International Journal of Mathematical Archive-3(7), 2012, 2815-2818
Available online through www.ijma.info ISSN 2229 – 5046

g*s-connectedness and g*s-locally closed sets in topological spaces

K. Anitha*
Department of Mathematics, Sri Subramanya college of Engineering and Technology
Palani-624 615, Dindugal District, Tamilnadu state, India
&
A. Pushpalatha
Department of Mathematics, Government Arts College, Udumalpet-642 126, Tirupur District, Tamilnadu state, India

(Received on: 07-07-12; Revised & Accepted on: 30-07-12)

ABSTRACT

*A. Puspalatha and K. Anitha [9] introduced properties of g*s-closed sets in topological space. In this paper, we introduced g*s-connectedness, g*s-locally closed sets, g*s*-locally closed sets and g*s**-locally closed sets.*

Keywords: *g*s-connectedness, g*s-locally closed sets, g*s* -locally closed sets and g*s**-locally closed sets.*

AMS Mathematics Classification (2000): *51A05.*

1. Introduction

The study of generalization of closed sets has been found to ensure some new separation axioms which have been very useful in the study of certain objects of digital topology. In recent years many generalizations of closed sets have been developed by various authors.

Ganster and Reilly [5] introduced locally closed sets topological spaces, and studies three different notions of generalized continuity namely lc-continuity, lc-irresoluteness and sub lc-continuity. According to them, a subset of (x,τ) is locally closed in x if it is the intersection of an open subset of x and closed subset of x. Also they gave the decomposition that a function between two topological spaces is continuous if and only if it sub-lc continuous and nearby continuous. In 1996,

H. Maki, P. Sundaram and K. Balachandran [7] introduced the concept of generalized locally closed sets and obtained different notions of generalized continuities.

In this paper, we introduce the concept of g*s-connectedness, g*s-locally closed sets, g*s*-locally closed sets g*s**-locally closed sets and study their basic properties in topological spaces.

2. Preliminaries

Definition: 2.1 A subset A of a topological space (x,τ) is called a
(a) generalized closed (briefly g-closed) [10] if $cl(A)\sqsubseteq U$ whenever $A\sqsubseteq U$ and U is open in X.
(b) generalized semi closed (briefly gs-closed) [1] if $cl(A)\sqsubseteq U$ whenever $A\sqsubseteq U$ and U is open in X.
(c) Semi-generalised closed (briefly sg-closed) [2] $A\sqsubseteq U$ and U is semi-open in x. Every semi-closed set is sg-closed.
(d) Strongly generalized closed [10] if $cl(A)\sqsubseteq U$ whenever $A\sqsubseteq U$ and U is g-open in X
(e) g*s-closed set [9] if $scl(A)\sqsubseteq U$ whenever $A\sqsubseteq U$, U is gs-open.

Definition: 2.2 [10] A subset s of X is called a
(a) Locally closed if $S=P\cap Q$ where P is open and Q is closed in X.
(b) Generalised locally closed if $S= P\cap Q$ where P is g-open and Q is g-closed in X.
(c) gs-locally closed if $S=P\cap Q$ where P is gs-open and Q is gs-closed in x.
(d) sg-locally closed if $S=P\cap Q$ where P is sg-open and Q is sg-closed.

Definition: 2.3 A map $f:x\rightarrow y$ from a topological space x into a topological space y is called g*s-continuous [9] if the inverse image of every closed set in y is g*s-closed in x.

Corresponding author: *K. Anitha*, Department of Mathematics, Sri Subramanya college of Engineering and Technology Palani-624 615, Dindugal District, Tamilnadu state, India*

Definition: 2.4 [8] A topological space x is called a
(a) T_L-space if every g*s-closed set of X is closed in X.
(b) T_A-space if every gs-closed set of X is g*s-closed in X.

3. g*s-connectedness

In this section, we introduce a new class of topological space g*s-connected spaces and study some of their properties.

Definition: 3.1 A topological space X is called a g*s-connected if X cannot be written as a disjoint union of two nonempty g*s-open sets. A subset of X is g*s-connected if it is g*s-connected as subspace of X.

Theorem: 3.2 For a topological space X. The following are equivalent.
(a) X is g*s-connected
(b) The only subsets of X which are both g*s-open and g*s-closed are the empty set ∅ and X.
(c) Each g*s-continuous map of X into a discrete space Y with atleast two points is a constant map.

Proof: (a)⇒(b): Let U be g*s-open and g*s-closed subset of X. Then X-U is both g*s-open and g*s-closed. Since X is the disjoint union of the g*s-open sets U and X-U, one of these must be empty (ie) U=∅ (or) U=X.

(b)⇒(a): Suppose that X=A∪B where A and B are disjoint nonempty g*s-open subsets of X. Then A is both g*s-open and g*s-closed. By assumption, A=∅. Therefore X is g*s-connected.

(b)⇒(c): Let f:X→Y be a g*s-continuous map. Then X is covered by g*s-open and g*s-closed covering {f^{-1}(y): y∈Y}.

Suppose that there are two points in Y such that f^{-1}(y) = ∅. By assumption f^{-1}(y) = ∅ (or) X for each y∈Y. If f^{-1}(y)= ∅ for all y∈Y then f fails to be a map. Then there exists only one point y∈Y such that f^{-1}(y) = ∅ and hence f^{-1}(y) =X which shows that f is a constant map.

(c)⇒(b): Let U be both g*s-open and g*s-closed in X. Suppose that U≠ ∅. Define f:x→y by f(U)={y} and f(x-U)={w} for some distinct points y and w in Y then f is a g*s-continuous map. By assumption f is constant. There y= w and so U=x.

Theorem: 3.3 Every g*s-connected space is connected.

Proof: Let X be a g*s-connected space. If possible, let X be not connected Then X can be written as X=A∪B where A and B are disjoint nonempty open sets in X. Since every open set is g*s-open, X = A∪B where A and B are disjoint nonempty and g*s-open sets in X. This contradicts the fact that X is g*s-connected. Therefore X is connected.

The converse of the above theorem need not be true as such from the following example.

Example: 3.4 Let X= {a, b, c} with topology τ= { ∅, X} clearly (X,τ) is a connected space. But (X,τ) is not g*s-connected, because X can be written as X= {a} ∪{b, c},where {a} and {b, c} are g*s-open sets in X.

Theorem: 3.5 If X is a T_L-space and connected, then X is g*s-connected.

Proof: Let X be T_L-space and connected. Assume that X can be written in the form X= A∪B where A and B are nonempty disjoint and g*s-opensets in X. Since X is T_L-space, every g*s-open set is open and so X= A∪B where A and B are disjoint nonempty and open sets in X. This contradicts the fact that X is connected. Therefore X is g*s-connected.

Theorem: 3.6 If f:X→Y is a g*s-continuous surjection and X is g*s-connected, then Y is connected.

Proof: Suppose that Y is not connected. Let Y= A∪B where A and B are disjoint nonempty opensets in Y. Since f is g*s-continuous and onto, X= ff_{-1}(y)(A) ∪ ff_{-1}(B) Where f^{-1}(A) and f^{-1}(B) are disjoint nonempty g*s-open in X. This contradicts the fact that X is g*s-connected. Hence Y is connected.

Theorem: 3.7 If f:X→Y is g*s-continuous map from a connected space X into a topological space Y, then Y is g*s-connected.

Proof: If possible, let Y be not g*s-connected. Then Y can be written as Y= A∪B where A and B are disjoint nonempty g*s-open sets in Y. Since f is g*s-continuous, f^{-1}(A) and f^{-1}(B) are open sets in X. Also X=f^{-1}(y) = f^{-1}(A∪B) = f^{-1}(A) U f^{-1}(B). This contradicts the fact that X is connected. Therefore Y is g*s-connected.

4. g*s , g*s* and g*s** - locally closed sets in topological spaces

In the notion of a locally closed set in a topological space was introduced by Kuratowski and Sierpinski [6]. According to Bourbaki[3] a subset of a topological space X is locally closed in X if it is the intersection of an open set in X and a closed set in X. In this chapter, we introduce the concept of g*s-locally continuous functions and study some of their properties.

Definition: 4.1 A subset S of X is called g*s-locally closed set (g*sl-set) if S=A∩B where A is g*s-open in X and B is g*s-closed in X.

g*sL⊏(X) denote the class of all g*s-locally closed sets in X.

Theorem: 4.2 If a subset S of X is locally closed then it is g*s-locally closed but not conversely.

Proof: Let S=P∩ Q where P is open and Q is closed in X. Since every open set is g*s-open and every closed set is g*s-closed, S is locally g*s-closed set in X.

The converse need not be true as seen from the following example.

Example 4.3 Consider the topological space X={a,b,c} with topology τ={∅,X,{a},{c},{a,c}}. Then the sets {a} and {c} are g*s-locally closed but are not locally closed.

Theorem: 4.4 If a subset S of X is g*s-locally closed in X, then S is gs-locally closed but not conversely.

Proof: Let S=P∩Q where P is g*s-open and Q is a g*s-closed. Since g*s-open implies gs-open and g*s-closed implies gs-closed, S is gs-locally closed set in X.

Theorem:4.5 If A is g*s-locally closed in X and B is g*s-open in X, then A∩B is g*s-locally closed in X.

Proof: There exist a g*s-open set P and a g*s-closed set Q such that A=P∩Q. Now A∩B= (P∩Q)∩B=(P∩B)∩Q. Since P∩B is g*s-open, A∩B is g*s-locally closed in X.

Definition:4.6 A subset S of a topological space X is called g*s*-locally closed set if S= P∩Q where P is g*s-open in X and Q is called in X.

Definition:4.7 A subset S of a topological space X is called g*s**-locally closed set if S= P∩Q where P is open in X and Q is g*s-closed in X.

Theorem:4.8 If A is g*s*-locally closed set in X and B is g*s-open (or) (g*s-closed), then A∩B is g*s*-locally closed set in X.

Proof: Since A is g*s*-locally closed set, there exist a g*s-open set P and a closed set Q such that A= P∩Q. Now A∩B=(P∩Q)∩B=(P∩B)∩Q. Since P∩Q is g*s-open and Q is closed, A∩B is g*s*-locally closed set. In the case of B being a closed set, we have A∩B= (P∩Q)∩B=P∩ (Q∪B). Since P is g*s-open and Q∩B is closed, A∩B is a g*s*-locally closed set.

Theorem:4.9 A subset A of a topological space X is g*s*-locally closed set if and only if there exists a g*s-open set P such that A= P∩cl(A).

Proof: Assume that A is a g*s*-locally closed set. There exist a g*s-open set P and a closed set Q such that A= P∩Q. Since A⊑Q and Q is closed, A⊑cl(A) ⊑Q. Then A⊑P and A⊑cl(A), and hence A⊑P∩cl(A). To prove the reverse inclusion let x∈ P∩cl(A). Then x∈ P∩cl(A). Then x∈P and x∈ cl(A)∩Q and so x∈P ∩Q=A. Hence P∩cl(A)⊑A. Therefore A= P∩cl(A). Conversely, assume that there exist a g*s-open set P such that A= P∩cl(A). Now P is g*s-open set and cl(A) is closed. Therefore A is g*s*-closed set.

Theorem 4.10 If A and B are g*s*-locally closed sets in a topological space X then A∩B is g*s*-locally closed in X.

Proof: From the assumption there exist g*s-open sets P and Q such that A=P ∩ cl(A) and B=Q ∩ cl(B).Then A∩B=(P∩Q)∩cl(A)∩cl(B)). Since P∩Q is g*s-open set and cl(A)∩cl(B) is closed, A∩B is g*s*-locally closed.

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Source of support: Nil, Conflict of interest: None Declared