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TRANSIENT FREE CONVECTIVE FLOW PAST A MOVING VERTICAL CYLINDER WITH COMBINED EFFECTS OF HEAT AND MASS TRANSFER

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ABSTRACT

An analytical study of unsteady one-dimensional natural convective flow over an infinite moving vertical cylinder with combined effects of heat and mass transfer is performed. The exact solutions of dimensionless unsteady linear governing partial differential equations are obtained in terms of Bessel functions by usual Laplace transform technique. Graphical results for the velocity profile and skin friction are illustrated and discussed for various physical parametric values viz. thermal Grashof number, mass Grashof number, Prandtl number, Schmidt number and time.

Keywords: *Vertical moving cylinder, Laplace transform Heat and mass transfer.*

NOMENCLATURE

C'	species concentration
C	dimensionless species concentration
D	mass diffusion coefficient
Gr	thermal Grashof number
Gc	mass Grashof number
g	acceleration due to gravity
J_0	Bessel function of first kind and order zero
J_1	Bessel function of first kind and order one
K_0	Modified Bessel function of second kind and order zero
K_1	Modified Bessel function of second kind and order one
Nu	Nusselt number
p	Laplace transform variable of time
$\Pr$	Prandtl number
r	radial coordinate measured from the axis of the cylinder
R	dimensionless radial coordinate
Sc	Schmidt number
Sh	Sherwood number
t'	time
t	dimensionless time
T'	temperature
T	dimensionless temperature
u	x-component of velocity
U	dimensionless velocity
V	dummy real variable used in inverse transform of loop integral
Y_0	Bessel function of second kind and order zero
Y_1	Bessel function of second kind and order one
α	Thermal diffusivity of fluid

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ν kinematic viscosity
β volumetric coefficient of thermal expansion
β^* volumetric coefficient of expansion with concentration

1. INTRODUCTION

Many transport processes exist in nature and in industrial applications in which the transfer of heat and mass occurs simultaneously as a result of combined buoyancy effects of thermal diffusion and mass diffusion. The problem of free convection flow with combined effects of heat and mass transfer over moving vertical cylinders finds applications in the field of engineering and geophysics such as hot rolling, hot extrusion, nuclear reactor cooling system and underground energy system. Recently the problems of free convective flows driven by temperature and concentration difference have attracted the attention of many researches. When both temperature and concentration differences occur simultaneously, the free convective flow can become quite complex.

Sparrow and Gregg [15] first studied the heat transfer from vertical cylinders. Yang [16] made a study of unsteady laminar free convection on vertical plates and cylinders to establish necessary and sufficient conditions under which similarity solutions are possible. Goldstein and Briggs [11] presented an analysis of the transient free convective flow past vertical flat plate and circular cylinder to a surrounding initially quiescent fluid by employing Laplace transform technique. Bottemanne [1] gave the experimental results of pure and simultaneous heat and mass transfer by free convection about a vertical cylinder placed in still air. Chen and Yuh [3] presented an analysis on steady heat and mass transfer processes near cylinders under the combined buoyancy force effects of thermal and species diffusion.

Gorla [12] presented a numerical solution of combined forced and free convection in the boundary layer flow of a micropolar fluid on a continuous moving vertical cylinder. Ganesan and Rani [7] presented a numerical solution for the transient natural convection flow over a vertical cylinder under the combined buoyancy effects of heat and mass transfer by employing an implicit finite-difference scheme of Crank-Nicolson type. Thereafter, Ganesan and Loganathan [8, 9, 10] presented a numerical analysis of unsteady natural convective flow past a semi-infinite vertical cylinder with heat and mass transfer under different physical situations. Elgazery and Hassan [6] presented a numerical study of radiation effect on MHD transient mixed-convection flow over a moving vertical cylinder with constant heat flux through a porous medium. Reddy and Reddy [14] presented a numerical analysis to study the effects of radiation and mass transfer on unsteady MHD free convection flow of an incompressible viscous fluid past a moving vertical cylinder by finite-difference scheme of Crank-Nicolson type. Recently, Deka and Paul [12] presented analytic study on transient free convective flow about an infinite moving vertical circular cylinder with constant temperature. Again, Deka and Paul [11] studied transient free convective flow past a stationary vertical cylinder with heat and mass transfer by Laplace transform technique.

The flow past a moving vertical cylinder plays an important role in many industrial applications. No analytic work on transient free convection along moving vertical cylinder under the combined buoyancy effects of thermal and mass diffusion has been reported. This has motivated the present investigation. Here, we have presented an analytical solution of unsteady natural convection flow past an infinite moving vertical cylinder with heat and mass transfer. To solve the governing boundary layer equations, first they are converted into non-dimensional form and then solved in terms of Bessel functions by Laplace transform technique.

2. MATHEMATICAL ANALYSIS

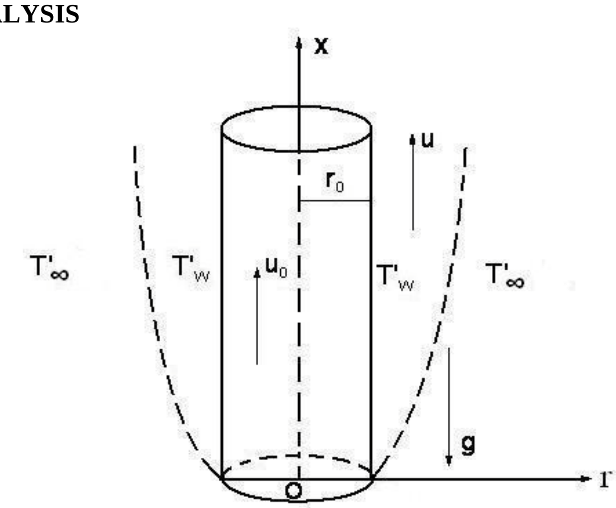

Figure 1: The physical model and co-ordinate system

Consider unsteady, laminar and incompressible viscous flow past an infinite moving vertical cylinder of radius r_0 with constant temperature and mass diffusion. The physical model of the problem is shown in figure 1. Here the x-axis is taken vertically upward along the axis of the cylinder and the redial co-ordinate r is taken normal to the cylinder. Initially it assumed that the cylinder and fluid are at same temperature T'_∞ and also the same concentration C'_∞. It is also assumed that at $t' \geq 0$, the cylinder starts to move in the vertical direction with constant velocity u_0 and temperature and the concentration near the cylinder raised to T'_w and C'_w respectively. Under these assumptions the governing boundary layer equations for momentum, energy and concentration for free convective flow with Boussinesq's approximation are as follows:

$$\frac{\partial u}{\partial t'} = \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + g\beta\left(T' - T'_\infty\right) + g\beta^*\left(C' - C'_\infty\right) \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \alpha\left(\frac{\partial^2 T'}{\partial r^2} + \frac{1}{r}\frac{\partial T'}{\partial r}\right) \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\left(\frac{\partial^2 C'}{\partial r^2} + \frac{1}{r}\frac{\partial C'}{\partial r}\right) \tag{3}$$

with initial and boundary conditions,

$$\left.\begin{array}{lllll} t' \leq 0: & u = 0, & T' = T'_\infty, & C = C_\infty & \forall\ r \\ t' > 0: & u = u_0, & T' = T'_w & C' = C'_w & at \quad r = r_0 \\ & u \to 0, & T' \to T'_\infty, & C' \to C'_\infty & as \quad r \to \infty \end{array}\right\} \tag{4}$$

Introducing the non dimensional variables

$$\left.\begin{array}{l} R = \dfrac{r}{r_0}, \quad U = \dfrac{u}{u_0} \quad t = \dfrac{t'\nu}{r_0^2}, \quad T = \dfrac{T' - T'_\infty}{T'_w - T'_\infty}, \quad C = \dfrac{C' - C'_\infty}{C'_w - C'_\infty} \\ \Pr = \dfrac{\nu}{\alpha}, \quad Gr = g\beta\, r_0^2\, \dfrac{T'_w - T'_\infty}{u_0\nu}, \quad \mathrm{Sc} = \dfrac{\nu}{D}, \quad \mathrm{Gc} = g\beta^* r_0^2\, \dfrac{C'_w - C'_\infty}{u_0\nu} \end{array}\right\} \tag{5}$$

the governing equations (1), (2) and (3) reduces to

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial R^2} + \frac{1}{R}\frac{\partial U}{\partial R} + GrT + Gc\,C \tag{6}$$

$$\Pr\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial R^2} + \frac{1}{R}\frac{\partial T}{\partial R} \tag{7}$$

$$Sc\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial R^2} + \frac{1}{R}\frac{\partial C}{\partial R} \tag{8}$$

with the initial and boundary conditions as:

$$\left.\begin{array}{lllll} t \leq 0: & U = 0, & T = 0, & C = 0 & \forall \quad R \\ t > 0: & U = 1, & T = 1 & C = 1 & at \quad R = 1 \\ & U \to 0, & T \to 0 & C \to 0 & as \quad R \to \infty \end{array}\right\} \tag{9}$$

2. SOLUTION TECHNIQUE

To solve the governing unsteady non dimensional boundary layer equations (6) – (8) subject to initial and boundary conditions (9), we apply Laplace transform technique.

Laplace transformations of equations (6) – (8) with initial conditions of (9) give

$$\frac{d^2\overline{U}}{dR^2}+\frac{1}{R}\frac{d\overline{U}}{dR}-p\overline{U}+Gr\overline{T}+Gc\overline{C}=0 \tag{10}$$

$$\frac{d^2\overline{T}}{dR^2}+\frac{1}{R}\frac{d\overline{T}}{dR}-p\Pr\overline{T}=0 \tag{11}$$

$$\frac{d^2\overline{C}}{dR^2}+\frac{1}{R}\frac{d\overline{C}}{dR}-pSc\overline{C}=0 \tag{12}$$

where p is the of Laplace transform parameter defined by L{$f(t)$} = $F(p)$, L being the Laplace operator and $\overline{U}$, $\overline{T}$ and $\overline{C}$ are Laplace transform of U , T and C respectively.

Solutions of the equations (11) and (12) subject to boundary conditions (9) give

$$\overline{T}=\frac{K_0\left(R\sqrt{p\Pr}\right)}{pK_0\left(\sqrt{p\Pr}\right)} \tag{13}$$

$$\overline{C}=\frac{K_0\left(R\sqrt{pSc}\right)}{pK_0\left(\sqrt{pSc}\right)} \tag{14}$$

Using the equations (13) and (14) with boundary conditions (9), we obtain the solution of the equation (10) as:

$$\overline{U}=\frac{K_0\left(R\sqrt{p}\right)}{pK_0\left(\sqrt{p}\right)}+\frac{(Gr+Gc)}{2p\sqrt{p}}\left\{R\frac{K_1\left(R\sqrt{p}\right)}{K_0\left(\sqrt{p}\right)}-\frac{K_1\left(\sqrt{p}\right)K_0\left(R\sqrt{p}\right)}{K_0^2\left(\sqrt{p}\right)}\right\}\qquad (\text{for } \Pr=Sc=1) \tag{15}$$

and

$$\overline{U}=\frac{K_0\left(R\sqrt{p}\right)}{pK_0\left(\sqrt{p}\right)}+\frac{Gr}{p^2(1-\Pr)}\left\{\frac{K_0\left(R\sqrt{p\Pr}\right)}{K_0\left(\sqrt{p\Pr}\right)}-\frac{K_0\left(R\sqrt{p}\right)}{K_0\left(\sqrt{p}\right)}\right\}+\frac{Gc}{p^2(1-Sc)}\left\{\frac{K_0\left(R\sqrt{pSc}\right)}{K_0\left(\sqrt{pSc}\right)}-\frac{K_0\left(R\sqrt{p}\right)}{K_0\left(\sqrt{p}\right)}\right\}\quad (\text{for } \Pr\neq 1,\ Sc\neq 1) \tag{16}$$

Now, using the theorem of inverse Laplace transform in equation (15), (16), (13) and (14) [following Deka and Paul [4, 5] we have

$$U=1+\frac{2}{\pi}\int_0^\infty e^{-V^2t}\Gamma(R,V)\frac{dV}{V}+\frac{Gr+Gc}{\pi}\int_0^\infty\left(1-e^{-V^2t}\right)\left[R\frac{J_1(RV)Y_0(V)-Y_1(RV)J_0(V)}{J_0^2(V)+Y_0^2(V)}+\frac{\{Y_1(V)J_0(RV)+Y_0(RV)J_1(V)\}\{J_0^2(V)-Y_0^2(V)\}}{\{J_0^2(V)+Y_0^2(V)\}^2}-\frac{2J_0(V)Y_0(V)\{J_1(V)J_0(RV)-Y_1(V)Y_0(RV)\}}{\{J_0^2(V)+Y_0^2(V)\}^2}\right]\frac{dV}{V^2}$$
$$(\text{for } \Pr=1,\ Sc=1) \tag{17}$$

and

$$U=1+\frac{2}{\pi}\int_0^\infty e^{-V^2t}\Gamma(R,V)\frac{dV}{V}+\frac{2Gr\Pr}{(\Pr-1)\pi}\int_0^\infty\left(1-e^{-\frac{V^2}{\Pr}t}\right)\left\{\Gamma\left(R,\frac{V}{\sqrt{\Pr}}\right)-\Gamma(R,V)\right\}\frac{dV}{V^3}+\frac{2Gc\,Sc}{(Sc-1)\pi}\int_0^\infty\left(1-e^{-\frac{V^2}{Sc}t}\right)\left\{\Gamma\left(R,\frac{V}{\sqrt{Sc}}\right)-\Gamma(R,V)\right\}\frac{dV}{V^3}\qquad (\text{for } \Pr\neq 1,\ \mathrm{Sc}\neq 1) \tag{18}$$

$$T = 1 + \frac{2}{\pi}\int_0^\infty \frac{e^{-\frac{V^2}{\Pr}t}}{V}\Gamma(R,V)\,dV \tag{19}$$

$$C = 1 + \frac{2}{\pi}\int_0^\infty e^{-\frac{V^2}{Sc}t}\Gamma(R,V)\frac{dV}{V} \tag{20}$$

Where $\Gamma(R,V) = \frac{J_0(RV)Y_0(V) - Y_0(RV)J_0(V)}{J_0^2(V) + Y_0^2(V)}$ (21)

3.1 Skin friction

Non-dimensional Skin friction $\tau = -\left.\frac{\partial U}{\partial R}\right]_{R=1}$ obtained from equation (17) and (18) for $\Pr = \mathrm{Sc} = 1$ and $\Pr \neq 1,\ Sc \neq 1$ respectively as:

$$\tau = \frac{2}{\pi}\int_0^\infty e^{-V^2 t}\Gamma_1(V)\,dV + \frac{Gr+Gc}{\pi}\int_0^\infty \left(1 - e^{-V^2 t}\right)\left[-\frac{2\{J_1(V))Y_0(V) - Y_1(V)J_0(V)\} + V\{J_0(V)Y_2(V) - Y_0(V)J_2(V)\}}{2\{J_0^2(V) + Y_0^2(V)\}} + 2V\frac{J_1(V)Y_1(V)\{J_0^2(V) - Y_0^2(V)\} - Y_0(V)J_0(V)\{J_1^2(V) - Y_1^2(V)\}}{\{J_0^2(V) + Y_0^2(V)\}^2}\right]\frac{dV}{V^2} \tag{22}$$

and

$$\tau = \frac{2}{\pi}\int_0^\infty e^{-V^2 t}\Gamma_1(V)dV + \frac{2Gr\Pr}{(\Pr-1)\pi}\int_0^\infty\left(1 - e^{-\frac{V^2}{\Pr}t}\right)\left\{\Gamma_1\left(\frac{V}{\sqrt{\Pr}}\right)\Big/\sqrt{\Pr} - \Gamma_1(V)\right\}\frac{dV}{V^2} + \frac{2Gc\,Sc}{(Sc-1)\pi}\int_0^\infty\left(1 - e^{-\frac{V^2}{Sc}t}\right)\left\{\Gamma_1\left(\frac{V}{\sqrt{Sc}}\right)\Big/\sqrt{Sc} - \Gamma_1(V)\right\}\frac{dV}{V^2} \tag{23}$$

where $\Gamma_1(V) = \frac{J_1(V)Y_0(V) - Y_1(V)J_0(V)}{J_0^2(V) + Y_0^2(V)}$ (24)

3. RESULTS AND DISCUSSIONS

In order to understand the effects of different physical parameters in the problem, velocity profiles and skin-friction have been discussed by assigning numerical values to various parameters, namely thermal Grashof number, mass Grashof number, Prandtl number, Schmidt number and time. Since water and air are the most commonly occurring fluids in nature, we basically restricted our observations for Pr=0.71 (air) and Pr=7 (water).

Velocity profiles represented by figure 2 exhibits the effects of Pr and Sc at Gr=Gc=5 and t=0.5. It is observed that velocity decreases with increasing values of Prandtl number and Schmidt number. Figure 3 exhibits the effects of Gr and Gc at Pr=0.71, Sc=0.6, t=0.5 and figure 4 shows the effect of t at Gr=5, Gc=10, pr=0.71 and Sc=0.6 on velocity profiles. It is observed from these figures that velocity increases with increase in thermal Grashof number or mass Grashof number or time.

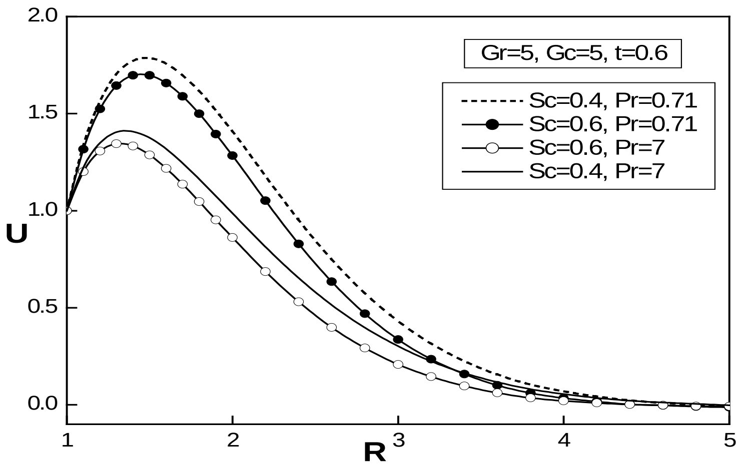

Figure 2: Effects of Pr and Sc on velocity profiles for Gr=Gc =5, t=0.6

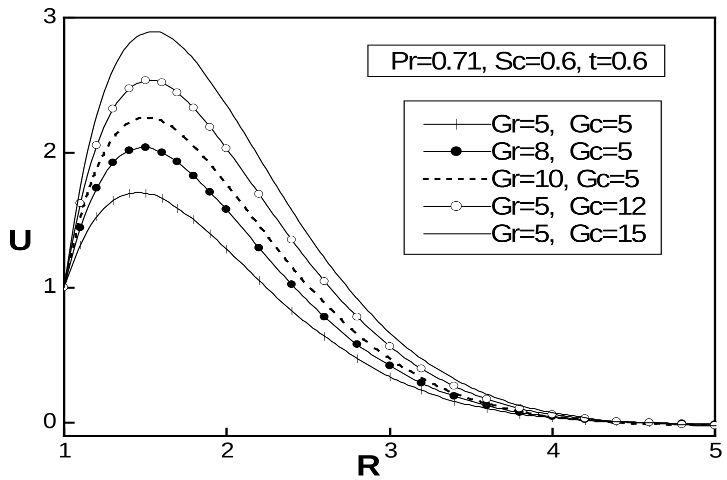

Figure 3: Effects of Gr and Gc on velocity profiles at Pr=0.71, Sc=0.6, t=0.6

Figure 5 and Figure 6 respectively reveals the effects of Pr, Sc and Gr, Gc on skin friction. It is observed from these figures that the skin friction increases with increase in Prandtl number and Schmidt number however it decreases with increase in thermal Grashof number or mass Grashof number.

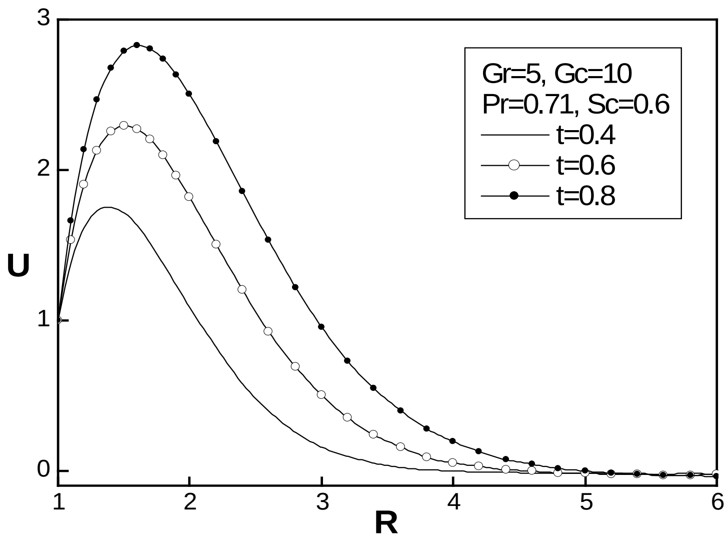

Figure 4: Effect of t on velocity profiles at Gr=5, Gc =10, Pr=0.71, Sc=0.6

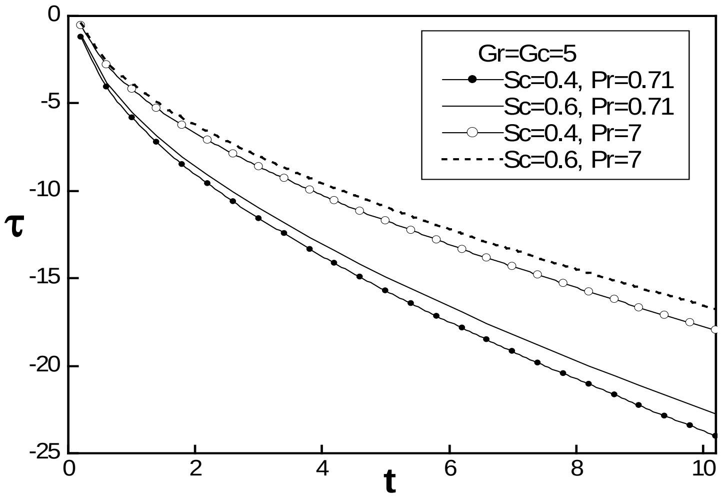

Figure 5: Effects of Pr and Sc on skin friction at Gr=Gc =5

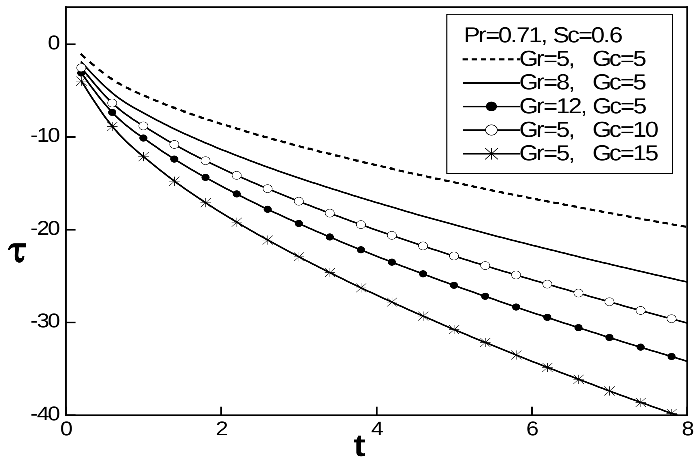

Figure 6: Effects of Gr and Gc on skin friction a Pr=0.71, Sc=0.6

4. CONCLUSIONS

An analysis is performed to study the effects of combined heat and mass transfer to the flow past an infinite isothermal vertical cylinder moving with uniform velocity. The exact solutions of the dimensionless governing equations are obtained by the usual Laplace transformation technique in terms of Bessel functions. The effects of different physical parameters like thermal Grashof number, mass Grashof number, Prandtl number and Schmidt number are studied graphically. The conclusions of the study are as follows:

- Velocity increases with an increase in thermal Grashof number or mass Grashof number.
- Velocity decreases with an increase in Prandtl number or Schmidt number.
- Skin friction decreases with increase in thermal Grashof number or mass Grashof number but increases with increase in Prandtl number or Schmidt number.

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