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THE TRUNCATED HETEROGENEOUS TWO-SERVER QUEUE: M/M/2/N WITH RENEGING AND GENERAL BALK FUNCTION

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ABSTRACT

This paper presents an analysis for M/M/2/N queue with heterogeneity, and two general different balk functions. By using the hyper geometric function, the given model will be discussed. Some measures of effectiveness are deduced. Also, some special cases in the given model are obtained. In fact this paper is a continuation and more generalization to the works done by many researchers such as Abou-El-Ata[1,2,3], *Singh*[12], *Krishnamorthi* [6] *and others.*

Keywords: *Exponential distribution, Non-increasing probability function, Steady-state difference equations, hyper geometric function.*

1. INTRODUCTION

Morse [7] introduced the concept of heterogeneity service and obtained the steady-state results for two cases. Staaty [8] has discussed Morse's problem and obtained the explicit expressions for the steady -state probabilities and the mean number in the system. Singh [12] has considered two heterogeneous server Markovian queues with balking and compared its efficiency with the corresponding homogeneous system. Abou-El-Ata [1,2] considered a Markovian queues with both balking and heterogeneity with a modified queue discipline of both Singh [12] and Krishnamoorthi [6]. Sharma and Dass [9] have also analyzed the busy period distributions for M/M/2/N queuing system with heterogeneous servers and obtained the probability density function of the busy period and its mean and variance. Sharma and Dass[10], Sharma and Maheswar [11] discussed M/M/2/N queuing system with un-identical services rates at the two channels and with correlated servers. The closed form solution is obtained and results for M/M/1/N model can be derived as a particular case by putting$\mu_2 = 0$. Also by putting $\mu = \mu_2$ obtained the results for M/M/2/N queue having equal service rates at both. Abou-El-Ata and Hariri[3], studied the queue with balking function in the number which in the queueb_n. Krishna Kumar et al. [5] discussed the transient solution of M/M/2 queue with heterogeneous servers and the possibility of catastrophes. El-Paoumy and Nabawey [4] studied the queue with the arriving customer joins the queue if the expected waiting time is short, and not join the expected waiting time in long. Thus this work was early studied by many researchers in case of only one property. Krishnamoorthi [6] considered Poisson queue with heterogeneity and two alternative queue disciplines one with slight and the other with a great modification of the classical one (FIFO). In this paper we adds a more third property (n-1) α due to the reneging factor. This means that we treats a truncated Poisson queue (capacity N) with balking function, reneging and heterogeneity. By using the hyper geometric function, the given model will be discussed. Some measures of effectiveness are deduced. Also, some special cases for different values of p and q such as: p, q=0,1 in the given model are obtained. Finally we draw some graphs using four Corollaries results in terms of the convergent hyper geometric functions of two basic measures in the steady-state case.

2. ANALYZING THE PROBLEM

Consider the truncated two-channel queue: M/M/2/N with finite capacity, heterogeneity, reneging and balking. Assume that the arrival units follow the Poisson process with arrival rate λ. The service time of the units is exponentially distribution with ratesμ_1, μ_2. The units are served according to the general discipline as follows:

1. If the two servers are busy, the units wait in the queue;

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2. If one server is free the head unit in the queue goes to it, and;

3. If both servers are free, the head unit of the queue chooses channel I with Probability π_1 or channel II with probability π_2; $\pi_1 + \pi_2 = 1$.

Assume the balk concept satisfies the non-increasing probability function:

$0 < b_{n+1} < b_n < 1;\ n \geq 2$ and $b_0, b_n = 1$ for $n = 0,1$

Where,

$b_n =$ Prob.{a unit joints the queue}

I.e. any unit balks with probability $(1 - b_n)$.

Also assume units renege according to a random variable T with probability function given by:

$$d(t) = \alpha e^{-\alpha t}; \alpha > 0, t \geq 0.$$

Let $r(n) = \alpha(n-2)$ be reneging effect when there are n units, $n > 2$ in the system and $r(0) = r(1) = r(2) = 0$ for $n = 0,1,2$.

Now let: $P_n(t)$ = Prob. {that there are n units in the system at time t}, and let

$P_{ij}(t)$ be probability when $i, j = 0$ only, i.e.

$P_0 = P_{00}$ = Prob. {system is empty};

P_{10} = Prob. {the second server is busy},

P_{01} = Prob. {the first server is busy }, also,

$P_1 = P_{01} + P_{10}$ *and* $P_2 = P_{11}$.

3. STEADY-STATE PROBABILITIES

The steady-state difference equations are:

$$-\lambda P_{00} + \mu_1 P_{10} + \mu_2 P_{01} = 0,\ n = 0 \tag{3.1}$$

$$\begin{cases} -(\lambda + \mu_1)P_{10} + \mu_2 P_{11} + \lambda \pi_1 P_{00} = 0 \\ -(\lambda + \mu_2)P_{01} + \mu_1 P_{11} + \lambda \pi_2 P_{00} = 0, \quad n = 1 \end{cases} \tag{3.2}$$

$$-(b_2\lambda + \mu)P_2 + (\mu + \alpha)P_3 + \lambda P_1 = 0,\quad n = 2 \tag{3.3}$$

i.e.

$$-\{b_n\lambda + \mu + (n-2)\alpha\}P_n + \{\mu + (n-1)\alpha\}P_{n+1} + b_{n-1}\lambda P_{n-1} = 0;\ 3 \leq n \leq N-1 \tag{3.4}$$

$$-\{\mu + (N-2)\alpha\}P_N + b_{N-1}\lambda P_{N-1} = 0, \tag{3.5}$$

Where, $\mu = \mu_1 + \mu_2$.

Adding the three relations of (3.1), (3.2) we have:

$$-\lambda P_{10} - \lambda P_{01} + \mu_2 P_{11} + \mu_1 P_{11} = 0;\quad \mu P_{11} = \lambda P_1$$

i.e.

$$P_2 = \rho P_1; \quad \rho = \frac{\lambda}{\mu} \le 1, \; \mu = \mu_1 + \mu_2 \tag{3.6}$$

From relation (3.1) we have

$$P_{10} = \frac{1}{\mu_1}\left(\lambda P_{00} - \pi_2 P_{01}\right).$$

Substituting in the second relation of (3.2) it is clear:

$$P_{11} = \frac{1}{\mu_1}\left((\lambda + \mu_2)P_{01} - \lambda \pi_2 P_{00}\right) \tag{3.7}$$

Substituting in the first relation of (3.2) we get:

$$-\left(\lambda + \mu_1\right)\frac{1}{\mu_1}\left(\lambda P_{00} - \pi_2 P_{01}\right) + \frac{\mu_2}{\mu_1}\left\{\left(\lambda + \mu_2\right)P_{01} - \lambda \pi_2 P_{00}\right\} + \lambda \pi_1 P_{00} = 0$$

$$\left(2\lambda + \mu_1 + \mu_2\right)\mu_2 P_{01} = \left\{\lambda\left(\lambda + \mu_1\right) - \lambda \mu_2 \pi_2 - \lambda \mu_2 \pi_1\right\} P_{00},$$

$$P_{01} = \frac{\lambda\left(\rho + \pi_2\right)}{\mu_2\left(2\rho + 1\right)} P_{00}$$

Also,

$$P_{01} = \frac{\lambda\left(\rho + \pi_1\right)}{\mu_1\left(2\rho + 1\right)} P_{00}$$

$$P_1 = P_{10} + P_{01} = \frac{\lambda[\rho(\mu_1 + \mu_2) + \mu_2 \pi_1 + \mu_1 \pi_2]}{\mu_1 \mu_2 (2\rho + 1)} P_{00} \quad = \Delta P_0 \tag{3.8}$$

where,

$$\Delta = \frac{\lambda[\rho(\mu_1 + \mu_2) + \mu_2 \pi_1 + \mu_1 \pi_2]}{\mu_1 \mu_2 (2\rho + 1)}.$$

From both relations (3.3) and (3.6) we deduce:

$$(\mu + \alpha)P_3 - b_2 \lambda P_2 = \mu P_2 - \lambda P_1 = 0. \tag{3.9}$$

Also from relations (3.4) and (3.9) we obtain:

$$\left\{\mu + \left(n-1\right)\alpha\right\} P_{n+1} - b_n \lambda P_n = \left\{\mu + \left(n-2\right)\alpha\right\} P_n - b_{n-1} \lambda P_{n-1} = \ldots = \left(\mu + \alpha\right) P_3 - b_2 \lambda P_2 = 0,$$

$$\{\mu + (n-1)\alpha\}P_{n+1} - b_n \lambda P_n = \{\mu + (n-2)\alpha\}P_n - b_{n-1}\lambda P_{n-1} = \cdots = (\mu + \alpha)P_3 - b_2 \lambda P_2 = 0,$$

$$\{\mu + (n-2)\alpha\}P_n - b_{n-1}\lambda P_{n-1} = 0; \qquad 2 \le n \le N-1$$

From (3.5) we have:

$$P_n = \frac{\lambda b_{n-1}}{\mu + (n-2)\alpha} P_{n-1}; \qquad 2 \le n \le N$$

$$P_n = \frac{\lambda b_{n-1}}{\mu+(n-2)\alpha}.\frac{\lambda b_{n-2}}{\mu+(n-3)\alpha}.\dots\ .\frac{\lambda b_2}{\mu+\alpha}.\frac{\lambda}{\mu}P_1$$

$$P_n = \frac{\gamma b_{n-1}}{\delta+(n-2)}.\frac{\gamma b_{n-2}}{\delta+(n-3)}.\dots\ .\frac{\gamma b_2}{\delta+1}\rho\Delta P_0;\ \gamma=\frac{\lambda}{\alpha},\delta=\frac{\mu}{\alpha}$$

$$P_n = \Delta\rho P_0\left[\frac{\gamma^{n-2}}{(1+\delta)_{n-2}}\prod_{r=2}^{n-1} b_r\right];\quad 2\le n\le N \tag{3.10}$$

From relations (3.8) and (3.10) we get:

$$P_n = \begin{cases}\Delta P_0, & n=1\\ \Delta\rho P_0\left[\frac{\gamma^{n-2}}{(1+\delta)_{n-2}}\right]\prod_{r=2}^{n-1} b_r & ,\quad 2\le n\le N\end{cases} \tag{3.11}$$

Let us use Abou El-Ata [2] more general balk function which is

$$b_r = \frac{\beta}{(r-1)^p}\left(1-\frac{r-1}{N}\right)^q;\ p,q\ge 0, r\ge 1, \text{are all + ve integers}$$

where $b_0, b_1 = 1$ and the number $r\le$ the number of servers. Also, $0\le b_r\le 1$ is a non-increasing function of r.

Therefore the relation (3.11) becomes

$$P_n = \Delta\rho P_0\left[\frac{\gamma^{n-2}}{(1+\delta)_{n-2}}\right]\prod_{r=2}^{n-1}\left[\frac{\beta}{(r-1)^p}\left(1-\frac{r-1}{N}\right)^q\right]$$

$$P_n = \Delta\rho P_0\left[\frac{\gamma^{n-2}}{(1+\delta)_{n-2}}\right]\prod_{r=2}^{n-1}\beta\left(\frac{-1}{N}\right)^q\left[\frac{(r-N-1)^q}{(r-1)^p}\right]$$

$$P_n = \Delta\rho P_0\left[\frac{(\alpha\lambda)^{n-2}}{(1+\delta)_{n-2}}\right]\prod_{r=2}^{n-1}\left[\frac{(r-N-1)^q}{(r-1)^p}\right];\quad \alpha=\beta\left(\frac{-1}{N}\right)^q$$

$$P_n = \Delta\rho P_0\left[\frac{(\alpha\lambda)^{n-2}}{(1+\delta)_{n-2}}\right]\left[\frac{\{(1-N)_{n-2}\}^q}{\{(n-2)\}^p}\right]$$

Then relation (3.11) becomes:

$$P_n = \begin{cases}\Delta P_0, & n=1\\ \Delta\rho P_0\left[\frac{\{(1-N)_{n-2}\}^q(\alpha\gamma)^{n-2}}{\{(n-2)!\}^p(1+\delta)_{n-2}}\right], & 2\le n\le N, \alpha=\beta\left(\frac{-1}{N}\right)^q\end{cases} \tag{3.12}$$

To get P_0 we use the boundary condition: $\sum_{n=0}^{N} P_n = 1$,

$$1 = P_0 + \Delta P_0 + \Delta\rho P_0\sum_{n=2}^{N}\frac{\{(1-N)_{n-2}\}^q(\alpha\lambda)^{n-2}}{\{(n-2)\}^p(1+\delta)_{n-2}}$$

Replace $(n-2)$ by the number n we obtain:

$$P_0^{-1} = 1+\Delta+\Delta\rho\sum_{n=2}^{N}\frac{\{(1-N)_n\}^q(\alpha\lambda)^n}{\{(n)\}^p(1+\delta)_n}$$

$$P_0^{-1} = 1 + \Delta + \Delta\rho \sum_{n=0}^{N-2} \frac{\left\{(1)_n (1-N)_n\right\}^q (\alpha\lambda)^n}{(n)\left\{(1)_n\right\}^p (1+\delta)_n}$$

$$P_0{}^{-1} = 1 + \Delta + \Delta\rho \quad {}_{q+1}F_{p+1}\left({2-N, 1-N, \dots, 1-N \atop 1+\delta, 1, \dots, 1} \;;\; -\alpha\gamma\right) \tag{3.13}$$

The expected number of units in the system is:

$$L = \sum_{n=0}^{N} nP_n = P_1 + \sum_{n=2}^{`N} nP_n = \Delta P_0 + \sum_{n=2}^{`N} (n-1) P_n + \sum_{n=2}^{`N} P_n$$

$$L = \Delta P_0 + \Delta\rho P_0 \sum_{n=2}^{N} (n-1) \frac{\left\{(1-N)_{n-2}\right\}^q (\alpha\gamma)^{n-2}}{\left\{(n-2)\right\}^p (1+\delta)_{n-2}} + \Delta\rho P_{0,q+1} F_{p+1}$$

Let us replace $(n-2)$ by n we have:

$$L = \Delta P_0 + \Delta\rho P_0 \sum_{n=2}^{N-2} (n+1) \frac{(1)_n \left\{(1-N)_n\right\}^q (\alpha\gamma)^n}{(n)\left\{(1)_n\right\}^p (1+\delta)_{n-2}} + \Delta\rho P_{0,q+1} F_{p+1}$$

$$L = \Delta P_0 + \Delta\rho P_0 \frac{d}{d(\alpha\gamma)} \left[\alpha\gamma \,{}_{q+1}F_{p+1}\right] + \Delta\rho P_0 \cdot{}_{q+1} F_{p+1}$$

$$L = \Delta P_0 \left[1 + 2\rho \cdot {}_{q+1} F_{p+1}\left({2-N, 1-N, \dots, 1-N \atop 1+\delta, 1, \dots, 1} \;;\; -\alpha\gamma\right) + \rho \frac{(2-N)(1-N)^q (\alpha\gamma)}{1+\delta} \cdot {}_{q+1} F_{p+1}\right] \tag{3.14}$$

This hyper geometric function which is convergent and it represents a finite summation since all terms of the series $U_n = 0$ at$n > N$.

4. SPICIAL CASES

To trace this model we discuss some special cases for different values.

Corollary 4.1: Let $p = q = 0,\ b_r = \beta;\ r \geq 2, b_0 b_1 = 1$, thus relations (3.12) and (3.13) become:

$$P_n = \begin{cases} \Delta P_0, & n = 1 \\ \Delta\rho P_0 \left[\dfrac{(\beta\gamma)^{n-2}}{(1+\delta)_{n-2}}\right], & 2 \leq n \leq N, \end{cases}$$

$$P_0{}^{-1} = 1 + \Delta + \Delta\rho \ \cdot_1 \mathrm{F}_1\left({2-N \atop 1+\delta} \;;\; -\beta\gamma\right) \tag{3.15}$$

Where, $\cdot_1 \mathrm{F}_1$ is called the confluent hyper geometric function.

$$L = \Delta P_0 \left[1 + 2\rho \cdot {}_1 F_1\left({2-N \atop 1+\delta} \;;\; |-\beta\gamma\right) + \rho\beta\gamma \left(\frac{2-N}{1+\delta}\right) \cdot {}_1 F_1\left({3-N \atop 2+\delta} \;;\; -\beta\gamma\right)\right] \tag{3.16}$$

Which are the same results as in Abou-El-Ata and Hariri[3].

Corollary 4.2: Let $p = 1, q = 0, \quad b_r = \frac{\beta}{r-1}; r \geq 2, \quad b_0,\ b_1 = 1.$

Hence relations (1.12) and (1.13) are:

$$P_n = \begin{cases} \Delta P_0, & n = 1 \\ \Delta\rho P_0 \left[\dfrac{(\beta\gamma)^{n-2}}{(n-2)!\,(1+\delta)_{n-2}}\right], & 2 \leq n \leq N, \end{cases}$$

$$P_0^{-1} = 1 + \Delta + \Delta\rho.\,{}_1F_2(2-N, 1,1+\delta;\ -\beta\gamma) \tag{3.17}$$

where, ${}_1F_2$ is a convergent hyper geometric function since it represents a finite summation due to Abou-El-Ata notation.

$$L = \Delta P_0\left[1 + 2\rho.{}_1F_2\left({2-N \atop 1,1+\delta};\ -\beta\gamma\right) + \rho\beta\gamma\left(\frac{2-N}{1+\delta}\right)\right] \tag{3.18}$$

Corollary 4.3: Let $p = 0, q = 1,\ b_r = \beta\left(1 - \frac{r-1}{N}\right);\ r \geq 2, b_0 b_1 = 1$, thus relations (3.12) and (3.13) reduce to:

$$P_n = \begin{cases} \Delta P_0, & n = 1 \\ \Delta\rho P_0\left[\dfrac{(1-N)_{n-2}(\alpha\gamma)^{n-2}}{(1+\delta)_{n-2}}\right], & 2 \leq n \leq N, \alpha = \dfrac{-\beta}{N} \end{cases}$$

$$P_0^{-1} = 1 + \Delta + \Delta\rho\ .{}_2F_1\left(2-N, 1-N, 1+\delta;\ \frac{\beta\gamma}{N}\right) \tag{3.19}$$

$$L = \Delta P_0\left[1 + 2\rho.{}_2F_1\left({2-N,1-N \atop 1+\delta};\ \frac{\beta\gamma}{N}\right) - \rho\beta\gamma\frac{(2-N)(1-N)}{N(1+\delta)}.{}_2F_1\left({3-N,2-N \atop 2+\delta};\ \frac{\beta\gamma}{N}\right)\right] \tag{3.20}$$

The hyper geometric function $._2F_1$ is really convergent. If β=1 we have the same results of Abou-El-Ata[1].

Corollary 4.4: Let $p = q = 1,\ b_r = \beta\left(1 - \frac{r-1}{N}\right);\ r \geq 2, b_0 b_1 = 1$, thus relations (3.12) and (3.13) are:

$$P_n = \begin{cases} \Delta P_0, & n = 1 \\ \Delta\rho P_0\left[\dfrac{(1-N)_{n-2}(\alpha\gamma)^{n-2}}{(n-2)!\,(1+\delta)_{n-2}}\right], & 2 \leq n \leq N, \alpha = \dfrac{-\beta}{N} \end{cases}$$

$$P_0^{-1} = 1 + \Delta + \Delta\rho\ .{}_2F_2\left(2-N, 1-N, 1,1+\delta;\ \frac{\beta\gamma}{N}\right) \tag{3.21}$$

$$L = \Delta P_0\left[1 + 2\rho.{}_2F_2\left({2-N,1-N \atop 1,1+\delta};\frac{\beta\gamma}{N}\right) - \rho\beta\gamma\frac{(2-N)(1-N)}{N(1+\delta)}.{}_2F_2\left({3-N,2-N \atop 2,2+\delta};\frac{\beta\gamma}{N}\right)\right] \tag{3.22}$$

I would like to draw some graphs by using the four corollaries' results in terms of the convergent hyper geometric functions of both the following two basic measures of effectiveness such as P_0, L in the steady-state which are:

a) The probability that there are no units in the system P_0,
b) The expected number of units in the system L.

We drew four different graphs for both the measures of assumes the values of the following parameters and constants:
i) The integers p, q given in (1984), (1995) general balk function

$$b_r = \frac{\beta}{(r-1)^p}\left(1 - \frac{r-1}{N}\right);\ b_0, b_1 = 1,\ r \geq 2 \text{ and } p, q = 0.1$$

ii) The constant balk function parameter $\beta = 0.5$.
iii) The capacity of the queue: $N = 10,15$.
iv) The capacity of the queue: $N = 10,15$.

Really he considered $\gamma = \frac{\lambda}{\alpha}, \delta = \frac{\mu}{\alpha}$ to have any arbitrary values which satisfy the following condition:

$$\rho = \frac{\lambda}{\mu},\ \mu = \mu_1 + \mu_2 \text{ where } \alpha = 5.$$

In fact he drew four different types of graphs two for each measure in the steady-state case. The two graphs of P_0 are shown in figures (1.1), (1.2) when $N = 10$ and 15 respectively in case of the four corollary's results. The other two graphs of L are shown in figures (1.3), (1.4) when $N = 10$ and 15 respectively in case of the four corollary's results also. Finally from figures (1.1), (1.2) it is clear that the graphs of are decreasing function of ρ, while from figures (1.3), (1.4) the graphs of L are increasing of ρ.

5. GRAPHS

We draw two graphs for P_0 and the measure of effectiveness L calculated in this model for different values of capacity $N = 10$ and 15. Let us substitute the values of

$$\rho = 0(0.1)0.9,\ \mu_1 = 6,\ \mu_2 = 5,\ \pi_1 = 0.4,$$
$$\pi_2 = 0.6,\ \alpha = 5,\ \lambda = (\mu_1 + \mu_2)\rho,$$
$$\gamma = \frac{\lambda}{\alpha},\ \delta = \frac{(\mu_1 + \mu_2)}{\alpha} \text{ and } \beta = 0.5.$$

In relations(1.15) → (1.22). We easily calculate the following subsidiary important Table (5.1) below. Clearly the required graphs of P_0, L are drawn against ρ as shown in Fig. 5.1, Fig. 5.2, Fig. 5.3 and Fig. 5.4 respectively.

N	10								15							
	cor.1		cor.2		cor.3		cor.4		cor.1		cor.2		cor.3		cor.4	
ρ	Po	L	Po	L	Po	L	Po	L	Po	L	Po	L	Po	L	Po	L
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
0.1	0.812	0.2043	0.8122	0.2042	0.8124	0.2041	0.8126	0.2039	0.8089	0.2065	0.8096	0.2065	0.8094	0.2065	0.81	0.2062
0.2	0.6462	0.4018	0.649	0.402	0.6485	0.4027	0.6511	0.4013	0.6263	0.4076	0.6338	0.4111	0.6289	0.4119	0.6362	0.4115
0.3	0.4966	0.5739	0.5062	0.5783	0.5016	0.5832	0.5108	0.5798	0.4483	0.5591	0.4724	0.587	0.4531	0.5847	0.4771	0.5933
0.4	0.3667	0.699	0.3858	0.719	0.3737	0.7299	0.3925	0.7274	0.2945	0.6149	0.3368	0.7079	0.2995	0.6686	0.3429	0.7279
0.5	0.2613	0.767	0.2892	0.8183	0.2684	0.8341	0.297	0.839	0.1813	0.5747	0.2334	0.7689	0.1845	0.7108	0.2395	0.8089
0.6	0.1815	0.7828	0.215	0.8791	0.1873	0.8976	0.2228	0.9163	0.108	0.4733	0.16	0.7821	0.1091	0.6828	0.1653	0.8452
0.7	0.1243	0.76	0.1596	0.9093	0.1282	0.9296	0.1667	0.9653	0.0638	0.3467	0.11	0.7641	0.0635	0.6316	0.1142	0.581
0.8	0.0846	0.7137	0.1189	0.9172	0.0888	0.9412	0.1251	0.993	0.0379	0.2168	0.0764	0.7283	0.037	0.5768	0.0796	0.8384
0.9	0.0577	0.6554	0.0892	0.9102	0.0585	0.9419	0.0944	1.0059	0.0229	0.0937	0.0538	0.6838	0.0218	0.5275	0.0562	0.8161

Table -5.1

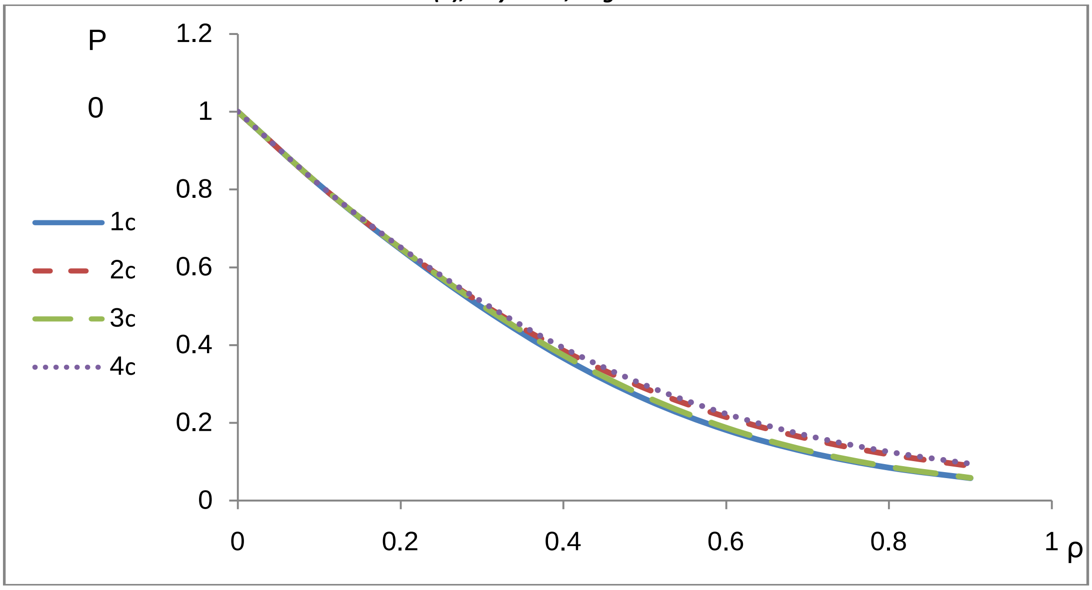

Fig. 5.1
N=10

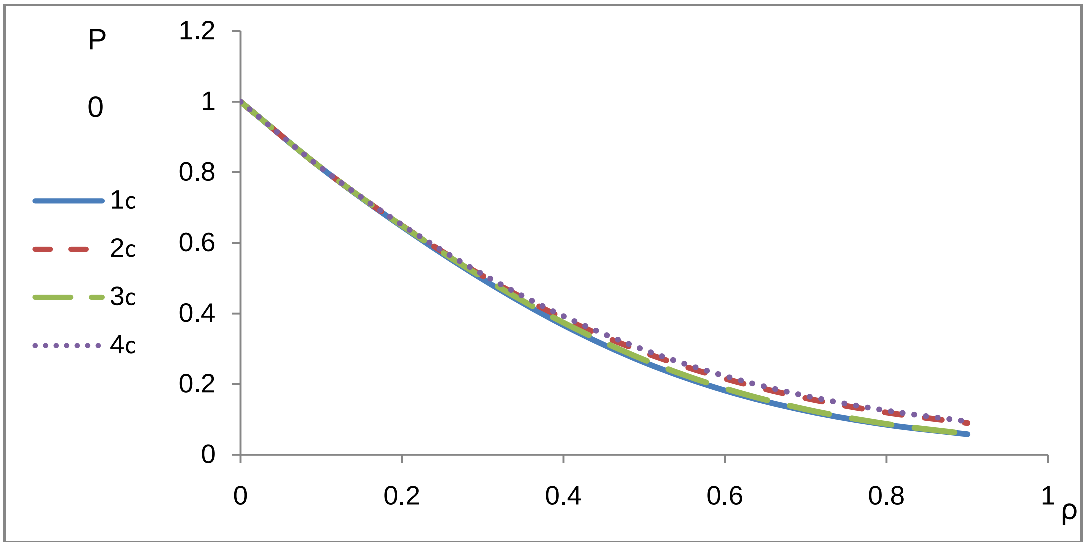

Fig. (5.2)
N=15

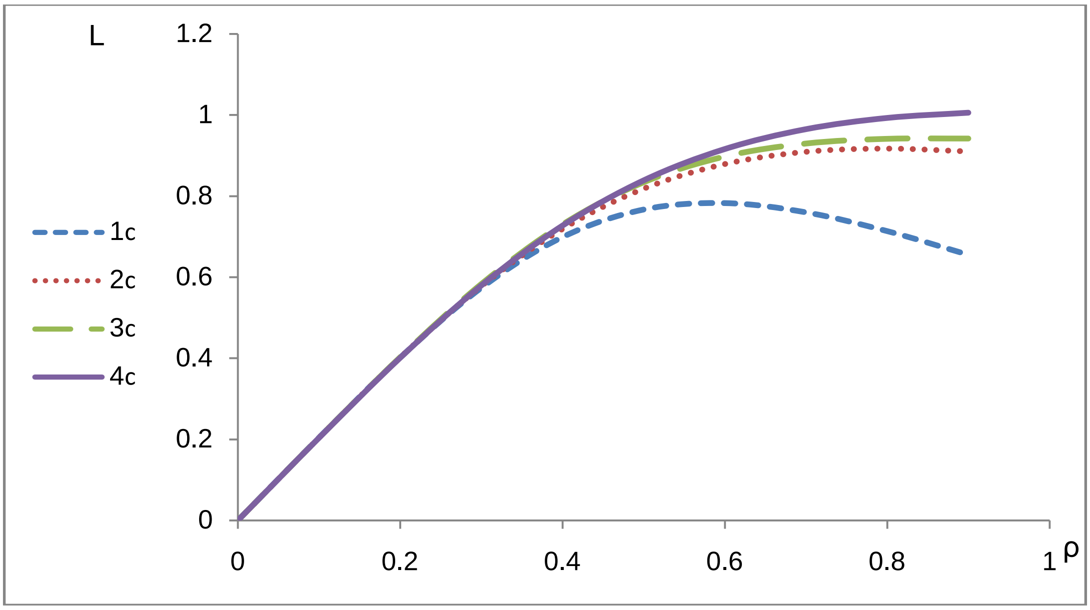

Fig. (5.3)
N=10

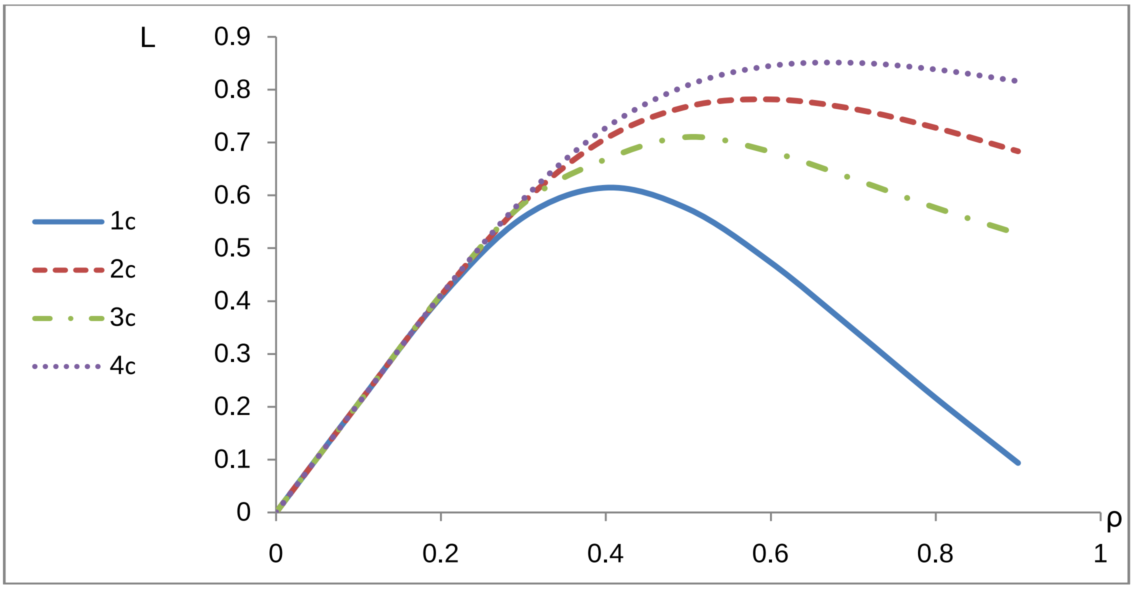

Fig. (5.4)
N=15

6. CONCLUSION

In this paper, the Model M/M/2/N is studied with heterogeneity, and two general different balk functions. By using the convergent of the hyper geometric function and from the boundary condition $\sum_{n=0}^{N} P_n = 1$, the given model will be discussed. Some measures of effectiveness are deduced. Also, some special cases for different values of p and q such as: p, q=0,1 in the given model are obtained. We draw some graphic for four corollaries results in terms of the convergent hyper geometric functions of two basic measures of effectiveness in the steady-state case.

Finally, it's worthwhile to mention that the proposed model can be applied other applications in futures. This our task.

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