the interactions of conducting fluids and magnetic fields that are present in and around heavenly bodies. Engineers employ MHD principles in the design of heat exchangers, MHD pumps and flow meters, in solving space vehicle propulsion, control and reentry problems; in creating novel power generating systems and in developing confinement schemes for controlled fusion. Soundalgekar and Bhat (1971) have investigated the MHD oscillatory flow of a Newtonian fluid in a channel with heat transfer. MHD flow of viscous fluid between two parallel plates with heat transfer was discussed by Attia, and Kotb (1996). Raptis et al. (1982) have analyzed the hydromagnetic free convection flow of a viscous fluid through a porous medium between two parallel plates. Aldoss et al. (1995) have studied mixed convection flow from a vertical plate embedded in a porous medium in the presence of a magnetic field. Khaled and Vafai (2004) have been discussed the effect of slip condition on stokes and coutte flows due to an oscillating wall. Makinde and Mhone (2005) have considered heat transfer to MHD oscillatory flow in a channel filled with porous medium. Makinde and Osalusi (2006) have discussed a MHD steady flow in a channel with slip at permeable boundaries. The effect of slip condition on unsteady MHD Oscillatory flow of a viscous fluid in a planer channel was studied by Mahmood and Ali (2007). Mostafa (2009) have studied thermal radiation effect on unsteady MHD free convection flow past a vertical plate with temperature dependent viscosity. Unsteady heat transfer to MHD oscillatory flow through a porous medium under slip condition was discussed by Hamza et al. (2011).

An important class of fluids differs from Newtonian fluids in that the relationship between the shear stress and the flow field is more complicated. Such fluids are non-Newtonian. Examples include various suspensions such as coal-water or coal-oil slurries, food products, inks, glues, soaps, polymer solutions, mud, blood at low shear rate, cosmetic products and many others. Al Khatib andWilson (2001) have studied the Poiseuille flow of a yield stress fluid in a channel. Flow of a visco-elastic fluid in a channel of slowly varying width was studied by Frigaard and Ryan (2004). Mokhtar et al. (2006) have studied the pulsatile MHD non-Newtonian fluid flow with heat and mass transfer through a porous medium between two permeable parallel plates. Mishra et al. (2008) investigated a flow and heat transfer of a MHD viscoelastic fluid in a channel with stretching walls. Ali and Asghar (2011) have analyzed by oscillatory channel flow for non-Newtonian fluid. Rita and Jyoti Das (2012) have studied the effect of heat transfer on MHD oscillatory viscoelastic fluid flow in a channel through a porous medium.

In view of these we studied the effect of heat transfer on MHD oscillatory flow of a Jeffrey fluid in a channel with slip effect at lower wall. The expressions are obtained for velocity and temperature analytically. The effects of various emerging parameters on the velocity and temperature are discussed through graphs in detail.

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2. MATHEMATICAL FORMULATION

We consider the slip effect on flow of a Jeffrey fluid in a channel of width h under the influence of electrically applied magnetic field and radiative heat transfer as depicted in Fig.1. It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. We choose the Cartesian coordinate system (x, y), where x - is taken along center of the channel and the y - axis is taken normal to the flow direction.

The constitute equation of S for Jeffrey fluid is

$$S = \frac{\mu}{1+\lambda_1}\left(\dot{\gamma} + \lambda_2 \ddot{\gamma}\right) \tag{2.1}$$

where μ is the dynamic viscosity, λ_1 is the ratio of relaxation to retardation times, λ_2 is the retardation time, $\dot{\gamma}$ is the shear rate and dots over the quantities denote differentiation with time.

The basic equations of momentum and energy governing such a flow, subject to the Boussinesq approximation, are:

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\mu}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2} - \sigma B_0^2 u + \rho g \beta \left(T - T_0\right) \tag{2.2}$$

$$\rho \frac{\partial T}{\partial t} = \frac{k}{c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{c_p}\frac{\partial q}{\partial y} \tag{2.3}$$

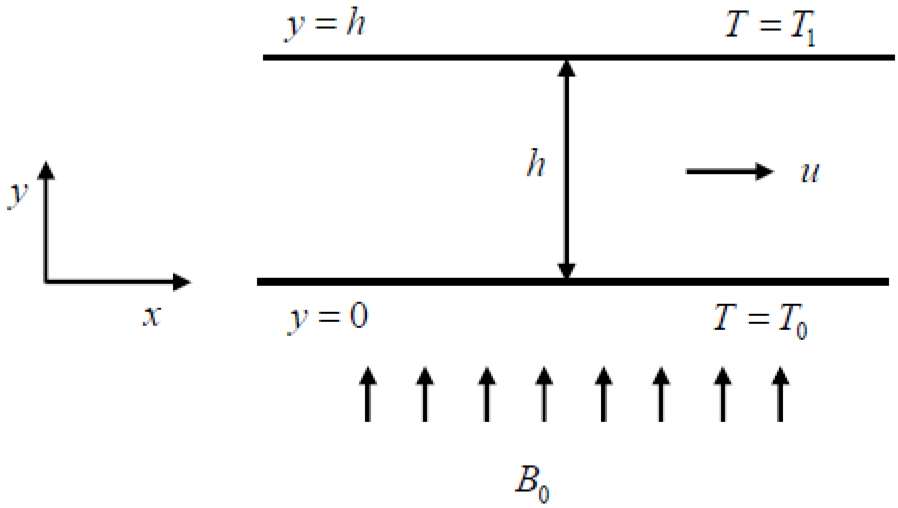

Fig. 1 Physical model of the problem

The boundary conditions are given by

$$u = \gamma \frac{\partial u}{\partial y}, \qquad T = T_0 \qquad \text{at} \qquad y = 0 \tag{2.4}$$

$$u = 0, \qquad T = T_1 \qquad \text{at} \qquad y = h \tag{2.5}$$

where u is the axial velocity, T is the fluid temperature, p is the pressure, ρ is the fluid density, B_0 is the magnetic field strength, σ is the conductivity of the fluid, g is the acceleration due to gravity, β is the coefficient of volume expansion due to temperature, c_p is the specific heat at constant pressure, k is the thermal conductivity, γ is slip parameter and q is the radiative heat flux. Following Cogley et al. (1968), it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q}{\partial y} = 4\alpha^2 \left(T_0 - T\right) \tag{2.6}$$

here α is the mean radiation absorption coefficient.

Introducing the following non-dimensional variables

$$\bar{x}=\frac{x}{h},\ \bar{y}=\frac{y}{h},\ \bar{u}=\frac{u}{U},\ \theta=\frac{T-T_0}{T_1-T_0},\ \bar{t}=\frac{tU}{h},\ \bar{p}=\frac{pa}{\mu U},\ M^2=\frac{\sigma h^2 B_0^2}{\mu},\ Gr=\frac{\rho g\beta\left(T_1-T_0\right)}{U\mu},$$

$$\mathrm{Re}=\frac{\rho hU}{\mu},\ Pe=\frac{\rho hUc_p}{k},\ N^2=\frac{4\alpha^2h^2}{k}$$

here U is the mean flow velocity, into the equations (2.2) and (2.3), we get (after dropping bars)

$$\mathrm{Re}\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\frac{1}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2}-M^2u+Gr\theta \tag{2.7}$$

$$Pe\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial y^2}+N^2\theta \tag{2.8}$$

where Re is the Reynolds number, M is the Hartmann number, Gr is the Grashof number, Pe is the Peclet number and N is the radiation parameter.

The corresponding non-dimensional boundary conditions are

$$u=0,\qquad \theta=0\qquad \text{at}\qquad y=0 \tag{2.9}$$

$$u=0,\qquad \theta=1\qquad \text{at}\qquad y=1 \tag{2.10}$$

3. SOLUTION

In order to solve equations (2.7) – (2.10) for purely oscillatory flow, let

$$-\frac{\partial p}{\partial x}=\lambda e^{i\omega t} \tag{3.1}$$

$$u\left(y,t\right)=u_0\left(y\right)e^{i\omega t} \tag{3.2}$$

$$\theta\left(y,t\right)=\theta_0\left(y\right)e^{i\omega t} \tag{3.3}$$

where λ is a real constant and ω is the frequency of the oscillation.

Substituting the equations (3.1) - (3.3) in to the equations (2.7) – (2.10), we get

$$\frac{d^2u_0}{dy^2}-m_2^2u_0=-\lambda\left(1+\lambda_1\right)-Gr\left(1+\lambda_1\right)\theta_0 \tag{3.4}$$

$$\frac{d^2\theta_0}{dy^2}+m_1^2\theta_0=0 \tag{3.5}$$

with the boundary conditions

$$u_0=0,\qquad \theta_0=0\qquad \text{at}\qquad y=0 \tag{3.6}$$

$$u_0=0,\qquad \theta_0=1\qquad \text{at}\qquad y=1 \tag{3.7}$$

in which $m_1=\sqrt{N^2-i\omega Pe}$ and $m_2=\sqrt{M^2+i\omega\mathrm{Re}}$.

Solving equations (3.4) and (3.5) using the boundary conditions (3.6) and (3.7), we obtain

$$u_0\left(y\right)=A_1\cosh m_2y+A_2\sinh m_2y+A_3+A_4\frac{\sin m_1y}{\sin m_1} \tag{3.8}$$

and $$\theta_0\left(y\right)=\frac{\sin m_1y}{\sin m} \tag{3.9}$$

where $A_1 = -\left(\dfrac{A_5 \sinh m_2 + \gamma m_2 A_6}{\sinh m_2 + \gamma m_2 \cosh m_2}\right)$, $A_2 = \dfrac{A_5 \cosh m_2 - A_6}{\sinh m_2 + \gamma m_2 \cosh m_2}$,

$A_3 = \dfrac{\lambda(1+\lambda_1)}{m_2^2}$, $A_4 = \dfrac{Gr(1+\lambda_1)}{(m_1^2 + m_2^2)}$, $A_5 = A_3 - A_4 \dfrac{\gamma m_1 \cos m_1}{\sin m_1}$ and $A_6 = A_3 + A_4$.

Therefore, the fluid velocity and temperature are given as

$$u(y,t) = \left(-A\cosh m_2 y + C\frac{\sinh m_2}{\sinh m_2} + A + B\frac{\sin m_1 y}{\sin m_1}\right)e^{i\omega t} \tag{3.10}$$

and
$$\theta(y,t) = \frac{\sin m_1 y}{\sin m} e^{i\omega t} \tag{3.11}$$

4. DISCUSSION OF THE RESULTS

Fig. 2 shows the effect of material parameter λ_1 on velocity u for $\mathrm{Re}=1$, $Gr=1$, $Pe=0.71$, $\lambda=1$, $\omega=1$, $t=0.1$, $N=1$ and $M=1$. It is observed that, the axial velocity u increases with increasing λ_1. Also, the maximum velocity occurs at the centerline of the channel while the minimum at the channel walls. Moreover, the velocity is more of Jeffrey fluid $(\lambda_1 > 0)$ than that of Newtonian fluid $(\lambda_1 \to 0)$.

Effect of Hartmann number M on velocity u for $\mathrm{Re}=1$, $Gr=1$, $Pe=0.71$, $\lambda=1$, $\omega=1$, $t=0.1$, $N=1$ and $\lambda_1=0.3$ is shown in Fig.3. It is found that, the axial velocity u decreases with increasing M.

Fig. 4 depicts the effect of radiation parameter N on velocity u for $\mathrm{Re}=1$, $Gr=1$, $Pe=0.71$, $\lambda=1$, $\omega=1$, $t=0.1$, $M=1$ and $\lambda_1=0.3$. It is noted that, the axial velocity u decreases with an increase in N.

Effect of Peclet number Pe on velocity u for $\mathrm{Re}=1$, $Gr=1$, $N=1$, $\lambda=1$, $\omega=1$, $t=0.1$, $M=1$ and $\lambda_1=0.3$ is shown in Fig. 5. It is noted that, the axial velocity u decreases with an increase in N.

Fig. 6 shows the effect of Grashof number Gr on velocity u for $\mathrm{Re}=1$, $M=1$, $Pe=0.71$, $\lambda=1$, $\omega=1$, $t=0.1$, $N=1$ and $\lambda_1=0.3$. It is observed that, the axial velocity u increases with increasing Gr.

Effect of Reynolds number Re on velocity u for $M=1$, $Gr=1$, $Pe=0.71$, $\lambda=1$, $\omega=1$, $t=0.1$, $N=1$ and $\lambda_1=0.3$ is depicted in Fig. 7. It is found that, the axial velocity u decreases with decreasing Re.

Fig. 8 shows the effect of λ on velocity u for $\mathrm{Re}=1$, $Gr=1$, $Pe=0.71$, $M=1$, $\omega=1$, $t=0.5$, $N=1$ and $\lambda_1=0.3$. It is noted that, the axial velocity u increases with an increase in λ.

Effect of ω on velocity u for $\mathrm{Re}=1$, $Gr=1$, $Pe=0.71$, $\lambda=1$, $M=1$, $t=0.1$, $N=1$ and $\lambda_1=0.3$ is shown in Fig. 9. It is observed that, the axial velocity u decreases with decreasing ω.

Fig. 10 shows the effect of N on temperature θ for $Pe=0.71$, $\omega=1$ and $t=0.5$. It is noted that, the temperature θ increases with an increase in N.

Effect of Pe on temperature θ for $N=1$, $\omega=1$ and $t=0.5$ is shown in Fig. 11. It is observed that, the temperature θ decreases with increasing Pe.

Fig. 12 depicts the effect of ω on temperature θ for $N=1$, $Pe=0.71$ and $t=0.5$. It is found that, the temperature θ decreases with an increase in ω.

5. CONCLUSIONS

In this chapter, we studied the effect of heat transfer on MHD oscillatory flow of Jeffrey fluid in a channel. The expressions for the velocity and temperature are obtained analytically. It is found that, the velocity u increases with increasing λ_1, Gr and λ, while it decreases with increasing M, N, Pe, Re and ω. Also, it is observed that the temperature θ increases with increasing N and Pe, while it decreases with increasing ω. Further, it is found that, the velocity is more for Jeffrey fluid than that of Newtonian fluid.

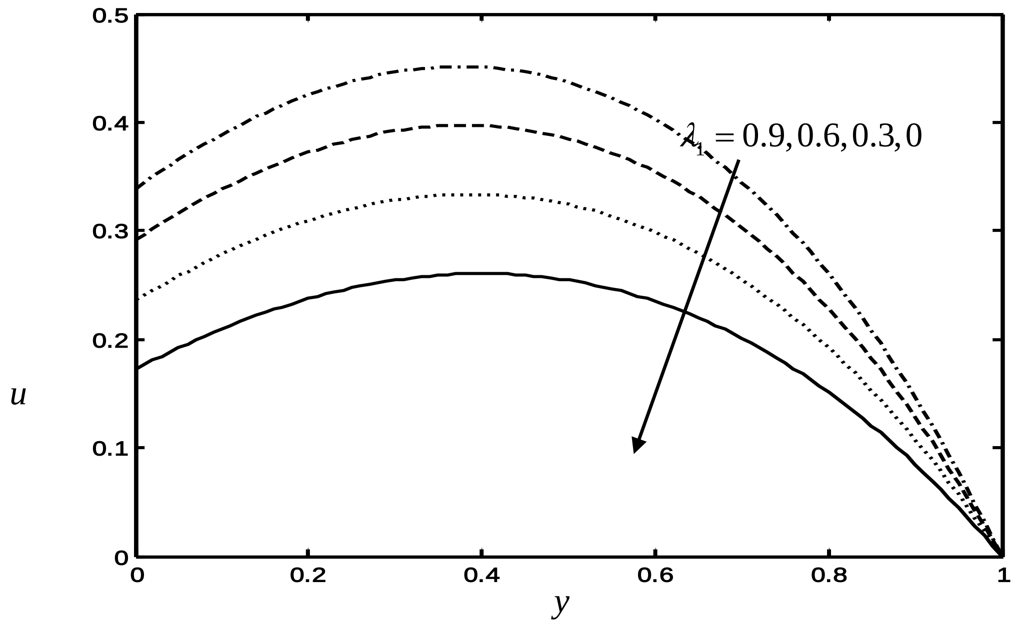

Fig. 2 Effect of material parameter λ_1 on velocity u for $\mathrm{Re}=1$, $Gr=1$, $Pe=0.71$, $\lambda=1$, $\omega=1$, $t=0.1$, $\gamma=1$, $N=1$ and $M=1$.

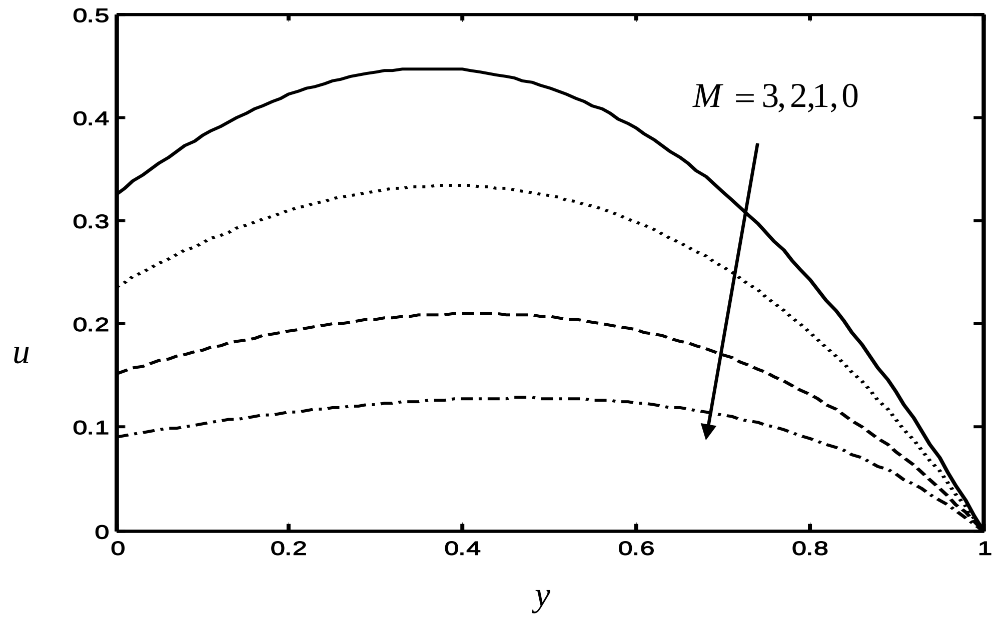

Fig. 3 Effect of Hartmann number M on velocity u for $\mathrm{Re}=1$, $Gr=1$, $Pe=0.71$, $\lambda=1$, $\omega=1$, $t=0.1$, $\gamma=1$, $N=1$ and $\lambda_1=0.3$.

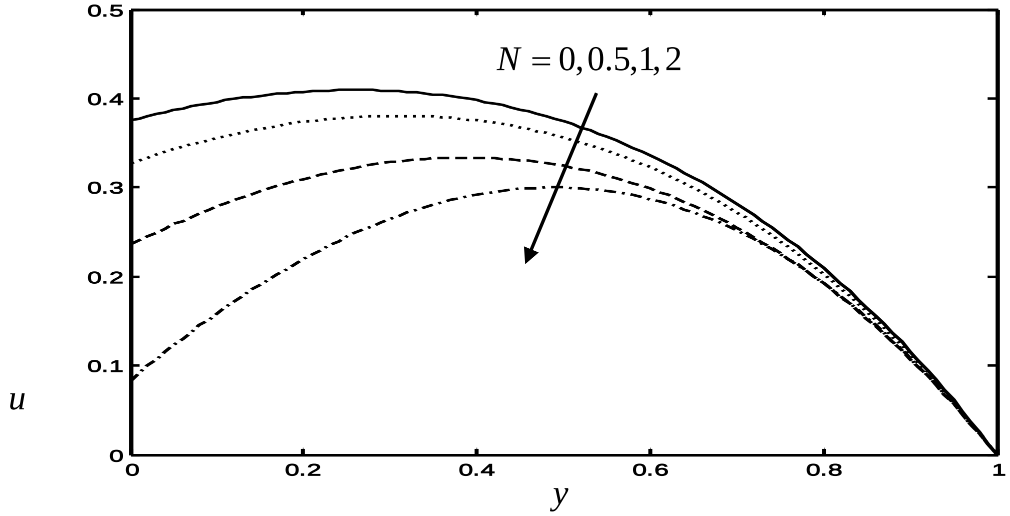

Fig. 4 Effect of radiation parameter N on velocity u for $\mathrm{Re} = 1$, $Gr = 1$, $Pe = 0.71$, $\lambda = 1$, $\omega = 1$, $t = 0.1$, $\gamma = 1$, $M = 1$ and $\lambda_1 = 0.3$.

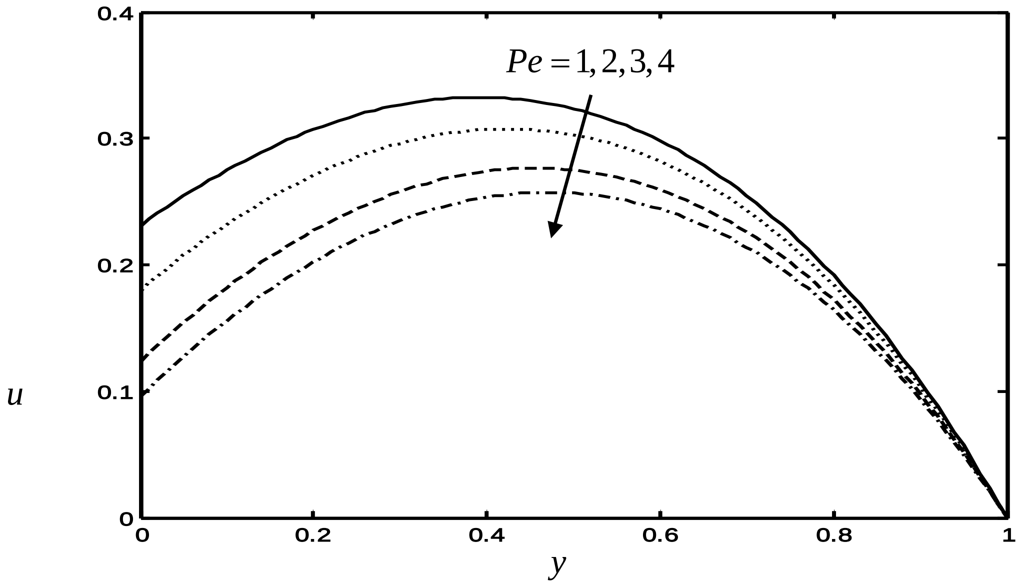

Fig. 5 Effect of Peclet number Pe on velocity u for $\mathrm{Re} = 1$, $Gr = 1$, $N = 1$, $\lambda = 1$, $\omega = 1$, $t = 0.1$, $\gamma = 1$, $M = 1$ and $\lambda_1 = 0.3$.

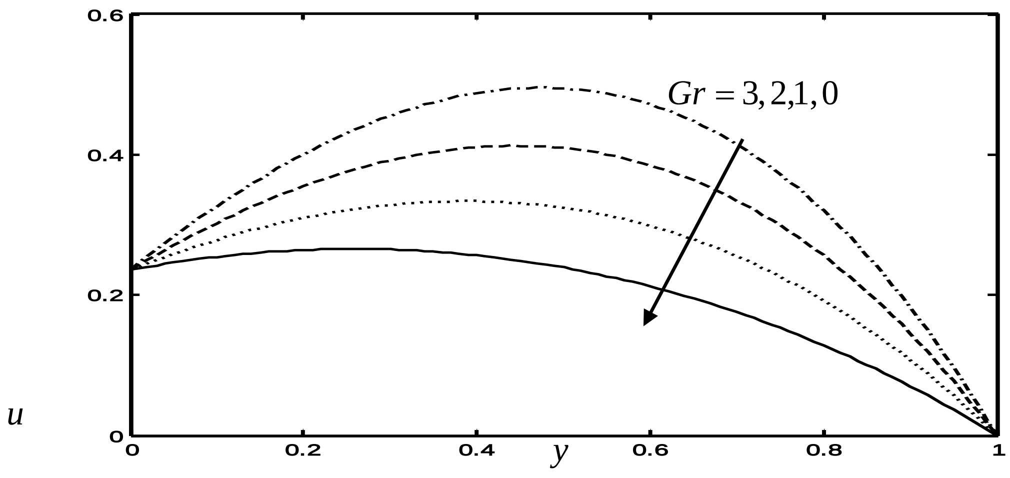

Fig. 6 Effect of Grashof number Gr on velocity u for $\mathrm{Re} = 1$, $M = 1$, $Pe = 0.71$, $\lambda = 1$, $\omega = 1$, $t = 0.1$, $\gamma = 1$, $N = 1$ and $\lambda_1 = 0.3$.

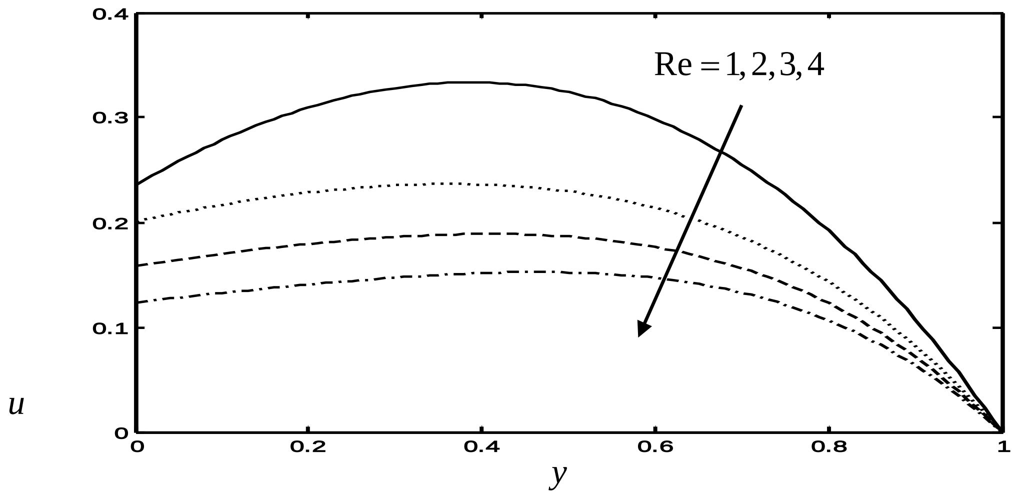

Fig. 7 Effect of Reynolds number Re on velocity u for $M = 1$, $Gr = 1$, $Pe = 0.71$, $\lambda = 1$, $\omega = 1$, $t = 0.1$, $\gamma = 1$, $N = 1$ and $\lambda_1 = 0.3$.

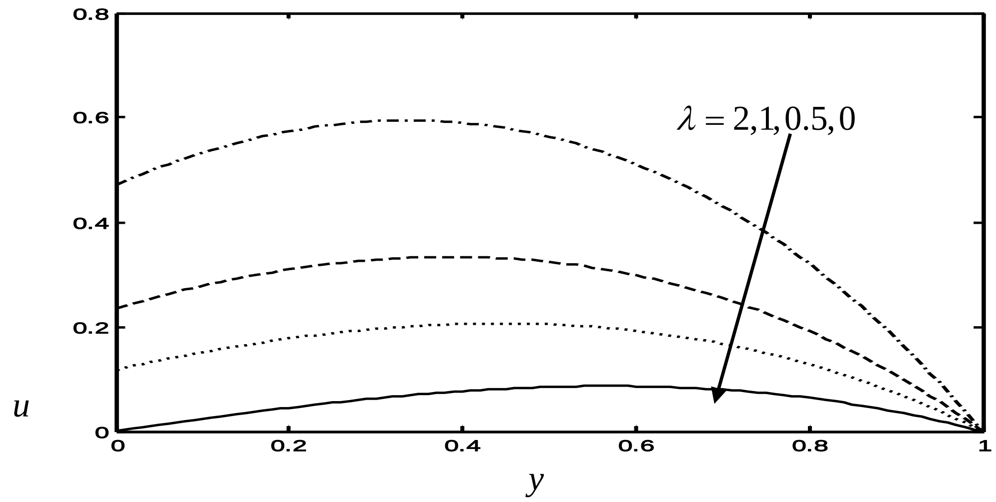

Fig. 8 Effect of λ on velocity u for $\mathrm{Re} = 1$, $Gr = 1$, $Pe = 0.71$, $M = 1$, $\omega = 1$, $\gamma = 1$, $t = 0.1$, $N = 1$ and $\lambda_1 = 0.3$.

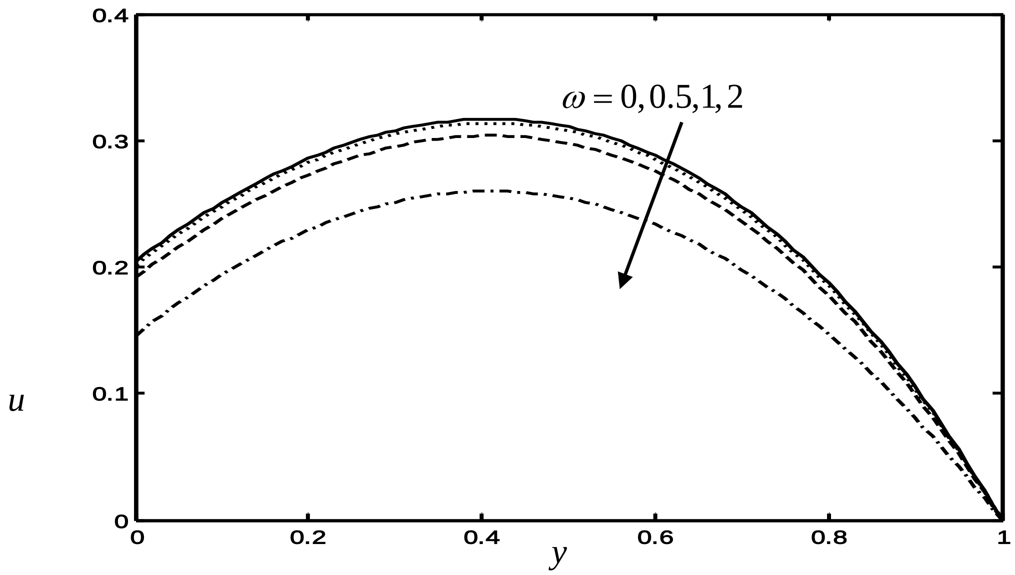

Fig. 9 Effect of ω on velocity u for $\mathrm{Re} = 1$, $Gr = 1$, $Pe = 0.71$, $\lambda = 1$, $M = 1$, $t = 0.1$, $\gamma = 1$, $N = 1$ and $\lambda_1 = 0.3$.

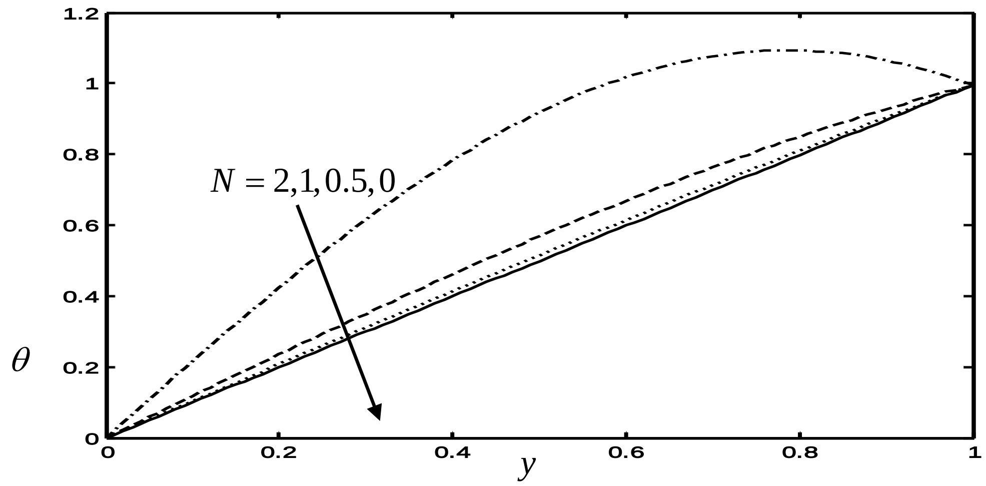

Fig. 10 Effect of N on temperature θ for $Pe = 0.7$, $\omega = 1$ and $t = 0.5$.

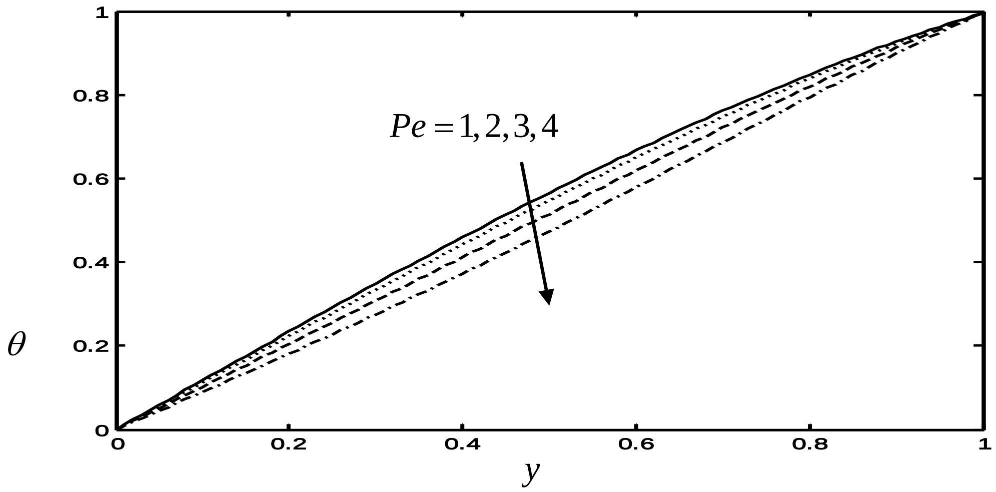

Fig. 11 Effect of Pe on temperature θ for $N = 1$, $\omega = 1$ and $t = 0.5$.

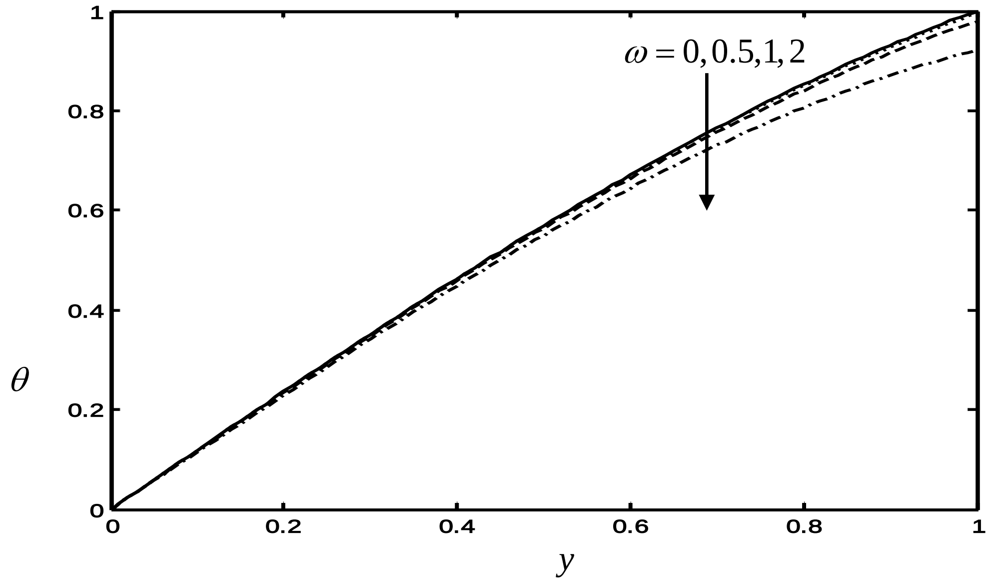

Fig. 12 Effect of ω on temperature θ for $N = 1$, $Pe = 0.71$ and $t = 0.5$.

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