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INVENTORY REPLENISHMENT DECISION RULE FOR A TWO-ECHELON INVENTORY SYSTEM WITH VARIOUS DEMAND TRENDS

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ABSTRACT

The growing global economy has caused a dramatic shift towards inventory management in recent years. Efficient and effective management of inventory will have a beneficial impact on a company's ability in serving its customers properly and to keep direct and indirect costs low. Too much inventory wastes the capital and increases the risk of obsolete goods. With too little inventory, there is a risk of lost sales, stock outs, and increased direct costs. The objective for a multi echelon inventory model is to coordinate the inventories at the various echelons so as to minimize the total cost associated with the entire multi echelon inventory system. Effective management of inventories at each stage offers a great prospective for increasing system efficiency, customer service level and minimization of total system costs. This paper discusses different models developed for analyzing two-echelon inventory system with linear trend and non-linear trend in the demand. The objective of the paper is to develop two echelon inventory models with time dependent demand with minimization of total inventory relevant cost of the system.

Key words: *Two-Echelon inventory system, Replenishment, Demand, Linear Trend, Non-Linear Trend, Total Relevant Cost.*

INTRODUCTION

Normally inventory models consider the situation of an item having a deterministic demand pattern and not allowing any shortages. The problem faced by the decision-maker is, the selection of the timing and sizes of replenishments so as to minimize the replenishment quantity and associated costs. This problem had been treated by Donaldson (1977), for the case of a positive trend and well-defined termination point (horizon) of the demand pattern. He used calculus considerations to establish a key property of the optimal replenishment pattern. Namely that " the quantity ordered at a replenishment point, i.e. a point at which actual inventory becomes zero, should be the product of the current instantaneous demand rate and the elapsed time since the last replenishment''. Donaldson then used this property to determine for a given demand pattern and horizon length, the best locations in time of a given number of replenishments. The computational effort is not simple. The tabular aids were developed but interpolation is needed. By introducing the cost parameters (fixed set-up cost per replenishment and unit inventory carrying cost), one can then establish the appropriate number of replenishments to use for the given demand pattern and horizon length. Diegel (1962) had used essentially the same approach and developed rather extensive tabular results. E.A. Silver (1997) had developed a simple inventory replenishment decision rule for linear trend in demand for a single echelon inventory model.

In this context, although several multi-echelon inventory models have been developed covering a wide spectrum of variations and assumptions, but the models with different trends in the demand are few. Hence an attempt is made **to develop two echelon inventory models** with time dependent demand. The assumptions and variations incorporate in these models suit the most practical situations of manufacturing or trading organizations. We take the special case of a positive linear trend and non-linear trend, which gives a very simple decision rule. They are illustrated with numerical examples

THE SILVER-MEAL HEURISTIC MODEL

GENERAL CASE

An analytical solution for the problem of determining lot sixe in cases of irregular demand has not been addressed until Donaldson made a beginning in 1977. Even though the Wagner Whitin (1958) method was available, practitioners have been using Wilson's (1929) square root formula only, mainly because of the simplicity. Donaldson observed that the

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analytical solution for the lot sixe problem could be studied easily, provided that the replenishment cycle T, is treated as a decision variable instead of the replenishment quantity, as done in the classical model. This approach has another advantage, as it facilitates a classification of several inventory items into groups based on the cycle time. It is a convenient method for material requirements planning (MRP). The demerit of this method is that, it solves for all points except at t_0. Sufficient conditions in case of linear trend pattern have been obtained by Donaldson using the properties of the Hessian matrix. A number of numerical illustrations have been developed by Donaldson and in each case it is observed that the duration of the successive replenishment decreases. An initial development in the results obtained by Donaldson is due to Henry (1979) which is a generalization of the class of demand functions which increases with time. For such functions, the optimal replenishment strategy gives reorder times which are successively close to each other. It is also true that the quantities of inventory taken at the reorder times increases from one order to the next.

In an independent study, Silver (1979) reexamined the problem posed by Donaldson and has extended to the case of linear trend in the demand, assuming that shortages are not allowed. The idea of Silver is to determine the optimal size of only the first replenishment in a forward looking fashion. It is determined in such a way to minimize cost per unit time over the first replenishment duration only. It is observed by Silver that, only a few iterations are required before convergence. Once T is determined, the size of the next replenishment interval is obtained by updating the demand function T and repeating the iterative method. This gives a sequence of T values which decrease as the number of replenishment increases.

ASSUMPTIONS

At time t = 0, the inventory is at zero level and a replenishment is necessary at that moment and the time between the replenishment is the decision variable.

Shortages are not allowed.

The replenishment is instantaneous

NOTATIONS USED

$f_D(t)$ be the Demand rate at time 't' at the distributor.
$f_R(t)$ be the Demand rate at time 't' at the retailer.
T be the time between replenishments
Q_D be the ordering quantity by the distributor.
Q_R be the ordering quantity by the retailer.
C_D be the fixed cost of replenishment per lot at the distributor.
C_R be the fixed cost of replenishment per lot at the retailer.
C_{hD} be the cost of holding inventory per unit per period at the distributor.
C_{hR} be the cost of holding inventory per unit per period at the retailer.

Let TRCUT (T) be the Total relevant cost per unit time.

The equation for total relevant cost may be written as given by Silver et al (1998)

$$\text{TRCUT}(T) = \frac{C_D}{T} + \frac{C_{hD}}{T} * \int_0^T t.\,f_D(t)\,dt + \frac{C_R}{T} + \frac{C_{hR}}{T} * \int_0^T t.\,f_R(t)\,dt \quad (1)$$

$$\text{also } Q_D = \int_0^T f_D(t)\,dt \quad (2)$$

$$\text{and } Q_R = \int_0^T f_R(t)\,dt \quad (3)$$

The two echelon inventory system can be written as give by Silver et al (1998)

$Q_D = n * Q_R$ (Where $n > 1$)

Similarly, the two echelon inventory system for this model can be written as

$$f_D(t) = n * f_R(t) \quad (4)$$

Substituting Eq (4) and Eq (1), we get

$$\text{TRCUT (T)} = \frac{C_D}{T} + \frac{C_{hD}}{T} * \int_0^T t.\,n.\,f_R\,(t)\,dt + \frac{C_R}{T} + \frac{C_{hR}}{T} * \int_0^T t.\,f_R\,(t)\,dt \quad (5)$$

a) Special case of a linear trend in demand

Here we have f (t) = a + bt (6)

Where a is the demand rate at time 0 and b is the trend value

Substituting (6) in (5), we get

$$\text{TRCUT (T)} = \frac{C_D}{T} + \frac{C_{hD}}{T} * \int_0^T t.\,n.\,(a+bt)\,dt + \frac{C_R}{T} + \frac{C_{hR}}{T} * \int_0^T t.\,(a+bt)\,dt \quad (7)$$

We wish to find the optimal value of T, so as to minimize the TRCUT (T)

A necessary condition is the $\frac{d\,[TRCUT(T)]}{dt} = 0$

$$\int_0^T t.\,f_D\,(t)\,dt = \int_0^T t.\,n.\,f_R\,(t)\,dt$$

$$= n\,.\int_0^T t.\,(a+bt)\,dt = n\,.\int_0^T (a.t + b.t^2)\,dt$$

$$= n\,[\int_0^T (a.t)\,dt + \int_0^T (b.t^2)\,dt\,]$$

$$= n\,[a\left[\frac{t^2}{2}\right]_0^T + b\left[\frac{t^3}{3}\right]_0^T] = n\,[a\,.\frac{T^2}{2} + b\,.\frac{T^3}{3}\,]$$

$$\int_0^T t.\,f_R\,(t)\,dt = \int_0^T t.\,(a+bt)\,dt = \int_0^T (a.t + b.t^2)\,dt$$

$$= a\left[\frac{t^2}{2}\right]_0^T + b\left[\frac{t^3}{3}\right]_0^T = [a\,.\frac{T^2}{2} + b\,.\frac{T^3}{3}\,]$$

Equation (7) can be written as

$$\text{TRCUT (T)} = \frac{C_D}{T} + \frac{C_{hD}}{T} * [\frac{n.a}{2}T^2 + \frac{n.b}{3}T^3] + \frac{C_R}{T} + \frac{C_{hR}}{T} * [\frac{a}{2}T^2 + \frac{b}{3}T^3]$$

$$= \frac{C_D}{T} + C_{hD} * [\frac{n.a}{2}T + \frac{n.b}{3}T^2] + \frac{C_R}{T} + C_{hR} * [\frac{a}{2}T + \frac{b}{3}T^2]$$

$$= \frac{C_D}{T} + (\frac{a}{2}T + \frac{b}{3}T^2) * (n.\,C_{hD} + C_{hR}) + \frac{C_R}{T} \quad (8)$$

Differentiating (8) with respect to ‘T’ , we get

$$[\text{TRCUT (T)}]^1 = \frac{-C_D}{T^2} + (\frac{a}{2} + \frac{2}{3}bT) * (n\,.\,C_{hD} + C_{hR}) - \frac{C_R}{T^2}$$

$$= \frac{-1}{T^2}(C_D + C_R) + (\frac{a}{2} + \frac{2}{3}bT) * (n\,.\,C_{hD} + C_{hR})$$

Equating $[\text{TRCUT (T)}]^1 = 0$, we get

$$\frac{-1}{T^2}(C_D + C_R) + (\frac{a}{2} + \frac{2}{3}bT) * (n\,.\,C_{hD} + C_{hR}) = 0$$

This equation yields

$$\left(\frac{a}{2} + \frac{2}{3}bT\right) T^2 = \frac{C_D + C_R}{n.C_{hD} + C_{hR}} \tag{9}$$

Thus (9) can be written as

$$\left(\frac{a}{2} + \frac{2}{3}bT\right) T^2 = M \tag{10}$$

i.e. a cubic equation in T.

for the case of positive (b≥ 0) Equation (10) can be effectively solved by iteration viz letting $T^{(k)}$ be the value of T obtained on the K^{th} iteration.

Equation (10) can be written as

$$T^{(k)} = \sqrt{\frac{M}{\frac{a}{2} + \frac{2}{3}bT}} \tag{11}$$

And we can initially set $T^{(0)} = 0$. Normally only a few iterations are required for convergence.

In addition, for the case of b > 0, the second derivative at Equation (7) shows that the solution of Equation (10) is a minimizing value of T.

Where a = 0, Equation (11) should not be used, but Equation (10) gives a closed form solution for T.

$$T = (3M / 2b)^{1/3}$$

The case of a negative (b < 0), trend is more complicated to analyze. In particular, the minimizing value of T can occur at the boundary value (-- a / b) where the demand rate goes to zero.

SIMULATION MODEL

A Computer programme is developed in "C – Language". It takes input values as a, b, C_{od}, C_{or}, C_{hr}, C_{hd} and n. The output values are n, M, T, Cost, Q_R and Q_D.

DEVELOPMENT OF SOFT-WARE PROGRAMME

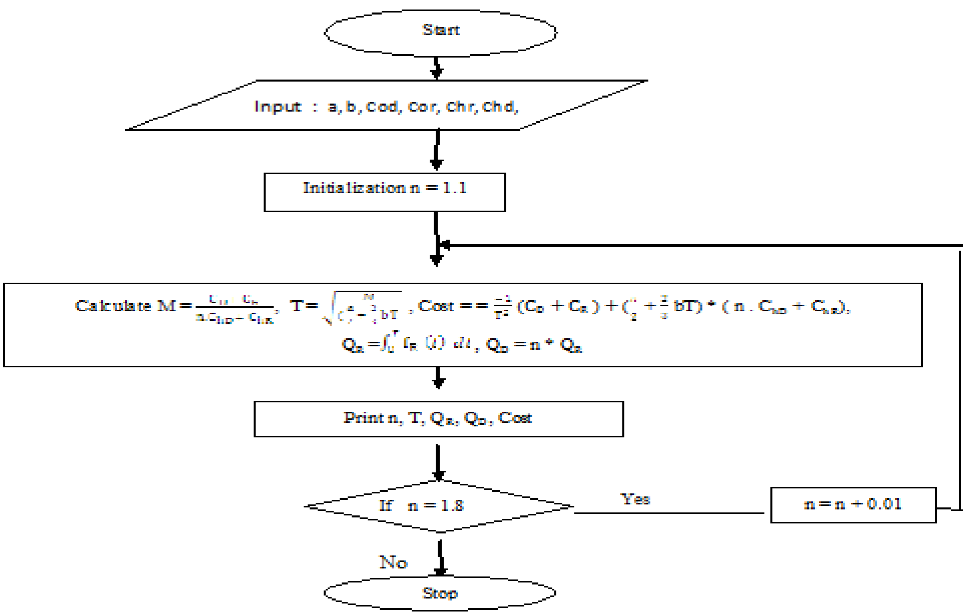

Fig. 5.1 Flow-diagram for Linear trend in the demand

b) Case of a non-linear trend in demand

Here we have $f(t) = a + bt + ct^2$ (6)

Where a is the demand rate at time 0 , b and c are constants

Substituting (6) in (5), we get

$$\text{TRCUT}(T) = \frac{C_D}{T} + \frac{C_{hD}}{T} * \int_0^T t\,.\,n.\,(a + bt + c\,t^2)\,dt + \frac{C_R}{T} + \frac{C_{hR}}{T} * \int_0^T t\,(a + bt + c\,t^2)\,dt \tag{7}$$

We wish to find the optimal value of T, so as to minimize the TRCUT (T)

A necessary condition is the $\frac{d\,[TRCUT(T)]}{dt} = 0$

$$\int_0^T t.f_D(t)\,dt = \int_0^T t.n.f_R(t)\,dt$$
$$= n.\int_0^T t\,(a + bt + c\,t^2)\,dt = n\int_0^T (a.t + b.t^2 + c.t^3)\,dt.$$
$$= n\left[\int_0^T (a.t)\,dt + \int_0^T (b.t^2)\,dt + \int_0^T (c.t^3)\,dt\right]$$
$$= n\left[a\left[\frac{t^2}{2}\right]_0^T + b\left[\frac{t^3}{3}\right]_0^T + c\left[\frac{t^4}{4}\right]_0^T\right] = n\left[a.\frac{T^2}{2} + b.\frac{T^3}{3} + C.\frac{T^4}{4}\right]$$

$$\int_0^T t.f_R(t)\,dt = \int_0^T t.(a + bt + c\,t^2)\,dt = \int_0^T (a.t + b.t^2 + c\,t^3)\,dt$$
$$= a\left[\frac{t^2}{2}\right]_0^T + b\left[\frac{t^3}{3}\right]_0^T + c\left[\frac{t^4}{4}\right]_0^T = \left[a.\frac{T^2}{2} + b.\frac{T^3}{3} + c.\frac{T^4}{4}\right]$$

Equation (7) can be written as

$$\text{TRCUT}(T) = \frac{C_D}{T} + \frac{C_{hD}}{T} * \left[\frac{n.a}{2}T^2 + \frac{n.b}{3}T^3 + \frac{n.c}{4}T^4\right] + \frac{C_R}{T} + \frac{C_{hR}}{T} * \left[a.\frac{T^2}{2} + b.\frac{T^3}{3} + c.\frac{T^4}{4}\right]$$
$$= \frac{C_D}{T} + C_{hD} * \left[\frac{n.a}{2}T + \frac{n.b}{3}T^2 + \frac{n.c}{4}T^3\right] + \frac{C_R}{T} + C_{hR} * \left[\frac{a}{2}T + \frac{b}{3}T^2 + \frac{c}{4}T^3\right]$$
$$= \frac{C_D}{T} + \left(\frac{a}{2}T + \frac{b}{3}T^2 + \frac{c}{4}T^3\right) * (n.\,C_{hD} + C_{hR}) + \frac{C_R}{T} \tag{8}$$

Differentiating (8) with respect to 'T', we get

$$[\text{TRCUT}(T)]^1 = \frac{-C_D}{T^2} + \left(\frac{a}{2} + \frac{2}{3}bT + \frac{3}{4}cT^2\right) * (n.\,C_{hD} + C_{hR}) - \frac{C_R}{T^2}$$
$$= \frac{-1}{T^2}(C_D + C_R) + \left(\frac{a}{2} + \frac{2}{3}bT + \frac{3}{4}cT^2\right) * (n.\,C_{hD} + C_{hR})$$

Equating $[\text{TRCUT}(T)]^1 = 0$, we get

$$\frac{-1}{T^2}(C_D + C_R) + \left(\frac{a}{2} + \frac{2}{3}bT + \frac{3}{4}cT^2\right) * (n.\,C_{hD} + C_{hR}) = 0$$

This equation yields

$$\left(\frac{a}{2}+\frac{2}{3}bT+\frac{3}{4}cT^2\right)T^2=\frac{C_D+C_R}{n.C_{hD}+C_{hR}} \tag{9}$$

Thus (9) can be written as

$$\left(\frac{a}{2}+\frac{2}{3}bT+\frac{3}{4}cT^2\right)T^2=M \tag{10}$$

i.e. a four degree equation in T.

for the case of positive $(b \geq 0)$ Equation (10) can be effectively solved by iteration viz letting $T^{(k)}$ be the value of T obtained on the K^{th} iteration.

Equation (10) can be written as

$$T^{(k)}=\sqrt{\frac{M}{\left(\frac{a}{2}+\frac{2}{3}bT+\frac{3}{4}cT^2\right)}} \tag{11}$$

SIMULATION MODEL

A Computer programme is developed in "C – Language". It takes input values as a, b, c, C_{od}, C_{or}, C_{hr}, C_{hd} and n. The output values are n, M, T, Cost, Q_R and Q_D.

DEVELOPMENT OF SOFT-WARE PROGRAMME

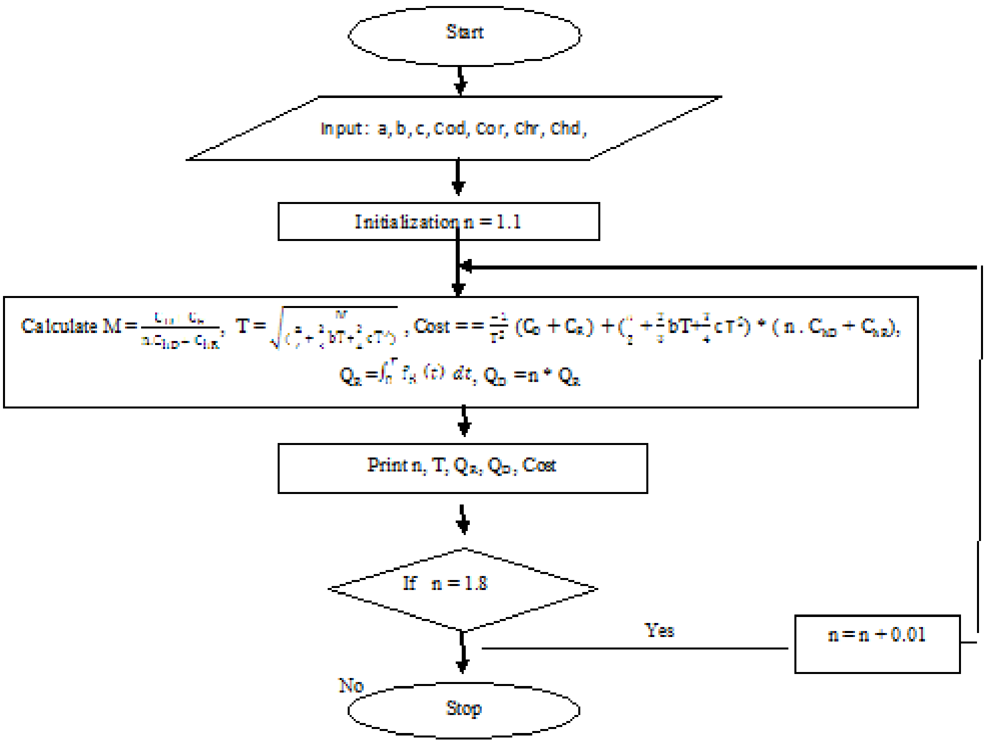

Fig. 5.2. Flow-diagram for non-Linear trend in the demand

NUMERICAL EXAMPLES

a) For Liner Trend in the Demand

Input values are

a = 50, 00,000; b = 29301; C_D = 13000; C_R = 12000; C_{hD} = 0.5; C_{hR} = 0.25

The output values are

S. No	Value of n	Value of T	Value of Q_R	Value of Q_D	Total relevant cost (Rs.)
1	1.0	0.12	577285.21	577285.21	433110.33
2	2.0	0.09	447174.58	894349.16	559114.63
3	3.0	0.08	377936.6	1133809.8	661535.47
4	4.0	0.07	333311.65	1333246.61	750097.64
5	5.0	0.06	301493.60	1507468.61	829253.84
6	6.0	0.06	277335.08	1664010.51	901485.47
7	7.0	0.05	258185.88	1807301.14	968343.47
8	8.0	0.05	242524.14	1940193.15	1030874.06
9	9.0	0.05	229405.46	2064649.14	1089822.39
10	10.0	0.04	218208.59	2182085.95	1145741.58
11	11.0	0.04	208505.93	2293565.19	1199055.53
12	12.0	0.04	199992.19	2399906.3	1250097.65
13	13.0	0.04	192442.86	2501757.17	1299135.76
14	14.0	0.04	185688.61	2599640.49	1346388.86
15	15.0	0.04	179599.1	2693985.06	1392038.75

Time between replenishments = 0.04 * 365 = 14.6 days

b) For Non-Liner Trend in the Demand

Input values are

a = 50, 00,000; b = 22527; c = - 13220; C_D = 10; C_R = 1000; C_{hD} = 2.0; C_{hR} = 3.0

The output values are

S. No	Value of n	Value of T	Value of QR	Value of QD	Total relevant cost (Rs.)
1	1.10	0.01	44072.49	48479.74	229175.45
2	1.15	0.01	43654.73	50202.93	231368.53
3	1.20	0.01	43248.62	51898.34	233541.03
4	1.25	0.01	42853.64	53567.05	234693.48
5	1.30	0.01	42469.29	55210.06	237826.48
6	1.35	0.01	42095.09	56828.37	239940.50

Time between replenishments = 0.01 * 365 = 3.65 days

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