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ON 4-PRODUCT CORDIAL GRAPHS

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ABSTRACT

Let f be a map from V(G) to{0,1,...k-1}where k is an integer, $2 \le k \le |V(G)|$.

For each edge uv assign the label f(u)f(v)(mod k). f is called a k- Product cordial labeling if
$|v_f(i) - v_f(j)| \le 1$ *and* $|e_f(i) - e_f(j)| \le 1$*,i,j* $\in$ *{0,1,..k-1},*

where $v_f(x)$ *and* $e_f(x)$ *denote the number of vertices and edges respectively labelled with x (x=0,1,2,3...k-1). We investigate the 4-Product cordial labeling behaviour of some standard graphs.*

Keywords: *Complete bipartite graph, Star, Wheel.*

1. INTRODUTION

The graphs considered here are finite, undirected and simple. The vertex set and edge set of a graph *G* are denoted by *V*(G) and *E*(G) respectively. Let join of two graphs G_1 and G_2 is a graph G_1+G_2 with $V(G_1+G_2)= V(G_1)\cup V(G_2)$ and $E(G_1+G_2)= E(G_1) \cup E(G2) \cup \{uv : u\in V(G_1) and\ v \in V(G_2)\}$.The graph obtained subdividing each edge of a graph G by a new vertex is denoted by *S(G)*.Product cordial and EP-cordial behaviour of graphs were studied extensively in[1] and[2]. The notion of k-Product cordial labeling of graphs has been introduced in [3].In this paper we investigate the 4- Product cordial labeling behaviour of Subdivision star $s(K_{1,n})$, wheel $W_n=C_n+K_1$, K_2+mK_1, $K_{2,n}$ etc. Terms not defined here are used in the sense of Harary[4].

2. K-PRODUCT CORDIAL LABELLING

Definition 2.1: Let *f* be a map from *V* (G) to {0,1….*k*-1} where *k* is an integer, $1 \le k \le |V(G)|$. For each edge uv, assign the label *f*(u)*f*(v) (mod k). *f* is called a *k*- Product cordial labeling of G if

$$|v_f(i) - v_f(j)| \le 1 \text{ and } |e_f(i) - e_f(j)| \le 1 \text{ , i,j } \in \{0,1,..k-1\}$$

where v_f(x) and e_f(x) respectively denote the number of vertices and edges respectively labelled with x(x=0,1,2…*k*-1).

3. ON STANDARD GRAPHS

Theorem 3.1: $K_{2,n}$ is 4-Product cordial iff n≡ 0,3(mod 4).

Proof: Let $V(K_{2,n})$={u, v, u_i; $1 \le i \le n$ } and $E(K_{2,n})$={uv, uu_i ,vv_i ; $1 \le i \le n$ }

Clearly f(u)≠*f*(v)≠0

Case (i): n≡ 0(mod 4).

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Let n=4t.Here $\left|V(K_{2,n})\right|$ =4t+2 and $\left|E(K_{2,n})\right|$ =8t. Define *f*(u)=1 ; *f*(v)=3

$f(u_i)=0$, $1\le i\le t$

$f(u_{t+i})=1$, $1\le i\le t$

$f(u_{2t+i})=2$, $1\le i\le t$

$f(u_{3t+i})=3$, $1\le i\le t$. Here $e_f(0)$=2t, $e_f(1)$=2t,$e_f(2)$=2t and $e_f(3)$=2t.

Case (ii) : n≡ 1(mod 4).

If possible let there be a 4-Product cordial labeling *f*. Let n=4t+1.Sothat $\left|V(K_{2,n})\right|$ =4t+3and $\left|E(K_{2,n})\right|$ =8t+2.

Clearly $v_f(0)$=t and $f(u)\neq 0$ and $f(v)\neq 0$

Subcase (i): *f*(u) = *f*(v) =1 .

Then $e_f(1)$ = 2t-2. Sothat $e_f(2)$ = 2t+2, $e_f(2)-e_f(1)$ =4, an impossibility.

Subcase (ii): *f*(u)=1 ; *f*(v)=2 .

Then $e_f(1)$ = t , $e_f(2)$ =3t+1, $e_f(2)-e_f(1)$ =t+1≥2, an impossibility.

Subcase (iii): *f*(u)=1 ; *f*(v)=3 .

Then $e_f(1)$ = 2t , $e_f(2)$ =2t+2, $e_f(2)-e_f(1)$ =2, an impossibility.

Subcase (iv): *f*(u)= *f*(v)=2.

Then $e_f(1)$ = 0 , $e_f(2)$ =4t+4. Sothat $e_f(2)-e_f(1)$ =4t+4, an impossibility.

Subcase (v): *f*(u)=2 ; *f*(v)=3.

Then $e_f(1)$ = t, $e_f(2)$ =2t+1, $e_f(2)-e_f(1)$ =t+1, an impossibility.

Subc ase (vi): *f*(u)= *f*(v)=3.

Then $e_f(1)$ =2t-2 , $e_f(2)$ =2t+2.Here $e_f(2)-e_f(1)$ =4, an impossibility. Thus there can not exists a 4-Product cordial labeling *f*.

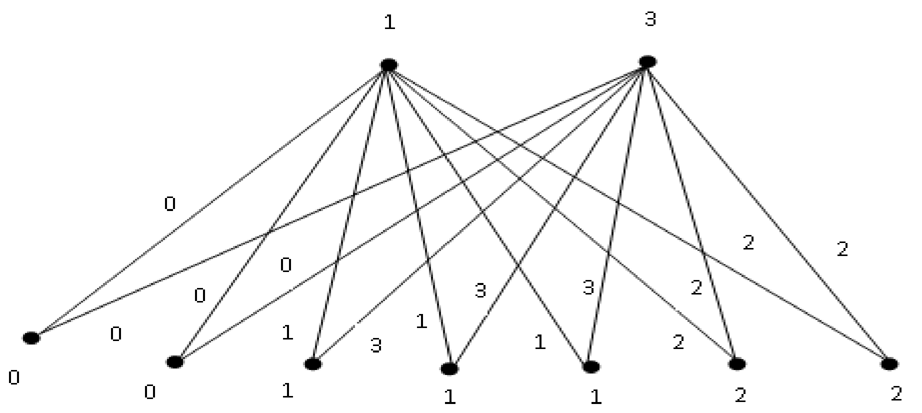
Case (iii): n≡ 2(mod 4).

Let n=4t+2. Here $\left|V(K_{2,n})\right|$ =4t+4 and $\left|E(K_{2,n})\right|$ =8t+4.either *f*(u) nor *f*(v) is 0.Therefore $v_f(0)$ =t+1, $e_f(0)$ =2t+2, a contradiction, since the size of $K_{2,n}$ is 8t+4.

Case (iv): n≡ 3 (mod 4).

Let n=4t+3.Sothat $\left|V(K_{2,n})\right|$ =4t+5, $\left|E(K_{2,n})\right|$ =8t+6. Define *f*(u)=1,*f*(v)=3 label the vertices u_i,1≤i≤n-3 as in case (i). Then label the vertices u_{n-2},u_{n-1},u_n by 2,0,1 respectively to get a 4-Product cordial labeling.

Illustration3.2: 4-Product cordial labels' of $K_{2,7}$ is



Theorem 3.3: $S(K_{1,n})$ is 4- Product cordial.

Proof: Let $V(K_{1,n}) = \{u, u_i : 1 \le i \le n\}$ and $E(K_{1,n}) = \{uu_i : 1 \le i \le n\}$

Let the edge uu_i be subdivided by the vertex v_i .

Case (i): $n \equiv 0 \pmod 4$.

Let n=4t.

$f(u_i) = 0$, $0 \le i \le 2t$

$f(u_{2t+i}) = 2$, $1 \le i \le 2t$

$f(v_{2i}) = 3$, $1 \le i \le 2t$

$f(v_{2i+1}) = 1$, $0 \le i \le 2t-1$

Define f (u) =3.Clearly f is a 4-Product cordial labeling since $v_f(0)=v_f(1)=v_f(2)=2t$ and

$v_f(3)=2t+1, e_f(0)=e_f(1)=e_f(2)=e_f(3)=2t$.

Case (ii): $n \equiv 1 \pmod 4$.

Assign the label v_i and u_i ($1 \le i \le n-1$) as in case(i) and then assign1and 0 to v_n and u_n respectively.

Case (iii): $n \equiv 2 \pmod 4$.

Assign the label v_i and u_i($1 \le i \le n-1$) as in case(ii) and then assign3and 2 to v_n and u_n respectively.

Case (iv): $n \equiv 3 \pmod 4$.

Assign the label v_i and u_i ($1 \le i \le n-1$) as in case(iii) and then assign1and 0 to v_n and u_n respectively. Hence $S(K_{1,n})$ is 4-Product cordial.

4. ON JOIN OF GRAPHS

Theorem4.1: K_2+mK_1 is 4-Product cordial iff $m \equiv 0,3 \pmod 4$.

Proof: Let $V(K_2+mK_1) = \{u, v, u_i : 1 \le i \le n\}$ and
$E(K_2+mK_1) = \{uv, uu_i, vu_i : 1 \le i \le n\}$

Case (i): $m \equiv 0 \pmod 4$.

Let m=4t, Define f (u) =1 and f (v) =3

$f(u_i) = 0$, $1 \le i \le t$

$f(u_{t+i}) = 1$, $1 \le i \le t$

$f(u_{2t+i}) = 2$, $1 \le i \le t$

$f(u_{3t+i}) = 3$, $1 \le i \le t$.Here $e_f(0)=2t$, $e_f(1)=2t, e_f(2)=2t$ and $e_f(3)=2t+1$.Therefore f is a 4-Product cordial labeling.

Case (ii): $m \equiv 3 \pmod 4$.

Let m=4t+3, Define f(u)=1 , f(v)=3 label the vertices u_i , $1 \le i \le m-3$ as in case(i).Then label the vertices u_{m-2}, u_{m-1}, u_m by 0 , 2 ,1 respectively, Clearly f is a 4-Product cordial labeling.

Case (iii) : $m \equiv 1 \pmod 4$.

If possible let there be a 4-Product cordial labeling. Let $m=4t+1$, Clearly $f(u) \ne 0, f(v) \ne 0$. Also $v_f(0)=t$
Therefore $e_f(0)=2t$.

Sub case (i): $f(u)=f(v)=1$ .Then $e_f(1)=2t-1, e_f(2)=2t+2, e_f(2)-e_f(1)=3$, an impossibility.

Sub case (ii): $f(u)=1$; $f(v)=3$. Then $e_f(1)=2t$,$e_f(2)=2t+2$, $e_f(2)-e_f(1)=2$, an impossibility.

Sub case (iii): $f(u)= f(v)=3$.Then $e_f(1)=2t-1$,$e_f(2)=2t+2$, $e_f(2)-e_f(1)=3$, an impossibility.

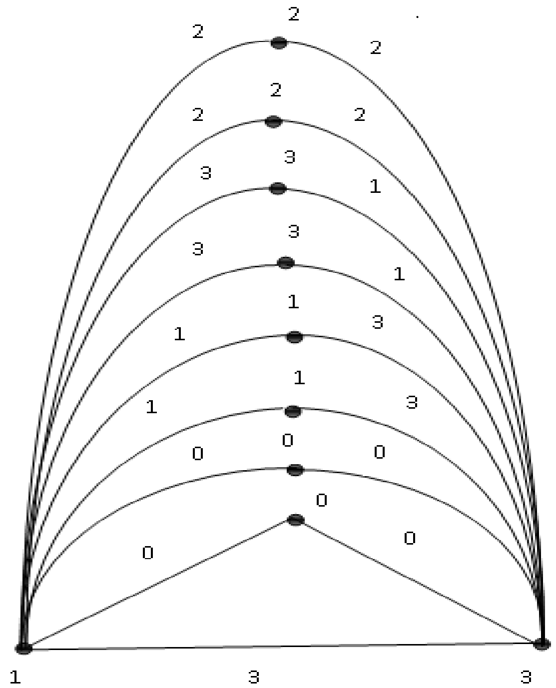
Sub case (iv): $f(u)=2$; $f(v)=3$. Then $e_f(1)=t$,$e_f(2)=3t+1$, $e_f(2)-e_f(1)=2t+1$, an impossibility.

Sub case (v): $f(u)= f(v)=2$.Then $e_f(1)=0$,$e_f(2)=4t+4$, $e_f(2)-e_f(1)=4t+4$, an impossibility.

Sub case (vi): $f(u)=1$; $f(v)=2$. Then $e_f(1)=t$,$e_f(2)=3t+1$, $e_f(2)-e_f(1)=2t+1$, an impossibility.

Case (iv): $m\equiv 2 \pmod 4$.
Let $m=4t+2$,Clearly $f(u)\neq 0$, $f(v)\neq 0$.Clearly $v_f(0)= t$ or $t+1$. Here $e_f(0)= 2t$ or $2t+2$, a contradiction Since the size of K_2+mK_1 is 8t+4.

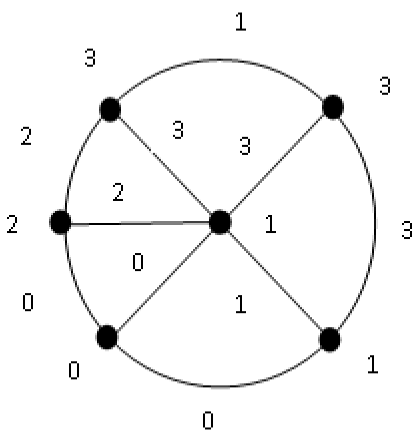
Illustration4.2: 4-Product cordial labels of K_2+8K_1 is

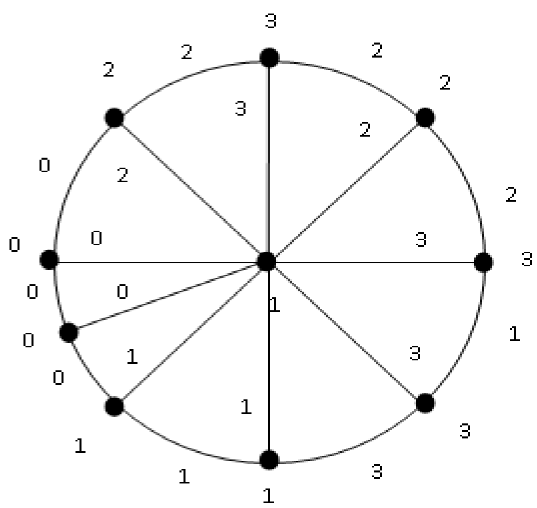


Theorem4.3: Wheel $W_n=C_n+K_1$ is 4-Product Cordial iff n=5 or 9.

Proof: Let C_n be the cycle $u_1,u_2....u_n,u_1$ and $V(W_n)=V(C_n) \cup \{u\}$ and $E(W_n)=E(C_n) \cup \{uu_i : 1\leq i\leq n\}$

Case (i): n=5 or 9
A 4-Product cordial labeling of W_5 and W_9 are given below.





Case (ii): $n \equiv 0 \pmod 4$.

Clearly *f(u)≠0, f(v)≠2*.If possible let there be a 4-product cordial labeling. Let n=4t, Hence $|V(W_n)|$=4t+1, $|E(W_n)|$=8t.Then $e_f(0) \geq 2t+1$. This is not possible.

Case (iii): $n \equiv 1 \pmod 4$.

Let n=4t+1, Hence $|V(W_n)|$=4t+2, $|E(W_n)|$=8t+2.To get the edge label 3,1and3should be the labels of adjacent vertices.

Sub case (i): f(u)=1

Sub case(i)a: $v_f(3)=t$ *and* $v_f(1)=t+1$.From the spokes we get t edges with label 3.Then when t is odd to get the edges with label 3from the rim, atleast $\frac{t+1}{2}$ 3's and $\frac{t+1}{2}$ 1's are used alternatively as vertex labels. Therefore remaining $t-\left(\frac{t+1}{2}\right)$ 3's are labelled consecutively. Similarly remaining $t-\left(\frac{t+1}{2}\right)$ 1's are labelled consecutively.

$e_f(1) \leq \left(\frac{t-1}{2}-1\right)+\left(\frac{t-1}{2}-1\right)+t \leq 2\left(\frac{t-1}{2}\right)-2+t \leq t-3+t \leq 2t-3$, a contradiction. Similarly when t is even $e_f(1) \leq 2t-3$, a contradiction.

Sub case(ii)b: $v_f(3)$=t+1 and $v_f(1)$=t+1.From the spokes we get t+1 edges with label 3.Then when t is odd to get the edges with label 3from the rim, atleast $\frac{t}{2}$ 3's and $\frac{t}{2}$ 1's are used alternatively as vertex labels.Therefore remaining 3's are $t+1-\left(\frac{t}{2}\right)=\left(\frac{t+2}{2}\right)$. Similarly remaining 1's are $\left(\frac{t}{2}\right)$, $e_f(1) \leq \left(\frac{t+2}{2}-1\right)+\left(\frac{t}{2}-1\right)+t+1 \leq 2t$ .Then some 0 appears as a vertex label consecutively. Then $e_f(0) \geq 2t+2$, an impossibility. Similarly we get a contradiction, when t is even also.

Sub case (iii)c: $v_f(1)$=t and $v_f(3)$=t.

From the spokes we get t edges with label 3.Then when t is odd to get the edges with label 3from the rim, atleast $\frac{t+1}{2}$ 3's and $\frac{t+1}{2}$ 1's are used alternatively as vertex labels. Remaining 3's $t-\left(\frac{t+1}{2}\right)$ are labelled consecutively.

Similarly remaining $t-1-\left(\frac{t+1}{2}\right)$ 1's are labelled consecutively.

$e_f(1) \leq \left(\frac{t-1}{2}-1\right)+\left(\frac{t-3}{2}-1\right)+t \leq \frac{2t-4}{2}-2+t \leq 2t-4$, an impossibility. Similarly when t is even we get a contradiction.

Sub case (ii): f(u)=3.

Similar to subcase(i),we get a contradiction.

Case (iv): $n \equiv 2 \pmod 4$.

Let n=4t+2, Hence $|V(W_n)|$=4t+3, $|E(W_n)|$ =8t+4.Clearly $v_f(0)$=t.

Sub case (i): f(u)=1.

Clearly $v_f(1)$=t+1, $v_f(3)$=t+1,as in subcase (i)b, $e_f(0) \geq 2t+2$, again an impossibility.

Case (v): $n \equiv 3 \pmod 4$.

Let n=4t+3, Hence $|V(W_n)|$ =4t+4, $|E(W_n)|$ =8t+6.Similar to case(ii) an impossibility.

Theorem4.2: $K_n^{\ c} + 2K_2$ is 4-Product Cordial iff n≤2.

Proof: Let V($K_n^{\ c} + 2K_2$) ={u_i,u,v,w,z ;1≤i≤n}and E($K_n^{\ c} + 2K_2$)={$uu_i, vu_i, wu_i, zu_i, uv, wz$; 1≤i≤n}.

Case (i): n=1 or 2.

A 4-Product cordial labeling of $K_1^{\ c} + 2K_2$ and $K_2^{\ c} + 2K_2$ are given below

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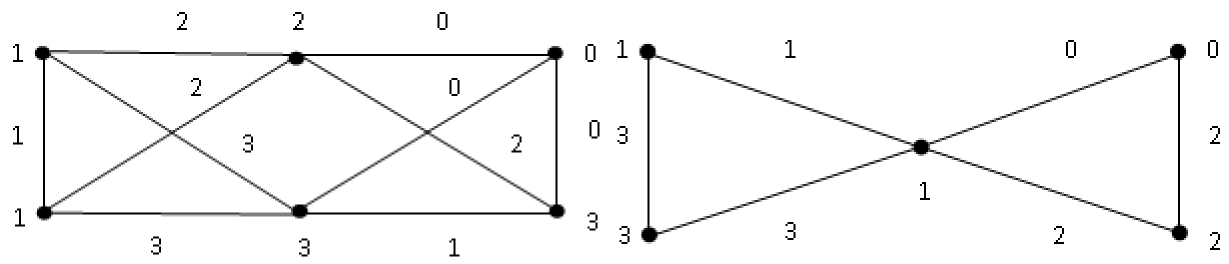

Case (ii): n≡0(mod 4) .

Let n=4t (t≥1), $\left|E(K_n^{\ c} + 2K_2)\right|$=4(4t)+2=16t+2.Clearly f(u),f(v),f(w) and f(z) are not equal to zero, Without loss of generality assume that $f(u_i)=0, 1 \le i \le t+1$.Then $e_f(0)$=4(t+1)=4t+4, a contradiction.

Case (iii): n≡1(mod 4).

Let n= 4t+1(t≥1),$\left|E(K_n^{\ c} + 2K_2)\right|$ = 4(4t+1)+2=16t+6.Here also $e_f(0) \ge 4t+4$, an impossibility.

Case (iv): n≡2(mod 4).

Let n= 4t+2(t≥1),$\left|E(K_n^{\ c} + 2K_2)\right|$ = 4(4t+2)+2=16t+10.Here also $e_f(0) \ge 4t+4$, an impossibility.

Case (v): n≡3(mod 4).

Let n= 4t+3(t≥1),$\left|E(K_n^{\ c} + 2K_2)\right|$ = 4(4t+3)+2=16t+14. Here clearly $v_f(0)$=t+1 and $f(u_i)$=0,1≤i≤t+1 $e_f(0) \ge 4t+4$. Also *f(u),f(v),f(w),f(z)* are not equal to 2, otherwise $e_f(0)>4t+4$.

Hence all 2's are labelled for the vertices u_i. Therefore $e_f(2)$=4(t+2)=4t+8>4t+4, an impossibility.

5. CONCLUSION

In this paper we have studied 4-Product cordial behaviour of graph obtained from two given graphs using graph operations. The authors are of the opinion that the study of *k*-Product cordial labeling behaviour of graph (where *k* is an integer $5 \le k \le |v|$) will be quiet interesting and also will lead to never results.

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