

$g^{**}I$ – closed and $g^{**s}I$ – closed sets

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ABSTRACT

In this paper $g^{**}I$ -closed sets and $g^{**s}I$ -closed sets are defined and their properties are investigated.

Key words: $g^{**}I$ – closed, $g^{**s}I$ – closed sets, $*$ – countably additive, $*$ – additive, $*s$ – additive, $*s$ – countably additive $*s$ – finitely additive ideal spaces, locally finite and locally countable family of sets.

1. INTRODUCTION

Ideal topological spaces have been first introduced by K. Kuratowski [2] in 1930. Vaidyanathaswamy [6] introduced local function in 1945 and defined a topology τ . M.E. Abd El Monsef, E. F. Lashien and A. a Nasef [1] introduced semi local function in 1992 and defined a topology τ^{*s} .

In this paper $g^{**}I$ -closed sets and $g^{**s}I$ -closed sets are defined and their properties are investigated.

2. PRELIMINARIES

Definition 2.1: An ideal [2] I on a non empty set X is a collection of subsets of X which satisfies the following properties. (i) $A \in I, B \in I \Rightarrow A \cup B \in I$ (ii) $A \in I, B \subset A \Rightarrow B \in I$. A topological space (X, τ) with an ideal I on X is called an ideal topological space and is denoted by (X, τ, I) . Let Y be a subset of X . $I_Y = \{I \cap Y / I \in I\}$ is an ideal on Y and by $(Y, \tau/Y, I_Y)$ we denote the ideal topological subspace.

Definition 2.2: Let $P(X)$ be the power set of X , then a set operator $(\)^*$: $P(X) \rightarrow P(X)$ called the local function [6] of A with respect to τ and I is defined as follows: For $A \subset X$, $A^*(I, \tau) = \{x \in X / U \cap A \notin I \text{ for every open set } U \text{ containing } x\}$. We simply write A^* instead of $A^*(I, \tau)$ in case there is no confusion. A

Kuratowski closure operator $cl^*(\)$ for a topology $\tau^*(I, \tau)$, called the τ^* - topology is defined by $Cl^*(A) = A \cup A^*$

For A, B in (X, τ, I) we have

- (i) If $A \subset B$ then $A^* \subset B^*$
- (ii) $(A^*)^* \subseteq A^*$
- (iii) $A^* \cup B^* = (A \cup B)^*$
- (iv) $(A \cap B)^* \subseteq A^* \cap B^*$ (v) If $I = \{\emptyset\}$, $A^* = cl(A)$ and $cl^*(A) = cl(A)$

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(vi) If $I = P(X)$ then $A^* = \phi$ and $cl^*(A) = A$ (vii) $A^* = cl(A^*) \subset cl(A)$ and A^* is a closed subset of $cl(A)$.

Definition 2.3: A subset A of a space (X, τ) is said to be semi-open [3] if $A \subset cl(int(A))$

Definition 2.4: A set operator [1] $(\)^{*S} : P(X) \rightarrow P(X)$ called a semi local function and $cl^{*S}(\)$ of A with respect to τ and I are defined as follows: For $A \subset X$, $A^{*S}(I, \tau) = \{x \in X / U \cap A \notin I \text{ for every semi open set } U \text{ containing } x\}$. and $Cl^{*S}(A) = A \cup A^{*S}$.

For a subset A of X , $cl(A)$ (resp. $scl(A)$) denotes the closure (resp. semi closure) of A in (X, τ) . Similarly $cl^*(A)$ and $int^*(A)$ denote the closure of A and interior of A in (X, τ^*) .

Definition 2.5: A subset A of X is called $*$ closed [6] (resp. $*s$ closed [1]) if $A^* \subseteq A$ (resp. $A^{*S} \subseteq A$).

Definition 2.6: A subset A of X is called $*$ - dense [6] in itself (resp. $*s$ - dense [1]) if $A \subset A^*$ (resp. $A \subset A^{*S}$).

Definition 2.7: A subset A of X is called $*$ - perfect [6] (resp. $*s$ - perfect [1]) if $A = A^*$ (resp. $A = A^{*S}$)

LEMMA 2.5: [1] For A, B in (X, τ, I) we have

- (i) If $A \subset B$ then $A^{*S} \subset B^{*S}$
- (ii) $(A^{*S})^{*S} \subseteq A^{*S}$
- (iii) $A^{*S} \cup B^{*S} \supseteq (A \cup B)^{*S}$
- (iv) $(A \cap B)^{*S} \subseteq A^{*S} \cap B^{*S}$
- (v) If $I = \{\phi\}$, $A^{*S} = scl(A)$ and $cl^{*S}(A) = scl(A)$
- (vi) If $I = P(X)$ then $A^{*S} = \phi$ and $cl^{*S}(A) = A$
- (vii) $A^{*S} = scl(A^{*S}) \subset scl(A)$ and A^{*S} is semi closed.

A subset A of an ideal space (X, τ, I) is said to be g - closed [4], if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X . The complement of g - closed set is said to be g - open.

A subset A of an ideal space (X, τ, I) is said to be g^* - closed [7], if $cl^*(A) \subseteq U$ whenever $A \subseteq U$ and U is g - open in X . The complement of g^* - closed set is said to be g^* - open.

A subset A of an ideal space (X, τ, I) is said to be g^{**} - closed [5], if $cl^{**}(A) \subseteq U$ whenever $A \subseteq U$ and U is g^* - open in X . The complement of g^{**} - closed set is said to be g^{**} - open

3. $g^{**}I$ - closed and $g^{**s}I$ - closed sets

Definition 3.1: subset A of an ideal space (X, τ, I) is said to be $g^{**}I$ closed, if $cl^{**}(A) \subseteq U$ whenever $A \subseteq U$ and U is g^* - open in X . The complement of $g^{**}I$ - closed set is said to be $g^{**}I$ - open. A subset A of an ideal space (X, τ, I) is said to be $g^{**s}I$ closed, if $cl^{**s}(A) \subseteq U$ whenever $A \subseteq U$ and U is g^* - open in X . The complement of $g^{**s}I$ - closed set is said to be $g^{**s}I$ - open. The collection of all $g^{**}I$ - closed sets (resp. $g^{**s}I$ - closed sets) is denoted by $G^{**}IC(X)$ (resp. $G^{**s}IC(X)$) Similarly the collection of all $g^{**}I$ - open sets (resp. $g^{**s}I$ - open sets) is denoted by $G^{**}IO(X)$ (resp. $G^{**s}IO(X)$)

Remarks: 3.2:

1. If $I = P(X)$ then $cl^{*S}(A) = cl^*(A) = A$ for all $A \subseteq X$ and hence every subset of X is $g^{**}I$ - closed and $g^{**s}I$ - closed
2. Since $A^{*S} = \{\phi\} = A^*$ for every $A \in I$, every member of I is $g^{**}I$ - closed and $g^{**s}I$ - closed
3. Every closed set A is $g^{**}I$ - closed since $cl^*(A) \subseteq cl(A) = A$ and every semi closed set A is $g^{**s}I$ - closed since $cl^{*S}(A) \subseteq scl(A) = A$
4. Every $*$ -closed set is $g^{**}I$ - closed since $cl^*(A) = A$
5. Finite union of $g^{**}I$ - closed sets is $g^{**}I$ - closed since $cl^*\left[\bigcup_{i=1}^n A_i\right] = \bigcup_{i=1}^n cl^*(A_i)$

In general, $(A \cup B)^{*S} \neq A^{*S} \cup B^{*S}$ for subsets A and B in X as seen from the following example.

Example 3.3: Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $I = \{\phi\}$. Then $\{a\}^{*S} = \{a\}$, $\{b\}^{*S} = \{b\}$ and $\{a, b\}^{*S} = X$. This shows $(A \cup B)^{*S} \neq A^{*S} \cup B^{*S}$ in general.

Moreover in general $\left[\bigcup_{i=1}^{\infty} A_i\right]^* \neq \bigcup_{i=1}^{\infty} (A_i)^*$ and $\left[\bigcup_{i=1}^{\infty} A_i\right]^{*S} \neq \bigcup_{i=1}^{\infty} (A_i)^{*S}$ For arbitrary indexing set Ω ,
 $\left[\bigcup_{\alpha \in \Omega} A_\alpha\right]^* \neq \bigcup_{\alpha \in \Omega} (A_\alpha)^*$ and $\left[\bigcup_{\alpha \in \Omega} A_\alpha\right]^{*S} \neq \bigcup_{\alpha \in \Omega} (A_\alpha)^{*S}$ as seen from the following examples.

Example 3.4: Let $X=Z$, τ be the cofinite topology in X and $I = \{\phi\}$. Then $A^* = A = A^{*S}$ if A is finite and $A^* = Z = A^{*S}$ if A is infinite. $G^{**}IC(X) = \{X, \phi, \text{all finite subsets}\}$.

Let $A_n = \{-n, -n+1, \dots, -1, 1, \dots, n-1, n\}$ for every positive integer n . Then $A_n^* = A_n^{*S} = A$ for every n .

$$\left[\bigcup_{i=1}^{\infty} A_i\right]^* = Z = \left[\bigcup_{i=1}^{\infty} A_i\right]^{*S} \text{ and } \bigcup_{i=1}^{\infty} (A_i)^* = Z - \{0\} = \bigcup_{i=1}^{\infty} (A_i)^{*S}$$

$$\text{Therefore } \left[\bigcup_{i=1}^{\infty} A_i\right]^* \neq \bigcup_{i=1}^{\infty} (A_i)^* \text{ and } \left[\bigcup_{i=1}^{\infty} A_i\right]^{*S} \neq \bigcup_{i=1}^{\infty} (A_i)^{*S}$$

Here A_n is $g^{**}I$ -closed and $g^{**s}I$ - closed for every n . But $\left[\bigcup_{i=1}^{\infty} A_i\right]$ is not $g^{**}I$ - closed and $g^{**s}I$ -closed.

Definition 3.5: An ideal space (X, τ, I) is said to be

- (i) $*$ - countably additive if $\left[\bigcup_{i=1}^{\infty} A_i\right]^* = \bigcup_{i=1}^{\infty} (A_i)^*$
- (ii) $*$ - additive if $\left[\bigcup_{\alpha \in \Omega} A_\alpha\right]^* = \bigcup_{\alpha \in \Omega} (A_\alpha)^*$ for all indexing sets Ω .
- (iii) $*$ - additive if $\left[\bigcup_{i=1}^n A_i\right]^{*S} = \bigcup_{i=1}^n (A_i)^{*S}$ for every positive integer n .
- (iv) $*S$ - countably additive if $\left[\bigcup_{i=1}^{\infty} A_i\right]^{*S} = \bigcup_{i=1}^{\infty} (A_i)^{*S}$
- (v) $*S$ - additive if $\left[\bigcup_{\alpha \in \Omega} A_\alpha\right]^{*S} = \bigcup_{\alpha \in \Omega} (A_\alpha)^{*S}$ for all indexing sets Ω .

Remark: 3.6: In an ideal topological space (X, τ, I) which is *S – finitely additive we have following results:

1. $cl^{*S}(\phi) = \phi$
2. $cl^{*S}(X) = X$
3. $A \subseteq cl^{*S}(A)$
4. $cl^{*S}(A \cup B) = cl^{*S}(A) \cup cl^{*S}(B)$
5. $cl^{*S}(cl^{*S}(A)) = cl^{*S}(A)$ for all subsets A, B in X.

Therefore $cl^{*S}(\)$ satisfies Kuratowski Closure axioms and hence it defines a topology τ^{*S} whose closure operation is given as $cl^{*S}(A) = A \cup A^{*S}$. Note that $\tau \subseteq \tau^* \subseteq \tau^{*S}$. $cl^{*S}(A)$ and $int^{*S}(A)$ denote the closure and interior of A in (X, τ^{*S}) .

Theorem3.7 In a * – countably additive (resp * – additive) ideal topological space (X, τ, I) Countable union (resp arbitrary union) of $g^{**}I$ – closed sets is $g^{**}I$ – closed.

Proof: It follows since in * – countably additive space $cl^* \left[\bigcup_{i=1}^{\infty} A_i \right] = \bigcup_{i=1}^{\infty} cl^*(A_i)$ and in * – additive space $cl^* \left[\bigcup_{\alpha \in \Omega} A_{\alpha} \right] = \bigcup_{\alpha \in \Omega} cl^*(A_{\alpha_i})$

Theorem 3.8: In a *s – finitely additive (resp *s -countably additive, *s - additive) ideal topological space (X, τ, I) finite union (resp countable union, arbitrary union) of $g^{**}I$ – closed sets is $g^{**}I$ – closed

Proof: It follows since in *s finitely additive space $cl^{*s} \left[\bigcup_{i=1}^n A_i \right] = \bigcup_{i=1}^n cl^{*s}(A_i)$ and in *s -countably additive space $cl^{*s} \left[\bigcup_{i=1}^{\infty} A_i \right] = \bigcup_{i=1}^{\infty} cl^{*s}(A_i)$ and in *s – additive space $cl^{*s} \left[\bigcup_{\alpha \in \Omega} A_{\alpha} \right] = \bigcup_{\alpha \in \Omega} cl^{*s}(A_{\alpha_i})$

Definition 3.9: $\{A_{\alpha} / \alpha \in \Omega\}$ is said to be a locally finite (resp locally countable) family of sets in (X, τ, I) if for every $x \in X$, there exists an open set U in X containing x that intersects only a finite (resp countable) number of members $A_{\alpha_1}, \dots, A_{\alpha_n}$ (resp $A_{\alpha_i}, i = 1, \dots, \infty$) of $\{A_{\alpha} / \alpha \in \Omega\}$.

Theorem 3.10: Let (X, τ, I) be an ideal space, and let $\{A_{\alpha} / \alpha \in \Omega\}$ be a locally finite family of $g^{**}I$ -closed sets in (X, τ, I) . Then $\left(\bigcup_{\alpha \in \Omega} A_{\alpha} \right)$ is also $g^{**}I$ - closed.

Proof: $A_{\alpha} \subseteq \bigcup A_{\alpha}$ implies $A_{\alpha}^* \subseteq \left(\bigcup A_{\alpha} \right)^*$ for every α . Therefore $\bigcup_{\alpha \in \Omega} (A_{\alpha})^* \subseteq \left(\bigcup_{\alpha \in \Omega} A_{\alpha} \right)^*$ ----- (1)

On the otherhand, if $x \in \left(\bigcup_{\alpha \in \Omega} A_{\alpha} \right)^*$ then there exists an open set U containing x , that intersects only finite number of members $A_{\alpha_1}, \dots, A_{\alpha_n}$. Let V be an open set containing x . Then $U \cap V$ is an open set containing x . which implies $(U \cap V) \cap \left(\bigcup_{\alpha \in \Omega} A_{\alpha} \right) \notin I$.

i.e. $\left[(U \cap V) \cap \left(\bigcup_{\alpha \neq \alpha_i} A_\alpha \right) \right] \cup \left[(U \cap V) \cap \left(\bigcup_{i=1}^n A_{\alpha_i} \right) \right] \notin I$ i.e. $\{\emptyset\} \cup \left[(U \cap V) \cap \left(\bigcup_{i=1}^n A_{\alpha_i} \right) \right] \notin I$ and this

implies $V \cap \left(\bigcup_{i=1}^n A_{\alpha_i} \right) \notin I$ Therefore $x \in \left(\bigcup_{i=1}^n A_{\alpha_i} \right)^* = \bigcup_{i=1}^n (A_{\alpha_i})^* \subseteq \bigcup_{\alpha \in \Omega} (A_\alpha)^*$

Therefore $\left(\bigcup_{\alpha \in \Omega} A_\alpha \right)^* \subseteq \bigcup_{\alpha \in \Omega} (A_\alpha)^*$ -----(2) From (1) and(2) we get $\left(\bigcup_{\alpha \in \Omega} A_\alpha \right)^* = \bigcup_{\alpha \in \Omega} (A_\alpha)^*$

Let $\bigcup_{\alpha \in \Omega} A_\alpha \subseteq U$ and $U - g^*$ - open in X . Then $A_\alpha \subseteq U \forall \alpha \in \Omega$ implies $cl^*(A_\alpha) \subseteq U \forall \alpha \in \Omega$.

Then $cl^*\left(\bigcup_{\alpha \in \Omega} A_\alpha\right) = \left(\bigcup_{\alpha \in \Omega} A_\alpha\right) \cup \left(\bigcup_{\alpha \in \Omega} A_\alpha\right)^* = \bigcup_{\alpha \in \Omega} cl^*(A_\alpha) \subseteq U$ Therefore $\bigcup_{\alpha \in \Omega} A_\alpha$ is $g^{**}I$ - closed.

Theorem 3.11: Let (X, τ, I) be an ideal space which is $*$ - countably additive, and let $\{A_\alpha / \alpha \in \Omega\}$ be a locally countable family of $g^{**}I$ - closed. sets in (X, τ, I) . Then $\left(\bigcup_{\alpha \in \Omega} A_\alpha\right)$ is also $g^{**}I$ - closed.

Proof: Similar to proof of above Theorem since in $*$ - countably additive space $\left[\bigcup_{i=1}^{\infty} A_i\right]^* = \bigcup_{i=1}^{\infty} (A_i)^*$

Theorem 3.12: Let the ideal space (X, τ, I) be $*s$ - finitely additive (resp. $*s$ - countably additive), and let $\{A_\alpha / \alpha \in \Omega\}$ be a locally finite (resp locally countable) family of sets in (X, τ, I) . If each A_α is $g^{**s}I$ - closed then $\bigcup_{\alpha \in \Omega} A_\alpha$ is also $g^{**s}I$ - closed

Proof: Similar to proof of above Theorem, since in $*S$ - finitely additive and $*S$ - countably additive spaces, $\left[\bigcup_{i=1}^n A_i\right]^* = \bigcup_{i=1}^n (A_i)^*$ and $\left[\bigcup_{i=1}^{\infty} A_i\right]^* = \bigcup_{i=1}^{\infty} (A_i)^*$ respectively

Remark 3.13: In general intersection of two $g^{**}I$ - closed sets need not be $g^{**}I$ - closed as seen from the following example.

Example 3.14: Let $X = \{a, b, c, d\}$ $\tau = \{\emptyset, X, \{a, b\}\}$ $I = \{\emptyset\}$, Then $A = \{a, c\}$ and $B = \{a, d\}$ are $g^{**}I$ -closed and $g^{**s}I$ - closed but $A \cap B = \{a\}$ is not - $g^{**}I$ -closed and $g^{**s}I$ - closed

Theorem 3.15: A subset A of an ideal space (X, τ, I) is $g^{**}I$ - open if and only if $F \subset Int^*(A)$ whenever $F \subseteq A$ and F is a g^* - closed subset of X .

Proof: Let A be $g^{**}I$ - open and F be a g^* - closed subset of X contained in A . Then $(X - F)$ is a g^* - open set containing $X - A$ which implies $X - Int^*(A) = cl^*(X - A) \subset X - F$. So $F \subset Int^*(A)$

Conversely, let $F \subset Int^*(A)$ whenever $F \subseteq A$ and F is a g^* - closed subset of X . Let U be a g^* - open and $X - A \subset U$. Then $X - U \subset Int^*(A) = X - cl^*(X - A)$. Therefore $cl^*(X - A) \subseteq U$ which proves $X - A$ is $g^{**}I$ - closed. So A is $g^{**}I$ - open.

Theorem 3.17: A subset A of a $*s$ - finitely additive ideal space (X, τ, I) is $g^{**s}I$ - open if and only if $F \subset Int^{*s}(A)$ whenever $F \subseteq A$ and F is a g^* - closed subset of X .

Proof: similar to the proof of the above theorem since in $*s$ - finitely additive ideal space $Int^{*s}(A) = X - cl^{*s}(X - A)$

Theorem 3.18: For each $x \in (X, \tau, I)$ either $\{x\}$ is g^* - closed or $\{x\}^c$ is $g^{**}I$ - closed in X .

Proof: Suppose $\{x\}$ is not g^* - closed. then $\{x\}^c$ is not g^* - open.

Therefore the only g^* - open set containing $\{x\}^c$ is X and $cl^*(\{x\}^c) \subseteq X$ which proves that $\{x\}^c$ is $g^{**}I$ - closed.

Theorem 3.19: For each $x \in (X, \tau, I)$ either $\{x\}$ is g^* - closed or $\{x\}^c$ is $g^{**s}I$ - closed in X .

Proof: Similar to the above proof.

Theorem 3.20: In an ideal space (X, τ, I) , if U is open and A is $g^{**}I$ - open, then $U \cap A$ is $g^{**}I$ - open.

Proof follows from (5) of remark (3.2) since every open set is $g^{**}I$ - open

Theorem 3.21: In an ideal space (X, τ, I) which is finitely $*s$ - additive, if U is semi open and A is $g^{**s}I$ - open, then $U \cap A$ is $g^{**s}I$ - open.

Proof follows from theorem (3.8) since every semi open set is $g^{**s}I$ - open

Theorem 3.22: If B is a subset of an ideal space (X, τ, I) such that $A \subset B \subset cl^*(A)$ and A is $g^{**}I$ - closed, then B is also $g^{**}I$ - closed in X .

Proof: Let U be g - open and $B \subset U$. Then $A \subset U$ and this implies $cl^*(A) \subset U$. Therefore $cl^*(B) \subset cl^*(cl^*(A)) \subset cl^*(A) \subset U$ which proves B is $g^{**}I$ - closed.

Theorem 3.23: If B is a subset of a finitely $*s$ - additive ideal space (X, τ, I) such that $A \subset B \subset cl^{*s}(A)$ and A is $g^{**s}I$ - closed, then B is also $g^{**s}I$ - closed in X .

Proof is similar to the proof of above theorem.

Theorem 3.24: Let (X, τ, I) be an ideal space and A be a $g^{**}I$ - closed subset of X . Then

- (i) $cl(A^*) \subseteq U$ for all g^* - open set U containing A .
- ii) $cl^*(A) - A$ contains no non empty g^* - closed set.
- (ii) $cl(A^*) - A$ contains no non empty g^* - closed set.
- (iii) $(A^*) - A$ contains no non empty g^* - closed set.

Proof:

(i) Let U be g^* - open set containing A . Then $cl A^* = A^* \subseteq cl^*(A) \subseteq U$,

(ii) Suppose that there exists a non empty g^* - closed set F such that $F \subset cl^*(A) - A$.

then $A \subseteq X - F$ which is g^* - open. So $cl^*(A) \subseteq X - F$ and this implies $F \subset X - cl^*(A)$. Hence $F \subseteq (X - cl^*(A)) \cap (cl^*(A) - A) = \{\emptyset\}$

(iii) It follows from (ii) since $cl(A^*) = A^* \subseteq cl^*(A)$

(iv) It follows from (ii) since $A^* - A \subseteq cl^*(A) - A$.

Theorem 3.25: Let (X, τ, I) be an ideal space and A a $g^{**s}I$ - closed subset of X . Then

(i) $scl(A^{*s}) \subseteq U$ for all g^* - open set U containing A .

(ii) $cl^{*s}(A) - A$ contains no non empty g^* - closed set.

(iii) $scl(A^{*s}) - A$ contains no non empty g^* - closed set.

(iv) $(A^{*s}) - A$ contains no non empty g^* - closed set.

Proof is similar to the proof of above theorem.

Theorem 3.26: Let (X, τ, I) be an ideal space and $A \subseteq Y \subseteq X$. If A is $g^{**}I$ - closed in $(Y, \tau/Y, I/Y)$, Y is open and τ^* - closed in X then A is $g^{**}I$ - closed in X .

Proof: Let U be g^* - open set in X containing A . Then $A^*(I, \tau) \cap Y = A^*(I_Y, \tau_Y) \subseteq U \cap Y$. Then $Y \subseteq U \cup (X - A^*(I, \tau))$. Since Y is τ^* - closed, $A^* \subseteq Y^* \subseteq Y \subseteq U \cup (X - A^*(I, \tau))$.

Therefore $A^* \subseteq U$ and this implies $cl^*(A) = A \cup A^* \subseteq U$

Theorem 3.27: Let (X, τ, I) be an ideal space and $A \subseteq Y \subseteq X$. If A is $g^{**s}I$ - closed in $(Y, \tau/Y, I/Y)$, Y is open and τ^{*s} - closed in X then A is $g^{**s}I$ - closed in X .

Proof is similar to the proof of above theorem.

Theorem 3.28: Let (X, τ, I) be an ideal space and $A \subseteq X$. If A is $g^{**}I$ - closed then $A \cup (X - A^*)$ is $g^{**}I$ - closed.

Proof: Let U be g^* - open and $A \cup (X - A^*) \subset U$. Then $X - U \subset X - [A \cup (X - A^*)] = A^* - A$. Since A is $g^{**}I$ - closed, $A^* - A$ contains no non empty g^* - closed set. Therefore $X - U = \phi$ which implies $X = U$. Thus X is the only g^* - open set containing $A \cup (X - A^*)$ which proves $A \cup (X - A^*)$ is $g^{**}I$ - closed.

Theorem 3.29: Let (X, τ, I) be an ideal space and $A \subseteq X$. If A is $g^{**s}I$ - closed then $A \cup (X - A^{*s})$ is $g^{**s}I$ - closed.

Proof is similar to the proof of above theorem.

Theorem 3.30: Let (X, τ, I) be an ideal space. If every g^* - open set is g^* - closed, then every subset of X is $g^{**}I$ - closed.

Proof: Let $A \subset U$ and U a g^* - open set in X . Then $cl^*(A) \subset cl^*(U) = U$ which proves A is $g^{**}I$ - closed

Theorem 3.31: Let (X, τ, I) be an ideal space. If every g^* - open set is $g^{**s}I$ - closed, then every subset of X is $g^{**s}I$ - closed.

Proof is similar to the proof of above theorem.

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