

EFFECTS OF VISCOSITY VARIATION DUE TO ADDITIVES ON SQUEEZE FILM
CHARACTERISTICS OF LONG PARTIAL JOURNAL BEARING:
COUPLE STRESS FLUID MODEL

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ABSTRACT

In this paper, the effects of viscosity variation on the squeeze film characteristics of long partial journal bearing lubricated with couple stress fluid are investigated. Using the Stokes constitutive equations, the modified Reynolds equation applicable to couple stress fluids is derived by considering viscosity variation along the film thickness. The film pressure distribution is solved and other squeeze film characteristics such as the load carrying capacity and time – height relationship are obtained. The results indicate that lubrication by couple stress fluids will increase the load carrying capacity and lengthen the squeezing time. It is also found that viscosity variation parameter tends to decrease the load carrying capacity.

Keywords: *couple stress fluids; viscosity variation; squeeze films; load carrying capacity; squeezing time.*

1. INTRODUCTION

Hydrodynamic squeeze films play an important role in engineering practice. The application of squeeze film action is commonly seen in gyroscopes, gears, aircraft engines, automotive engines and the mechanics of synovial joints in human beings and animals. The squeeze film behavior arises from the phenomenon of two lubricated surfaces approaching each other with a normal velocity. Because of the viscous lubricant present between the two surfaces, it takes a certain time for these to come into contact. Since the viscous lubricant has a resistance to extrusion, a pressure is built up during that interval, and the lubricant film then supports the load. If the applied load acts for a short enough time, it may happen that the two lubricated surfaces will not meet at all. Therefore, the analysis of squeeze film action focuses on the load carrying capacity and rate of approach.

Conventionally, the prediction of squeeze film motion [5, 10, 11, 7, 6, 16] assumes that the lubricant behaves as a Newtonian viscous fluid. But the Newtonian fluid constitutive approximation is not a satisfactory engineering approach to many lubrication problems. Hence, the use of non-Newtonian fluids as lubricants has gained importance in modern industry. The experimental results show that the addition of a small amount of a long-chain polymer solution to a Newtonian fluid gives the most desirable lubricant [18, 20, 21]. A number of microcontinuum theories have been developed to explain the peculiar behavior of the fluids containing a structure such as polymeric fluids [8, 9]. The microcontinuum theory derived by Stokes [4] is the simplest generalization of the classical theory of fluids, which allows for the polar effects such as the presence of couple stresses and body couples. A number of studies have applied to the Stokes couple stress fluid model to analyze various hydrodynamic lubrication problems. Ramanaiah [13] analyzed the squeeze film behavior between finite plates of various shapes lubricated with couple stress fluids. N.M. Bujurke & Jayaraman [15] predicted the characteristics in a squeeze film configuration with reference to synovial joints. J.R. Lin [22, 23, 24] applied the couple stress fluid model to predict the pure squeeze film characteristics of a long partial bearing, short bearing and finite bearing. Also J.R. Lin, et al., [25] studied the pure squeeze film behavior of long journal bearings with couple stress fluids under dynamic loading. Naduvinamani, et al. [26, 27] examined the effects of couple stresses on the static and dynamic behavior of the squeeze film lubrication of narrow porous journal bearings.

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Recently Naduvinamani [28] have analyzed the effect of surface roughness on the couplestress squeeze film between a sphere and a flat plate.

Earlier theories were based on the assumption that the viscosity μ was constant, although it is a function of both pressure and temperature. The variation in viscosity with temperature is important in many practical applications, where lubricants are required to function over a wide range of temperature [2]. There is no fundamental mathematical relationship which will accurately predict the variation in the viscosity of oil with temperature. The formulae proposed for defining the viscosity – temperature relationship are purely empirical, and for accurate calculations the lubrication engineer requires experimental data. Generally, it is assumed that thermal equilibrium exists and that the viscosity varies with the temperature according to a given law. For practical application of the law, the temperature at each point should be known and this would require a complete thermal calculation. However, a viscosity-temperature relationship can be replaced by a viscosity-film thickness relationship as it has been verified experimentally that the highest temperature occurs in zones where the film thickness is lowest [3].

It is further assumed that the new material constant η for couple stress fluids also vary similar to the μ [12]. P. Sinha et al. [14] studied the effect of viscosity variation in journal bearings lubricated with micropolar fluids. D.F. Wilcock and O. Pinkus [17] examined the influence of turbulence and variable viscosity on the dynamic properties of journal bearings by assuming that the viscosity is to vary exponentially with the temperature. G. Jayachandra Reddy et.al, [29] analyzed the effect of viscosity variation on the static performance of a narrow journal bearing operating with couple stress fluids.

The present paper predicts theoretically the effects of viscosity variation on the squeeze film characteristics of a long partial journal bearing lubricated with couple stress fluid.

2. MODIFIED REYNOLDS EQUATION

Fig.1 represents the physical configuration of a long partial squeeze film journal bearing. The shaft of radius R approaches the bearing surface with velocity V . The film thickness h is a function of θ i.e., $h = c - e \cos \theta$, where c is the radial clearance and e is the eccentricity of the journal center. The lubricant in the system is taken to be a Stokes couple stress fluid. According to the Stokes microcontinuum theory, the constitutive equations of an incompressible fluid with couple stress are [4]

$$\rho \frac{DV}{Dt} = -\nabla p + \rho F + \frac{1}{2} \nabla \times (\rho C) + \mu \nabla^2 V - \eta \nabla^4 V \quad (1)$$

$$\nabla \cdot V = 0 \quad (2)$$

where the vectors V , F and C represent the velocity, body force per unit mass, and body couple per unit mass; ρ , the density; p , the pressure; μ is the shear viscosity, and η is a new material constant responsible for the couple stress fluid property.

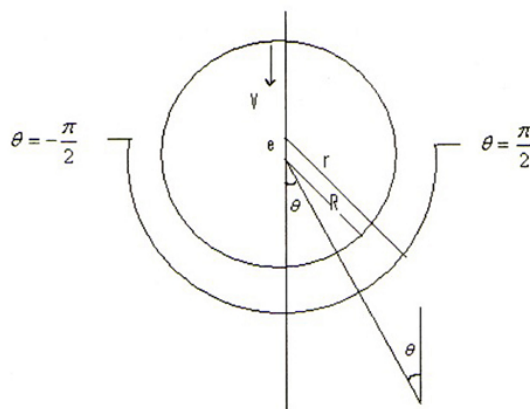


Fig. 1: Long partial journal Bearing Configuration

Under the usual assumptions of hydrodynamic lubrication applicable to thin films, the equations of motion and the equation of continuity in the Cartesian coordinates are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (3)$$

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2} - \eta \frac{\partial^4 u}{\partial y^4} \quad (4)$$

$$\frac{\partial p}{\partial y} = 0 \quad (5)$$

$$\frac{\partial p}{\partial z} = \mu \frac{\partial^2 w}{\partial y^2} - \eta \frac{\partial^4 w}{\partial y^4} \quad (6)$$

where u , v and w denote the velocity components in the x , y and z - directions respectively. The boundary conditions at the bearing surface are

$$\left. \begin{aligned} u(x, 0, z) = v(x, 0, z) = w(x, 0, z) = 0 \\ \frac{\partial^2 u}{\partial y^2} \Big|_{y=0} = \frac{\partial^2 w}{\partial y^2} \Big|_{y=0} = 0 \end{aligned} \right\} \quad (7)$$

and at the journal surface are

$$\left. \begin{aligned} u(x, h, z) = w(x, h, z) = 0 \\ v(x, h, z) = \frac{dh}{dt} \\ \frac{\partial^2 u}{\partial y^2} \Big|_{y=h} = \frac{\partial^2 w}{\partial y^2} \Big|_{y=h} = 0 \end{aligned} \right\} \quad (8)$$

where h is the fluid film thickness. By applying the above boundary conditions the velocity components u and w are solved from the Eq. (4) and Eq. (6) respectively:

$$u = U \frac{y}{h} + \frac{1}{2\mu} \frac{\partial p}{\partial x} \left\{ y(y-h) + 2l^2 \left[1 - \frac{\cosh\left(\frac{2y-h}{2l}\right)}{\cosh\left(\frac{h}{2l}\right)} \right] \right\} \quad (9)$$

$$w = \frac{1}{2\mu} \frac{\partial p}{\partial z} \left\{ y(y-h) + 2l^2 \left[1 - \frac{\cosh\left(\frac{2y-h}{2l}\right)}{\cosh\left(\frac{h}{2l}\right)} \right] \right\} \quad (10)$$

where $l = (\eta / \mu)^{1/2}$

By replacing the velocity components u and w with their expressions and integrating the continuity equation (3) with respect to y and using the boundary conditions (7) and (8), the modified Reynolds equation is:

$$\frac{\partial}{\partial x} \left[f(h, l) \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial z} \left[f(h, l) \frac{\partial p}{\partial z} \right] = 1 - 2\mu \frac{\partial h}{\partial t} \quad (11)$$

$$\text{where } f(h, l) = h^3 - 12l^2 \left[h - 2l \tanh\left(\frac{h}{2l}\right) \right] \quad (12)$$

It is noticed that, for Newtonian fluid, $\eta = 0$ and $l = 0$. As $l \rightarrow 0$, the function $f(h, l)$ defined in (12) approaches h^3 and the modified Reynolds equation (11) reduces to the classical form of the Newtonian lubricant case [1].

Now it is assumed that, the Newtonian viscosity μ is varying along the fluid film thickness h according to [3] as

$$\mu = \mu_1 \left(\frac{h}{h_1} \right)^Q \quad (13)$$

where μ_1 is the inlet viscosity at $h = h_1 = c$. The exponent Q may be determined using the relation:

$$Q = \frac{\log\left(\frac{\mu_1}{\mu_2}\right)}{\log\left(\frac{h_1}{h_2}\right)} \quad (14)$$

where μ_2 is the outlet viscosity with film thickness h_2 .

The parameter Q ($0 \leq Q \leq 1$) depends on the particular lubricant used; for perfect Newtonian fluids $Q = 0$, whereas for perfect gasses $Q = 1$. For mathematical simplicity the couple stress parameter l is assumed to be independent of viscosity variation. This can be done by assuming that η is varying in the same way as μ .

3. LONG BEARING ANALYSIS

In order to simplify the problem and to obtain a closed form solution for the fluid pressure, a long bearing approximation is assumed. Since $\lambda^2 \geq 2$ for long bearing approximation, the axial variations of pressure can be neglected as compared to the circumferential variation. Then the modified Reynolds equation (11) reduces to

$$\frac{d}{dx} \left[\frac{f(h, l)}{\mu} \frac{dp}{dx} \right] = 12 \frac{\partial h}{\partial t} \quad (15)$$

It is noted that, for $l \rightarrow 0$, equation (15) reduces to the classical form of the Newtonian- lubricant case.

The Newtonian viscosity μ is assumed to vary along the fluid film thickness h according to

$$\mu = \mu_1 \left(\frac{h}{c} \right)^Q \quad (16)$$

where μ_1 is the inlet viscosity coefficient and c is the radial clearance of the bearing, then equation (15) reduces to

$$\frac{d}{dx} \left[c^Q f_1(l, h) \frac{dp}{dx} \right] = 12 \mu_1 \frac{\partial h}{\partial t} \quad (17)$$

where

$$f_1(l, h) = h^{3-Q} \left[1 - 12 \left(\frac{l}{h} \right)^2 + 24 \left(\frac{l}{h} \right)^3 \tanh\left(\frac{h}{2l}\right) \right] \quad (18)$$

and $\frac{\partial h}{\partial t} = -c \frac{d\varepsilon}{dt} \cos \theta$

Introducing the non – dimensional variables and parameters:

$$\begin{aligned} \bar{p} &= \frac{pc^2}{\mu_1 R^2 (d\varepsilon/dt)}, \quad \theta = \frac{x}{R}, \\ \bar{h} &= \frac{h}{c} = 1 - \varepsilon \cos \theta, \quad \bar{l} = \frac{l}{c} \end{aligned} \quad (19)$$

The modified Reynolds equation can now be written in a non – dimensional form as:

$$\frac{d}{d\theta} \left[\bar{f}_1(\bar{h}, \bar{l}) \frac{d\bar{p}}{d\theta} \right] = -12 \cos \theta \quad (20)$$

where

$$\bar{f}_1(\bar{h}, \bar{l}) = (\bar{h})^{3-\rho} \left[1 - 12 \left(\frac{\bar{l}}{\bar{h}} \right)^2 + 24 \left(\frac{\bar{l}}{\bar{h}} \right)^3 \tanh \left(\frac{\bar{h}}{2\bar{l}} \right) \right] \quad (21)$$

3.1 Squeeze film pressure

For 180° partial journal bearing the boundary conditions for the pressure are:

$$\begin{aligned} \bar{p} &= 0 \text{ at } \theta = -\frac{\pi}{2} \\ \bar{p} &= 0 \text{ at } \theta = +\frac{\pi}{2} \\ \frac{d\bar{p}}{d\theta} &= 0 \text{ at } \theta = 0 \end{aligned} \quad (22)$$

Integrating the non-dimensional modified Reynolds equation with respect to θ by applying the above boundary conditions, the film pressure is obtained as:

$$\bar{p} = -12 \int_{\theta=-\pi/2}^{\theta=\theta} \frac{\sin \theta}{\bar{f}_1(\bar{h}, \bar{l})} d\theta \quad (23)$$

With the film pressure known, the bearing characteristics such as the load-carrying capacity and the time- height relationship can now be calculated.

3.2 Load-carrying capacity

The load-carrying capacity is evaluated by integrating the film pressure acting on the journal rotor:

$$W = \int_{\theta=-\pi/2}^{\theta=\pi/2} p \cos \theta \cdot R d\theta \quad (24)$$

where W represents the load-carrying capacity per unit length of the bearing generated by the squeeze film pressure. Let the non – dimensional load carrying capacity be:

$$\bar{W} = \frac{Wc^2}{\mu_1 R^3 (d\varepsilon/dt)} \quad (25)$$

As a consequence, equation (22) can be expressed in the non – dimensional form as

$$\bar{W} = 12 \int_{-\pi/2}^{\pi/2} \frac{\sin^2 \theta}{f_1(\bar{h}, \bar{l})} d\theta = g(\varepsilon, \bar{l}) \quad (26)$$

The non-dimensional load carrying capacity \bar{W} in (26) cannot be obtained by direct integration. It can be numerically evaluated by the method of Gaussian Quadrature [19].

3.3 Time-height relationship

For constant load W , the time taken by journal center to move from $\varepsilon = 0$ to $\varepsilon = \varepsilon_1$ is obtained by integrating equation (26) with respect to time. Introducing the non – dimensional response time:

$$\tau = \frac{Wc^2}{\mu_1 R^3} t \quad (27)$$

We have the time-height relationship expressed as:

$$\frac{d\varepsilon}{dt} = \frac{1}{g(\varepsilon, \bar{l})} \quad (28)$$

Equation (28) is a first-order nonlinear differential equation. The initial condition for ε is:

$$\varepsilon = 0 \text{ at } \tau = 0 \quad (29)$$

The above differential equation can be solved using fourth-order Runge-Kutta method [19].

4. RESULTS AND DISCUSSION

In couple stress fluids, the dimensionless parameter, \bar{l} defined by $\bar{l} = l/c$, where $l = \left(\frac{\eta}{\mu}\right)^{1/2}$ characterizes the couple stress property of the fluid and also distinguishes it from the Newtonian fluid. This parameter l has dimensions of length and can be identified with a property which depends on the size of the fluid molecule of a polar additive in a non-polar lubricant. Thus the parameter \bar{l} can be considered as a characterization of the interaction of the fluid with the bearing geometry. Therefore the parameter \bar{l} provides a mechanism which might be helpful in explaining some of the rheological abnormalities that are commonly observed in certain additive containing fluids when the flows are confined to narrow passages. When $\bar{l} \rightarrow 0$, the lubricant becomes Newtonian in character. This situation happens when $\eta \rightarrow 0$, which in turn implies that the additive molecule has vanishing chain length.

In the present analysis, the influence of viscosity variation on the couple stress squeeze film characteristics of a long partial journal bearing is presented. All the squeeze film characteristics are functions of the eccentricity ratio, couple stress parameter and viscosity variation parameter.

4.1. Selection of design parameters

(i) **Eccentricity ratio (ε):** The eccentricity ratio range in practice is 0.4 to 0.6. Minimum eccentricity ratio 0.1 and maximum is 0.4 in steps of 0.1 have been taken for analysis.

(ii) **Couple stress parameter (\bar{l}):** The couple stress fluid is characterized by this non-dimensional parameter \bar{l} . The value of this couple stress parameter depends upon the characteristic material length of the polar suspensions l or the radial clearance c . The values of \bar{l} are taken as 0.0, 0.1, 0.2, and 0.25.

(iii) **Viscosity variation parameter (Q):** It usually lies between 0 and 1 according to the nature of the lubricant. Numerical values of 0.0, 0.25, 0.5, 0.75 and 1.0 are assumed for the Q in order to discuss the effects of viscosity

variation in the present analysis. The limiting case of $Q \rightarrow 0$ represents the corresponding case studied by Lin [34]. As both the couple stress parameter and viscosity variation parameter approach to zero, the problem reduces to the Newtonian case.

4.2 Squeeze film pressure

Fig. 2. shows the non-dimensional pressure \bar{p} as a function of circumferential coordinate θ (in degrees) on the mid-plane $\bar{z}=0$ at the eccentricity ratio $\varepsilon = 0.4$ and viscosity variation parameter $Q=0.1$ for different couple stress parameter. The curve for $\bar{l} = 0$ represents the pressure of a Newtonian lubricant in the squeeze film motion, whereas the curves for $\bar{l} = 0.1, 0.2$ and 0.25 show the results of couple stress fluids. It is observed that the couple stress parameter is dominant. The presence of couple stress offers an increase in squeeze film pressure.

Fig.3. shows dimensionless pressure versus circumferential angle for different values of eccentricity ratio ε . It is observed that the couple stress effects are predominant for higher values of \bar{l} . As the value of couple stress parameter decreases, the fluid film pressure built up tends towards the Newtonian lubricant case. For higher values of \bar{l} , the hydrodynamic fluid film pressure is substantially higher.

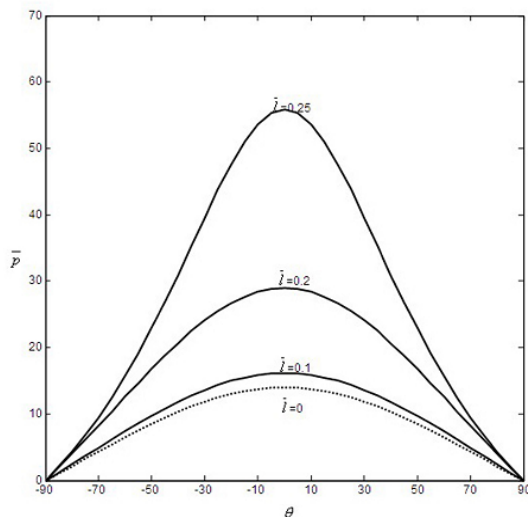


Fig 2: Dimensionless film pressure Vs θ for different \bar{l} with $\varepsilon = 0.4, Q = 0.1$.

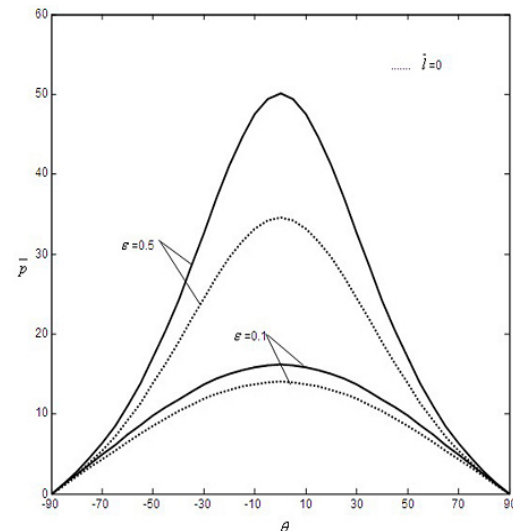


Fig 3: Dimensionless film pressure Vs θ for ε different with $\bar{l} = 0.1, Q = 0.1$.

4.3 Load carrying capacity

Fig. 4 illustrates the dimensionless load carrying capacity \bar{W} versus \bar{l} for different ε . It is observed that the effects of couple stresses produce an increase in the load carrying capacity. Moreover, a larger increment of the load is obtained with increasing value of ε . The variation of the dimensionless load carrying capacity with the couple stress parameter \bar{l} for different viscosity variation parameter Q is depicted in Fig. 5. It is observed that the load carrying capacity increases as \bar{l} increases and decreases as Q increases.

Fig. 6 shows the dimensionless load carrying capacity \bar{W} as a function of eccentricity ratio ε for different values of viscosity variation parameter Q . It is observed that the load carrying capacity increases as ε increases and it decreases as Q increases. Fig. 7 displays the dimensionless load carrying capacity \bar{W} as a function of minimum film thickness ε for different values of \bar{l} , couple stress parameter. Since the couple stress effects results in a higher film pressure, the integrated load carrying capacity is similarly affected. As the additives concentration increases, load carrying capacity increases.

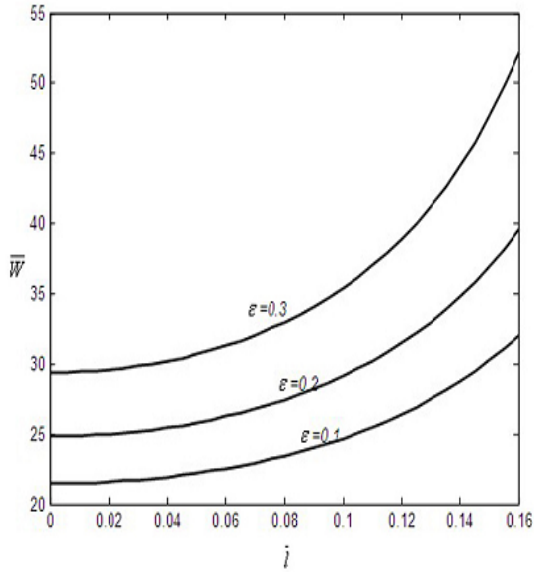


Fig. 4: Dimensionless load Vs i for different ϵ with $Q = 0.1$.

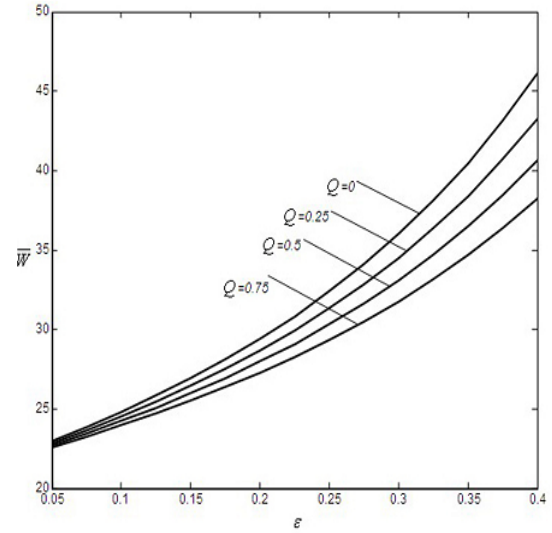


Fig. 6: Dimensionless load Vs ϵ for different q with $i = 0.1$

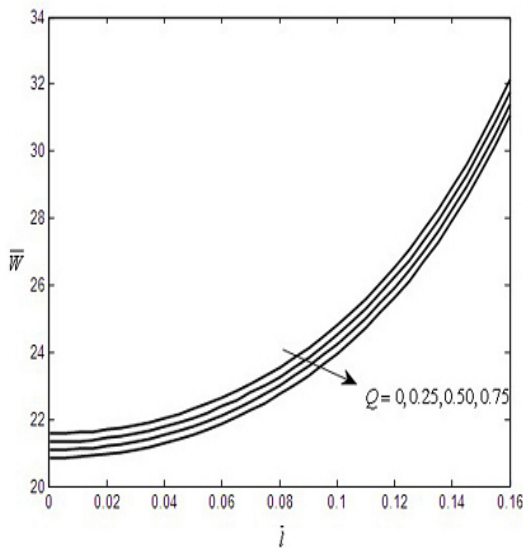


Fig. 5: Dimensionless load Vs i for different Q with $\epsilon = 0.1$

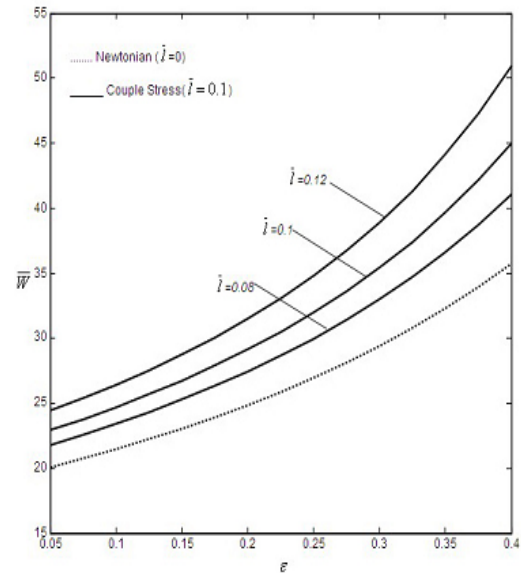


Fig. 7: Dimensionless load Vs ϵ for different i with $Q = 0.1$.

4.4 Squeeze time – eccentricity ratio relationship

The non-dimensional minimum permissible film thickness is an important factor in designing the squeeze film bearings for the sake of safe operation. Fig. 8 shows the dimensionless time versus eccentricity ratio with viscosity variation factor for both Newtonian fluid and couple stress fluid. It is observed that the presence of couple stress provides an increase in response time than Newtonian fluid. But the viscosity variation factor decreases the response time.

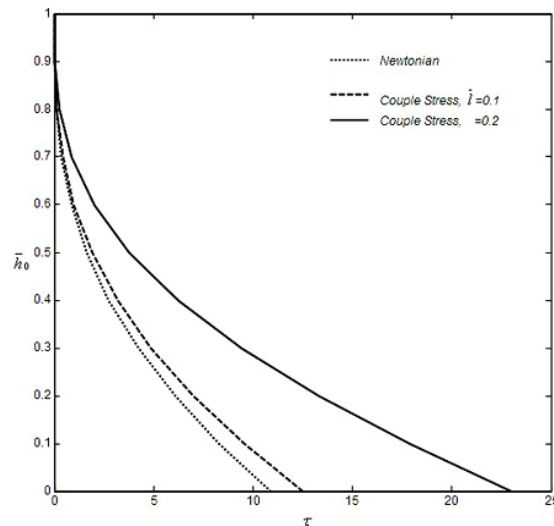


Fig.8. Minimum film thickness Vs τ for different \bar{l} with $Q = 0.1$

5. CONCLUSIONS

On the basis of the Stokes micro-continuum theory, modified Reynolds equation is derived to study the combined effects of the viscosity variation and couple stress parameters. As the value of the couple stress parameter approaches zero, the bearing performance predicted by this analysis approaches the Newtonian case. According to the results obtained, the presence of couple stresses provides an enhancement in the load carrying capacity and in squeeze film time as compared to the Newtonian lubricant case. The bearing lubricated with the couple stress fluids provides longer time to prevent the journal bearing contact and results in a longer bearing life. Increasing the value of viscosity variation parameter signify a decrease in viscosity, which may be a consequence of temperature rise. Hence as temperature increases the load carrying capacity decreases.

Nomenclature

c	Radial clearance
h	Film thickness
l	Couple stress parameter
p	Hydrodynamic pressure
Q	Viscosity variation parameter
R	Radius of the shaft
t	Squeezing time
W	Load carrying capacity
u, v, w	Fluid velocity components in x, y, z directions
x, y, z	Cartesian coordinates
μ	Classical viscosity coefficient
μ_1	Inlet viscosity coefficient
η	Material constant responsible for the couple stress property
λ	Length to diameter ratio
ε	Eccentricity ratio
θ	Circumferential coordinate
τ	Dimensionless response time

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