International Journal of Mathematical Archive-3(7), 2012, 2713-2727
IJMA Available online through www.ijma.info ISSN 2229 – 5046

The Onset of Double Diffusive Convection in a Couple Stress Fluid Saturated Anisotropic Porous Layer with Cross-Diffusion Effects

S. N. Gaikwad* & Dhanraj M.
Department of Mathematics, Gulbarga University, Jnana Ganga, Gulbarga - 585 106, India

(Received on: 10-07-12; Revised & Accepted on: 31-07-12)

ABSTRACT
The double diffusive convection in a horizontal couple stress fluid saturated anisotropic porous layer, which is heated and salted from below in the presence of Soret and Dufour effects, is studied analytically using linear stability analyses. The linear analysis is based on the usual normal mode technique. The modified Darcy equation that includes the time derivative term is used to model the momentum equation. The critical Rayleigh number for stationary and oscillatory modes and frequency of oscillations are obtained analytically using linear theory. The effects of anisotropy parameter, solute Rayleigh number, Lewis number, Vadasz number, couple stress parameter, Soret and Dufour parameter on the stationary and oscillatory Rayleigh number is shown graphically.

***Keywords**: Double diffusive convection, Porous layer, Anisotropy, Couple stress fluid, Soret and Dufour parameters.*

1. INTRODUCTION

The problem of convection induced by temperature and concentration gradients or by concentration gradients of two species, known as double diffusive convection. The problem of double-diffusive convection in porous media has attracted considerable interest during the last few decades because of its wide range of applications, from the solidification of binary mixtures to the migration of solutes in water-saturated soils. Other examples include geophysical systems, electrochemistry and the migration of moisture through air contained in fibrous insulation. A comprehensive review of the literature concerning double-diffusive natural convection in a fluid-saturated porous medium may be found in the book by Ingham and Pop [7, 8], Nield and Bejan [18], Vafai [31, 32], and Vadasz [30].

In a system where two diffusing properties are present, instabilities can occur only if one of the components are destabilizing. When heat and mass transfer occur simultaneously in a moving fluid, the relation between the fluxes and the driving potentials are of more intricate in nature. It has been found that an energy flux can be generated not only by temperature gradient but also by composition gradients as well. The energy flux caused by a composition gradient is called the DuFour or diffusion-thermo effect. On the other hand, mass fluxes can also be created by temperature gradients and this is the Soret or thermal-diffusion effect. If the cross-diffusion terms are included in the species transport equations, then the situation will be quite different. Due to the cross-diffusion effects, each property gradient has a significant influence on the flux of the other property.

There are many studies available on the effect of cross diffusions on the onset of double diffusive convection in a porous medium (see e.g., Nield and Bejan [18]). Thermal convection in a binary fluid driven by the Soret and Dufour effects has been investigated by Knobloch [10]. He has shown that equations are identical to the thermosolutal problem except a relation between the thermal and solute Rayleigh numbers. The double-diffusive convection in a porous medium in the presence of Soret and Dufour coefficients has been studied by Rudraiah and Malashetty [19] for a Darcy porous medium using linear analysis, which was extended to include weak nonlinear analysis by Rudraiah and Siddheshwar [20]. Recently, Mansour et al. [17] have investigated the multiplicity of solutions induced by thermosolutal convection in a square porous cavity heated from below and subject to horizontal solute gradient in the presence of Soret effect. More recently, Wang and Tan [33] have studied the stability analysis of Soret driven double diffusive convection of Maxwell fluid in a porous medium. Malashetty and Biradar [15] have investigated the onset of double diffusive convection in a binary Maxwell fluid saturated porous layer with cross diffusion effects.

It is well known that non-Newtonian fluids occur in a wide variety of industrial and natural flows, ranging from polymer processing and construction of oil wells, processing of food stuffs to lava and mud flows. There exist different types of non-Newtonian fluids. In particular, the theory of polar fluids has received wider attention in recent years because the traditional Newtonian fluids cannot precisely describe the characteristics of the fluid flow involved therein. These fluids deform and produce a spin field due to the microrotation of suspended particles forming micropolar fluid developed by Eringen [2]. The micropolar fluids take care of local effects arising from microstructure and as well as the

Corresponding author: *S. N. Gaikwad**
Department of Mathematics, Gulbarga University, Jnana Ganga, Gulbarga - 585 106, India

intrinsic motions of microfluidics. The spin field due to microrotation of freely suspended particles set up an anti-symmetric stress, known as couple stress, and thus forming couple-stress fluid. Thus, couple-stress fluid, according to Eringen [2], is a particular case of micropolar fluid when microrotation balances with the natural vorticity of the fluid. The couple-stress fluid has distinct features, such as polar effects and whose microstructure is mechanically significant. Besides, couple stresses are found to appear in noticeable magnitude in fluids with large molecules. For such a special kind of non-Newtonian fluids, the constitutive equations are given by Stokes [24] and the theory proposed by him is the simplest one for microfluids, which allows polar effects such as the presence of couple stress, body couples, and nonsymmetric tensors.

Sharma and Thakur [21] investigated the thermal stability of an electrically conducting couple stress fluid saturated porous layer in the presence of a magnetic field. They reported that the couple stress postponed the onset of stationary convection. Sunil et al. [27] investigated the effect of suspended particles on double diffusive convection in a couple stress fluid saturated porous medium. They reported that for the case of stationary convection, the stable solute gradient and couple stress have stabilizing effects, whereas the suspended particles and medium permeability have destabilizing effects. Siddheshwar and Pranesh [23] analytically studied linear and nonlinear convection in a couple stress fluid layer. Recently, Shivakumara [22] have studied onset of convection in a couple stress fluid saturated porous medium with nonuniform temperature gradients. Malashetty et al. [13] analyzed the double diffusive instability of a couple stress fluid saturated porous layer using linear and weakly nonlinear theories. More recently, Malashetty and Premila [16] studied the onset of double diffusive convection in a couple stress fluid saturated anisotropic porous layer using linear and weakly nonlinear theories.

Anisotropy is generally a consequence of preferential orientation of asymmetric geometry of porous matrix or fibers and is in fact encountered in numerous systems in industry and nature. Anisotropy is particularly important in a geological context, since sedimentary rocks generally have a layered structure; the permeability in the vertical direction is often much less than in the horizontal direction. Anisotropy can also be a characteristic of artificial porous materials like pelleting used in chemical engineering process and fiber material used in insulating purpose. The review of research on convective flow through anisotropic porous medium has been well documented by McKibbin [12] and Storesletten [25, 26]. Castinel and Combarnous [1] have conducted an experimental and theoretical investigation on the Rayleigh-Benard convection in an anisotropic porous medium. Tyvand and Storesletten [28] investigated the problem concerning the onset of convection in an anisotropic porous layer in which the principal axes were obliquely oriented to the gravity vector. Recently, many authors have studied the effect of anisotropy on the onset of convection in a porous layer (Govender [5]; Gaikwad et al. [3, 4]; Malashetty and Swamy [14]).

In most of the studies on double-diffusive convection, the Dufour (diffusion-thermo) and Soret (thermal-diffusion) effects were ignored on the basis that these are of smaller magnitude compared with the effects described by Fourier and Fick's laws. It is, however, known that there are exceptions when Dufour and Soret effects cannot be ignored; (see, for instance, Kafoussias and Williams [9] and references therein). Further, the effect of cross diffusions on the onset of double diffusive convection in a couple stress fluid saturated anisotropic porous layer is not available. The intent of the present paper is therefore to study the onset of double diffusive convection in a couple stress fluid saturated anisotropic porous layer in the presence of soret and dufour effects.

2. MATHEMATICAL FORMULATION

Here, we consider an infinite horizontal couple stress fluid saturated porous layer confined between the planes $z = 0$ and $z = d$, with the vertically downward gravity force $\mathbf{g}$ acting on it. A Cartesian frame of reference is chosen with the origin in the lower boundary and the z-axis vertically upwards. A uniform adverse temperature difference ΔT and a stabilizing concentration difference ΔS are maintained between the lower and upper boundaries. The porous medium is assumed to possess isotropy in the horizontal plane in mechanical property. The Darcy model with time derivative is employed for the momentum equation and both the cross-diffusion terms are included in the temperature and concentration equations. With these assumptions, the basic governing equations are:

$$\nabla \cdot \mathbf{q} = 0, \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\left(\frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q}\cdot\nabla)\mathbf{q}\right) = -\nabla p + \rho\mathbf{g} - (\mu - \mu_c \nabla^2)\mathbf{q}_a, \tag{2}$$

$$\gamma\frac{\partial T}{\partial t} + (\mathbf{q}\cdot\nabla)T = \kappa_{11}\nabla^2 T + \kappa_{12}\nabla^2 S, \tag{3}$$

$$\varepsilon\frac{\partial S}{\partial t} + (\mathbf{q}\cdot\nabla)S = \kappa_{21}\nabla^2 T + \kappa_{22}\nabla^2 S, \tag{4}$$

$$\rho = \rho_0(1-\beta_T(T-T_0)+\beta_S(S-S_0)), \tag{5}$$

where $\mathbf{q}$ is the Darcy velocity vector, p is the pressure, $\mathbf{g}$ is the acceleration due to gravity, μ is the viscosity, T is the temperature, S is the concentration, ε is the porosity. $\gamma = \dfrac{(\rho c)_m}{(\rho c)_f}$, where $(\rho c)_f$ is the volumetric heat capacity of the fluid and $(\rho c)_m = (1-\varepsilon)(\rho c)_s + \varepsilon(\rho c)_f$ is the volumetric heat capacity of the saturated medium as a whole, with the subscripts $f, s,$ and m denoting the properties of the fluid, solid, and porous matrix, respectively. κ_{11} and κ_{22} are effective thermal diffusivity and solutal diffusivity of the medium, κ_{12} and κ_{21} are the Dufour and Soret coefficients. Further, β_T and β_S are the thermal and solutal expansion coefficients in the medium.

2.1 Basic State

The basic state of the fluid is assumed to be quiescent and is given by

$$\mathbf{q}_b = (0,0,0),\ p = p_b(z),\ T = T_b(z),\ S = S_b\left(z\right),\ \rho = \rho_b(z). \tag{6}$$

Using Esq. (6), Eqs. (1) – (5) yields

$$\frac{dp_b}{dz} = -\rho_b g, \frac{d^2T_b}{dz^2} = 0, \frac{d^2S_b}{dz^2} = 0,\ \rho_b = \rho_0(1-\beta_T(T_b-T_0)+\beta_S(S_b-S_0)), \tag{7}$$

with boundary conditions

$$T = T_0 + \Delta T \text{ and } S = S_0 + \Delta S \text{ at } z = 0, \tag{8}$$
$$T = T_0 \text{ and } S = S_0 \text{ at } z = d. \tag{9}$$

The steady state solutions are given as

$$T_b(z) = T_0 + \Delta T\left(1-\frac{z}{d}\right), \quad S_b(z) = S_0 + \Delta S\left(1-\frac{z}{d}\right). \tag{10}$$

2.2 Perturbed State

On the basic state, we superpose small perturbations in the form

$$\mathbf{q} = \mathbf{q}_b + \mathbf{q}',\ T = T_b + T', S = S_b + S',\ p = p_b + p', \rho = \rho_b + \rho', \tag{11}$$

where the primes indicate perturbations. Substituting Eq. (11) into Eqs. (1)– (5), using the basic state solutions (10), we obtain the equations governing the perturbations in the form

$$\nabla\cdot\mathbf{q}' = 0, \tag{12}$$
$$\frac{\rho_0}{\varepsilon}\frac{\partial\mathbf{q}'}{\partial t} = -\nabla p' + \rho_0\mathbf{g}(\beta_T T' - \beta_S S') - (\mu - \mu_c\nabla^2)\mathbf{q}'_a, \tag{13}$$
$$\gamma\frac{\partial T'}{\partial t} + (\mathbf{q}'\cdot\nabla)T' - w'\frac{\Delta T}{d} = \kappa_{11}\nabla^2T' + \kappa_{12}\nabla^2S', \tag{14}$$
$$\varepsilon\frac{\partial S'}{\partial t} + (\mathbf{q}'\cdot\nabla)S' - w'\frac{\Delta S}{d} = \kappa_{21}\nabla^2T' + \kappa_{22}\nabla^2S'. \tag{15}$$

By operating curl twice on Eq. (13), we eliminate p' from it, and then render the resulting equation, and the Eqs. (14) and (15) dimensionless using the following transformations:

$$(x,y,z)=(x^*,y^*,z^*)d,\ t=t^*\frac{\gamma d^2}{\kappa_{11}},\ (u',v',w')=\frac{\kappa_{11}}{d}(u^*,v^*,w^*),$$
$$T'=(\Delta T)T^*,\ S'=(\Delta S)S^*, \tag{16}$$

to obtain non-dimensional equations as (on dropping the asterisks for simplicity),

$$\left[\frac{1}{\gamma Va}\frac{\partial}{\partial t}\nabla^2+\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)(1-C\nabla^2)\right]w-Ra_T\nabla_1^2T+Ra_S\nabla_1^2S=0, \tag{17}$$

$$\left(\frac{\partial}{\partial t}-\nabla^2\right)T-D_f\nabla^2S-w=0, \tag{18}$$

$$\left(\varepsilon_n\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)S-S_r\nabla^2T-w=0, \tag{19}$$

where $Ra_T=\dfrac{\beta_T\,g\,\Delta T\,d\,K_z}{\nu\,\kappa_{11}}$, $Ra_S=\dfrac{\beta_S\,g\,\Delta S\,d\,K_z}{\nu\,\kappa_{11}}$, $C=\dfrac{\mu_c}{\mu\,d^2}$, $Va=\dfrac{\varepsilon\,\nu\,d^2}{\kappa_{11}\,K_z}$, $Le=\dfrac{\kappa_{11}}{\kappa_{22}}$, $\xi=\dfrac{K_x}{K_z}$, $\varepsilon_n=\dfrac{\varepsilon}{\gamma}$, $D_f=\dfrac{\kappa_{12}\,\Delta S}{\kappa_{11}\Delta T}$, and $S_r=\dfrac{\kappa_{21}\Delta T}{\kappa_{11}\,\Delta S}$.

The dimensionless groups that appear are thermal Rayleigh number Ra_T, solute Rayleigh number Ra_S, couple-stress parameter C, Vadasz number Va, the Lewis number Le, mechanical anisotropy parameter ξ, the normalized porosity ε_n, the Dufour parameter D_f, and the Soret parameter S_r. It is worth mentioning here that the Vadasz number Va includes the Prandtl number $Pr=\dfrac{\nu}{\kappa_{11}}$, Darcy number $Da=\dfrac{K_z}{d^2}$ and porosity ε of the porous medium, that is $Va=\dfrac{\varepsilon\,Pr}{Da}$. The Prandtl number affects the stability of the porous system through the combined dimensionless group known as Vadasz number. The Vadasz number is also known as Darcy-Prandtl number in the literature. For detailed discussion on this dimensionless group one can refer to the excellent study of Vadasz [29].

Equations (17)–(19) are to be solved for impermeable, isothermal and isosolutal boundaries. Hence the boundary conditions for the perturbation variables are given by

$$w=T=S=0 \quad \text{at } z=0,1. \tag{20}$$

3. LINEAR STABILITY ANALYSIS

In this section, we predict the thresholds of both marginal and oscillatory convections using linear theory. The eigen value problem defined by Eqs. (17)– (19) subject to the boundary conditions (20) is solved using the time dependent periodic disturbances in a horizontal plane. We assume that the amplitudes are small enough and can be expressed as

$$\begin{pmatrix} w \\ T \\ S \end{pmatrix}=\begin{pmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \end{pmatrix}\exp[i(lx+my)+\omega t], \tag{21}$$

where l, m are the wavenumbers in the horizontal plane and ω is the growth rate. Substituting Eq. (21) into Eqs. (17)– (19), we obtain

$$\left[\frac{\omega}{Va}(D^2-a^2)+\left(\frac{D^2}{\xi}-a^2\right)(1-C(D^2-a^2))\right]W+a^2Ra_T\Theta-a^2Ra_S\Phi=0, \tag{22}$$

$$W+(D^2-a^2-\omega)\Theta+D_f(D^2-a^2)\Phi=0, \tag{23}$$

$$W+S_r(D^2-a^2)\Theta+\left(\frac{1}{Le}(D^2-a^2)-\varepsilon_n\omega\right)\Phi=0, \tag{24}$$

where $D=\dfrac{d}{dz}$ and $a^2=l^2+m^2$.

The boundary conditions (20) now read

$$W=\Theta=\Phi=0 \text{ at } z=0,1. \tag{25}$$

We assume the solutions of Eqs. (22)– (24) in the form

$$\begin{pmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \end{pmatrix}=\begin{pmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{pmatrix}\sin n\pi z, \qquad (n=1,2,3,....) \tag{26}$$

which satisfy the boundary conditions (25). The most unstable mode corresponds to $n=1$ (fundamental mode). Therefore, substituting Eq. (26) with $n=1$ into Eqs. (22)– (24), we obtain a matrix equation of the form

$$\begin{pmatrix} \delta^2\omega Va^{-1}+\delta_1^2(1+C\delta^2) & -a^2Ra_T & a^2Ra_S \\ 1 & -(\delta^2+\omega) & -D_f\delta^2 \\ 1 & -S_r\delta^2 & -(\delta^2Le^{-1}+\varepsilon_n\,\omega) \end{pmatrix}\begin{pmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \tag{27}$$

For the nontrivial solution of the above matrix equation (27), we require

$$Ra_T=\frac{\delta^2\omega Va^{-1}+\delta_1^2(1+C\delta^2)}{a^2}\left(\frac{(\delta^2+\omega)(\delta^2+\varepsilon_nLe\omega)-D_fS_rLe\delta^4}{\delta^2(1-D_fLe)+\varepsilon_n\,Le\omega}\right)+\left(\frac{\delta^2(1-S_r)+\omega}{\delta^2(1-D_fLe)+\varepsilon_nLe\omega}\right)Ra_SLe, \tag{28}$$

where $\delta^2=\pi^2+a^2$, and $\delta_1^2=\pi^2\xi^{-1}+a^2$. The growth rate ω is in general a complex quantity such that $\omega=\omega_r+i\omega_i$. The system with $\omega_r<0$ is always stable, while for $\omega_r>0$, it will become unstable. For neutral stability state $\omega_r=0$.

3.1 Stationary State

For the validity of principle of exchange of stabilities (i.e., steady case), we have ω = 0 (i.e., $\omega_r=\omega_i=0$) at the margin of stability. Then, Eq. (28) gives the Rayleigh number at which marginally steady mode exists is given by

$$Ra_T^{St}=\frac{(\pi^2+a^2)(\pi^2\xi^{-1}+a^2)[1+C(\pi^2+a^2)](1-D_fS_rLe)}{a^2(1-D_fLe)}+\left(\frac{1-S_r}{1-D_fLe}\right)Ra_SLe. \tag{29}$$

The minimum value of the Rayleigh number Ra_T^{St} occurs at the critical wavenumber $a=a_c^{St}$, where $a_c^{St}=\sqrt{x}$ satisfies the equation

$$x^3(2C\xi)+x^2(\xi+C\pi^2(1+2\xi))-\pi^4(1+C\pi^2)=0. \tag{30}$$

In the absence of Soret and Dufour effects, the stationary Rayleigh number given by Eq. (29) reduces to

$$Ra_T^{St} = \frac{(\pi^2 + a^2)(\pi^2\xi^{-1} + a^2)[1 + C(\pi^2 + a^2)]}{a^2} + Ra_S Le\,. \tag{31}$$

This result coincides with the results of Malashetty and Premila [16].

For an isotropic porous medium, that is when $\xi = 1$, Eq. (31) gives

$$Ra_T^{St} = \frac{1}{a^2}(\pi^2 + a^2)^2[1 + C(\pi^2 + a^2)] + Ra_S Le\,, \tag{32}$$

the result given by Malashetty et al. [13] for the onset of double diffusive convection for stationary mode in a couple stress fluid saturated isotropic porous medium. Further, in the absence of couple stresses, that is C = 0, the Eq. (32) reduces to

$$Ra_T^{St} = \frac{1}{a^2}(\pi^2 + a^2)^2 + Ra_S Le\,. \tag{33}$$

This is the classical result for double diffusive convection in a porous medium for the stationary mode (Nield and Bejan [18]). For single component couple stress fluid saturated porous medium, $Ra_S = 0$, the stationary Rayleigh number given by Eq. (31) reduces to

$$Ra_T^{St} = \frac{1}{a^2}(\pi^2 + a^2)(\pi^2\xi^{-1} + a^2)[1 + C(\pi^2 + a^2)]\,. \tag{34}$$

When C = 0, i.e., in the absence of couple stresses, Eq. (34) reduces to

$$Ra_T^{St} = \frac{1}{a^2}(\pi^2 + a^2)(\pi^2\xi^{-1} + a^2)\,, \tag{35}$$

the result given by Storesletten [25] for the case of single component fluid saturated anisotropic porous layer. Further, for an isotropic porous medium, $\xi = 1$, the above Eq. (35) reduces to

$$Ra_T^{St} = \frac{(\pi^2 + a^2)^2}{a^2}\,, \tag{36}$$

which has the critical value $Ra_{T,c}^{St} = 4\pi^2$, for $a_c^{St} = \pi$ obtained by Horton and Rogers [6], and Lapwood [11].

3.2 Oscillatory State

For overstable convection we allow for the possibility of oscillatory motion, and therefore, ω is represented in the form $\omega = \omega_r + i\omega_i$. At the marginal stability state $\omega_r = 0$. Therefore, we now set $\omega = i\omega_i$ in Eq. (28) and clear the complex quantities from the denominator to obtain

$$Ra_T = \Delta_1 + i\omega_i\Delta_2\,. \tag{37}$$

Since Ra_T is a physical quantity, it must be real. Hence, from Eq. (37) it follows that either $\omega_i = 0$ (steady onset) or $\Delta_2 = 0$ ($\omega_i \neq 0$, oscillatory onset).

For oscillatory onset $\Delta_2 = 0$ ($\omega_i \neq 0$) and this gives an expression for frequency of oscillations in the form (on dropping the subscript i)

$$\omega^2 = \left(\frac{\begin{array}{c}\delta^8(1-D_f Le)(-1+D_f Le\, S_r) - a^2 Le\, Ra_S Va\delta^2(1-D_f Le + Le(-1+S_r)\varepsilon_n) - \\ Va\delta^4(1+C\delta^2)(1+D_f Le(-1+Le(-1+S_r)\varepsilon_n))\delta_1^2\end{array}}{Le^2\varepsilon_n(\delta^4(D_f+\varepsilon_n)+Va(1+C\delta^2)\varepsilon_n\delta_1^2} \right). \tag{38}$$

Now Eq. (37) with $\Delta_2 = 0$ gives

$$Ra_T^{Osc} = \frac{\begin{array}{c}a^2 Le Ra_S Va((-1+D_f Le)(-1+S_r)\delta^4 + Le\varepsilon_n\omega^2) - \delta^6(1+D_f Le(-1+Le(-1+S_r)\varepsilon_n))\omega^2 \\ -Le^2\delta^2\varepsilon_n^2\,\omega^4 + Va\delta^2(1+C\delta^2)((-1+D_f Le)(-1+D_f LeS_r)\delta^4 + Le^2\varepsilon_n(D_f+\varepsilon_n)\omega^2)\delta_1^2)\end{array}}{a^2 Va((-1+D_f Le)^2\delta^4 + Le^2\varepsilon_n^2\,\omega^2)}. \tag{39}$$

The analytical expression for oscillatory Rayleigh number given by Eq. (39) is minimized with respect to the wavenumber numerically, after substituting for ω^2 (>0) from Eq. (38), for various values of physical parameters in order to know their effects on the onset of oscillatory convection.

4. RESULT AND DISCUSSION

The onset of double diffusive convection in a couple stress fluid saturated anisotropic porous layer with cross diffusion effects is investigated analytically using the linear theories. In the linear stability theory the expressions for the stationary and oscillatory Rayleigh number are obtained analytically along with the expression for frequency of oscillation.

The neutral stability curves in the (Ra_T, a) plane for various parameter values are as shown in Figs.1-8. We fixed the values for the parameters as $\xi = 0.2,\ C = 0.3,\ \varepsilon_n = 0.4,\ Ra_S = 100,\ Le = 20,\ Va = 10,\ D_f = 0.005,$ and $S_r = 0.05$ except the varying parameter. From these figures it is clear that the neutral curves are connected in a topological sense. This connectedness allows the linear stability criteria to be expressed in terms of the critical Rayleigh number, Ra_{Tc}, below which the system is stable and unstable above.

Fig.1 shows the neutral stability curves for different values of mechanical anisotropy parameter ξ and for fixed values of other parameters. We observed that increasing of the mechanical anisotropy parameter ξ decreases the minimum of the Rayleigh number for both stationary and oscillatory state, indicating that, the effect of increasing mechanical anisotropy parameter ξ is to advance the onset of stationary and oscillatory convection. Further, we find that the minimum of Rayleigh number shift towards the smaller values of the wavenumber with increasing mechanical anisotropy parameter. This indicates that the cell width increases with increasing mechanical anisotropy parameter.

Fig. 2 shows the neutral stability curves for different values of the couple stress parameter C and for fixed values of $\xi = 0.2,\ \varepsilon_n = 0.4,\ Ra_S = 100,\ Le = 20,\ Va = 10,\ D_f = 0.005,$ and $S_r = 0.05$. We observe from this figure that the minimum value of the Rayleigh number for both stationary and oscillatory modes increases with an increase in the value of the couple stress parameter C, indicating that the effect of the couple stress parameter is to stabilize the system.

Fig. 3(a) shows the marginal stability curves for stationary mode for different values of Lewis number Le are drawn. We find that the increase in the value of Lewis number increases the critical Rayleigh number for stationary mode. Fig. 3(b) shows the marginal stability curves for oscillatory mode for different values of Lewis number Le. We find that an increase in the value of Lewis number decreases the critical Rayleigh number for oscillatory mode. Therefore, the effect of Le is to advance the onset of oscillatory convection, whereas its effect is to inhibit the stationary onset.

The effect of the Vadasz number Va on the oscillatory neutral curves for the fixed values of other parameters is shown in Fig. 4. We find that increasing Vadasz number increases the oscillatory Rayleigh number, indicating that the increase of Vadasz number is to enhance the stability of the system.

Fig.5 shows the neutral stability curves for different values of solute Rayleigh number Ra_S and for fixed values of other parameters. We find from this figure that the minimum value of the Rayleigh number for both stationary and oscillatory modes increases with an increase in the values of solute Rayleigh number Ra_S, indicating that the effect of solute Rayleigh number is to stabilize the system.

Fig. 6(a) shows the neutral stability curves for stationary mode for different values of Dufour parameter D_f and for fixed values of other parameters. We find that the stationary Rayleigh number increases with an increase in the value of the Dufour parameter, indicating that the effect of Dufour parameter is to enhance the stability of the system. Fig. 6(b) shows the neutral stability curves for oscillatory mode for different values of Dufour parameter D_f and for fixed values of other parameters. We observe that the oscillatory Rayleigh number decreases with increasing the value of the Dufour parameter, indicating that Dufour coefficient advances the oscillatory convection.

In Fig. 7(a), the marginal stability curves for stationary mode for different values of Soret parameter S_r and for fixed values of other parameters are drawn. We observe that with an increase in the negative Soret parameter, the stationary Rayleigh number increases indicating that the negative Soret parameter stabilizes the system. On the other hand, for positive Soret parameter, the minimum of the stationary Rayleigh number decreases with an increase of the Soret parameter, indicating that the positive Soret parameter destabilizes the system. Fig. 7(b) shows the marginal stability curves for oscillatory mode for different values of Soret parameter S_r and for fixed values of other parameters. We find that with an increase in the negative Soret parameter, the oscillatory Rayleigh number decreases indicating that the negative Soret parameter destabilizes the system. On the other hand, for positive Soret parameter, the minimum of the oscillatory Rayleigh number increases with an increase of the Soret number, indicating that the positive Soret coefficient has a stabilizing effect.

Fig.8 shows the marginal stability curves for different values of normalized porosity parameter ε_n and for fixed values of other parameters. We observe that the effect of increasing the normalized porosity parameter decreases the minimum of the Rayleigh number, indicating that the effect of normalized porosity parameter is to advance the onset of double diffusive convection. Further, it is important to note that the effect of normalized porosity is significant for small ε_n.

The detailed behavior of stationary and oscillatory critical Rayleigh number with respect to the solute Rayleigh number is analyzed in the $(Ra_T)_c - Ra_S$ plane through Figs. 9-15.

The variation of the critical Rayleigh number for both stationary and oscillatory modes with the solute Rayleigh number for different values of mechanical anisotropy parameter ξ and for fixed values of other parameters is shown in fig. 9. We find from this figure that the critical Rayleigh number for both stationary and oscillatory modes decreases with increasing mechanical anisotropy parameter ξ indicating that the effect of mechanical anisotropy parameter is to advance the onset of stationary and oscillatory convection.

Fig. 10 shows the variation of the critical Rayleigh number for both stationary and oscillatory Rayleigh number for different values of the couple stress parameter C and for fixed values of other parameters. We observe that the critical Rayleigh number for both stationary and oscillatory modes increases with increasing couple stress parameter C indicating that the couple stress parameter stabilizes the system.

In fig. 11(a), the variation of critical stationary Rayleigh number $(Ra_T^{St})_c$ with solute Rayleigh number Ra_S for different values of Lewis number Le for fixed values of other parameters is shown. The critical Rayleigh number increases with increasing Le, indicating that the effect of Lewis number is to stabilize the system in the stationary mode. On the other hand, Fig. 11(b) shows the variation of critical Rayleigh number for the oscillatory mode $(Ra_T^{Osc})_c$ with solute Rayleigh number Ra_S for different values of Lewis number Le. We observe that the critical Rayleigh number decreases with increasing Lewis number Le, indicating that the effect of Lewis number is to destabilize the system in the oscillatory mode.

Fig. 12 shows the varitation of critical Rayleigh number with solute Rayleigh number for the oscillatory mode for different values of Vadasz number. We observe that the critical oscillatory Rayleigh number increases with an increase in the value of Vadasz number Va, indicating that the Vadasz number inhibit the onset of oscillatory convection.

Fig. 13(a) shows the variation of critical stationary Rayleigh number $(Ra_T^{St})_c$ with solute Rayleigh number Ra_S for different values of Dufour parameter D_f. We find that critical stationary Rayleigh number increases with an increase of D_f. Thus, the effect of Dufour parameter is to enhance the stability of the system. Fig. 13(b) shows the variation of $(Ra_T^{Osc})_c$ with Ra_S for different values of Dufour parameter D_f. We find that critical oscillatory Rayleigh number decreases with an increase of D_f, indicating that the effect of Dufour parameter is to advance the onset of oscillatory convection.

Fig. 14(a) shows the variation of $(Ra_T^{St})_c$ with Ra_S for different values of Soret parameter S_r and for fixed values of other parameters. We find that the negative Soret coefficient has a stabilizing effect, and on the other hand, the positive Soret coefficient has destabilizing effect. Fig. 14(b) shows the variation of $(Ra_T^{Osc})_c$ with Ra_S for different values of Soret parameter S_r is drawn. We observe that the negative Soret coefficient has destabilizing effect, and on the other hand, the positive Soret coefficient has a stabilizing effect.

The effect of normalized porosity parameter ε_n on the critical oscillatory Rayleigh number $(Ra_T^{Osc})_c$ and for fixed values of other parameters is shown in Fig. 15. We observe that the critical oscillatory Rayleigh number decreases with increasing normalized porosity parameter. Thus, the effect of normalized porosity parameter ε_n is to destabilize the system.

5. CONCLUSIONS

The onset of double diffusive convection in a couple stress fluid saturated anisotropic porous layer in the presence of Soret and Dufour effects, is studied analytically using linear stability analysis. The modified Darcy model that includes the time derivative term is used for the momentum equation. The effects of anisotropy parameter, solute Rayleigh number, Lewis number, Vadasz number, couple stress parameter, Soret and Dufour parameters on the stationary and oscillatory convection are shown graphically. The following conclusions are drawn:

1. The effect of increasing the mechanical anisotropy parameter is to advance the onset of both stationary and oscillatory convection.

2. The effect of couple stress parameter is to delay, both stationary and oscillatory convection.

3. The Lewis number has a stabilizing effect in the case of stationary convection while it has destabilizing effect in the case of the oscillatory convection.

4. The Vadasz number enhances the onset of oscillatory convection indicating that it has a stabilizing effect.

5. The solute Rayleigh number has a stabilizing effect on both stationary and oscillatory modes.

6. The Dufour parameter stabilizes the system in the stationary mode, while it destabilizes the system in the oscillatory mode.

7. The negative Soret parameter stabilizes the system and positive Soret parameter destabilizes the system in the stationary convection, while in the oscillatory convection the negative Soret coefficient destabilize the system and positive Soret coefficient stabilizes the system.

8. The normalized porosity parameter has a destabilizing effect in the case of oscillatory mode.

ACKNOWLEDGEMENTS

This work is supported by UGC, Research Fellowship in Science for Meritorious Students (RFSMS) under UGC-BSR Scheme No.4-1/2010(BSR) pt. IV, dated 06.01.2011 and partially under UGC Major Research Project No.F.37-174/2009 (SR), dated 12.01.2010. One of the authors (D.M.) acknowledges UGC for awarding the Research Fellowship.

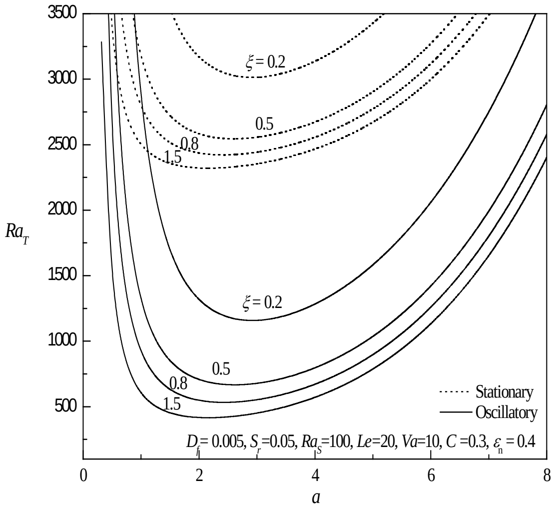

Fig.1. Neutral stability curves for different values of mechanical anisotropic parameter ξ

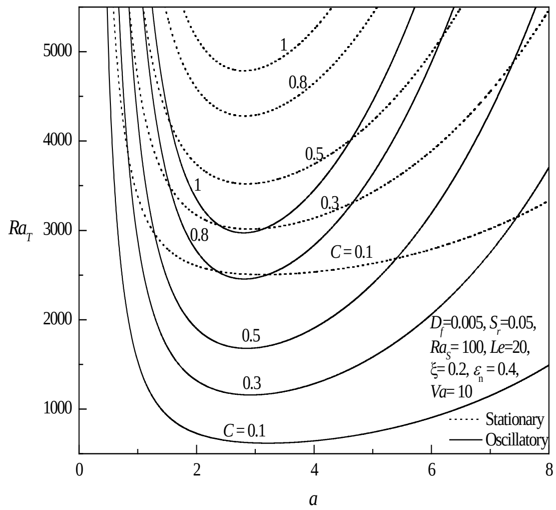

Fig.2. Neutral stability curves for different values of couple stress parameter C

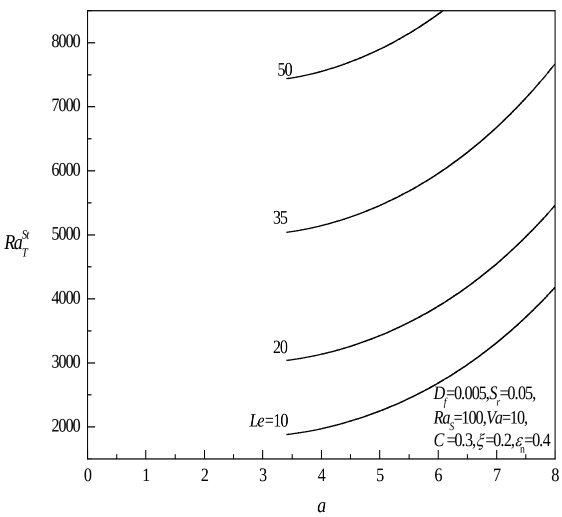

Fig.3(a). Stationary neutral stability curves for different values of Lewis number Le

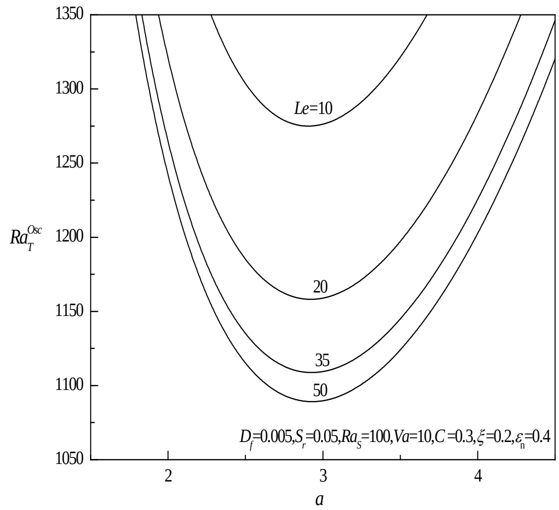

Fig.3(b). Oscillatory neutral stability curves for different values of Lewis number Le

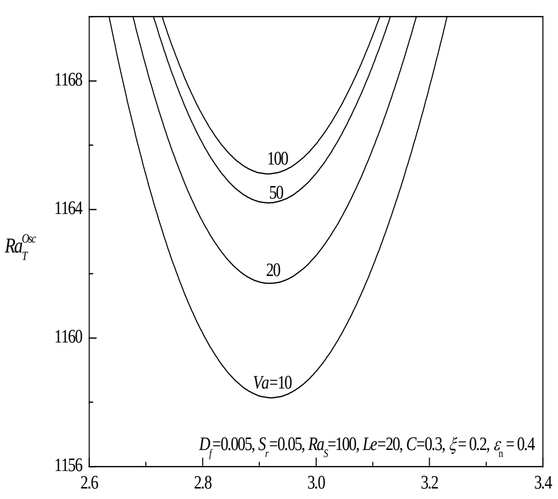

Fig.4. Oscillatory neutral stability curves for different values of Vadaz number Va

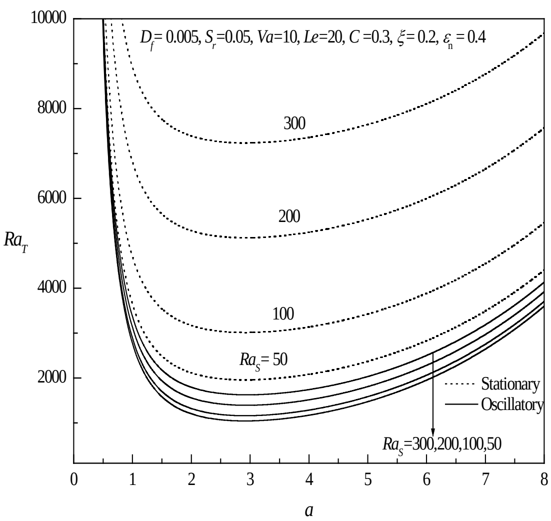

Fig.5. Neutral stability curves for different values of solute Rayleigh number Ra_S

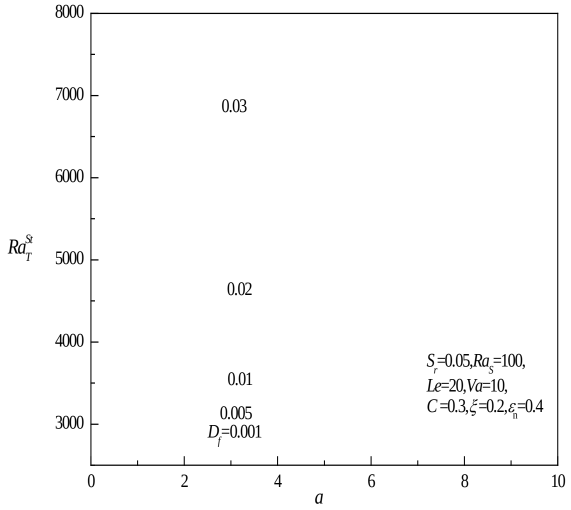

Fig.6(a). Stationary neutral stability curves for different values of Dufour parameter D_f

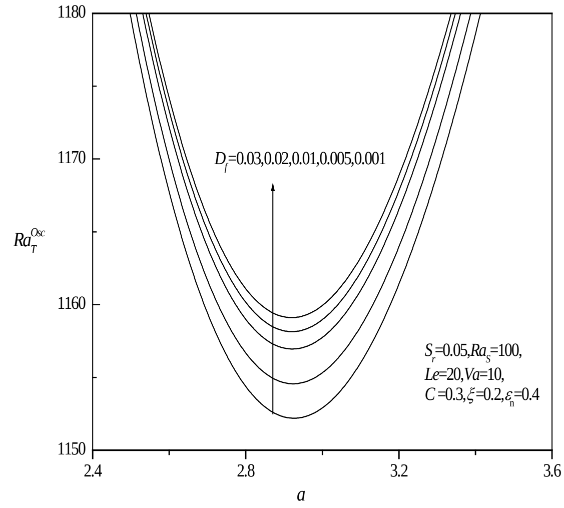

Fig.6(b). Oscillatory neutral stability curves for different values of Dufour parameter D_f

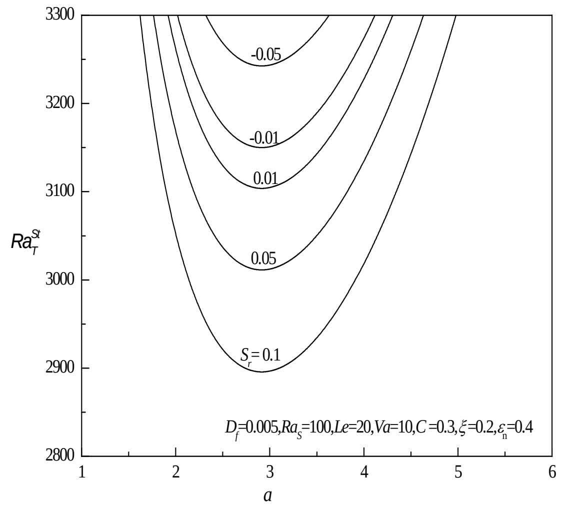

Fig.7(a). Stationary neutral stability curves for different values of Soret parameter S_r

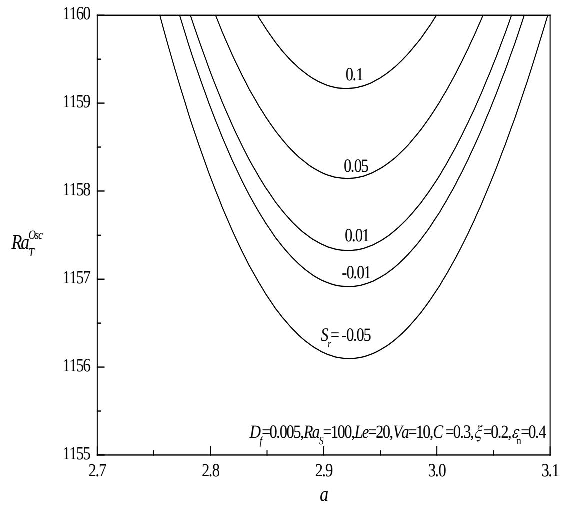

Fig.7(b). Oscillatory neutral stability curves for different values of Soret parameter S_r

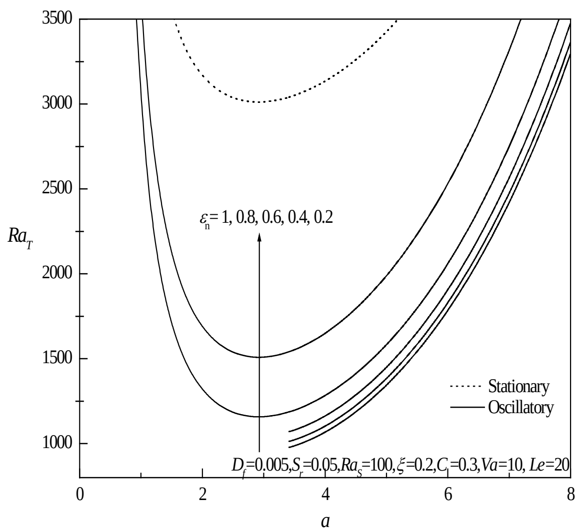

Fig.8. Neutral stability curves for different values of normalized porosity parameter ε_n

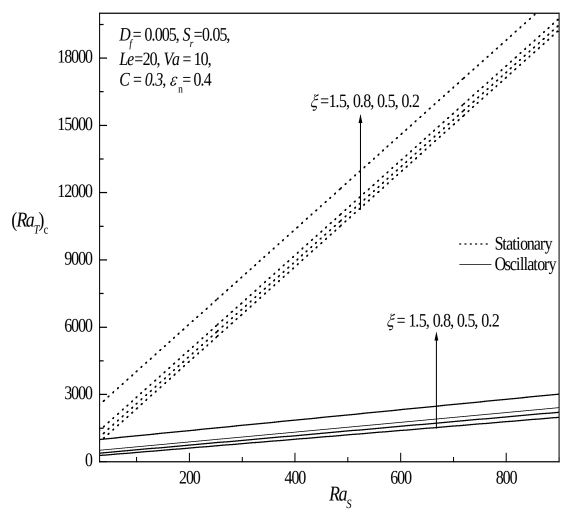

Fig.9. Variation of critical Rayleigh number with solute rayleigh number Ra_S for different values of mechanical anisotropy parameter ξ

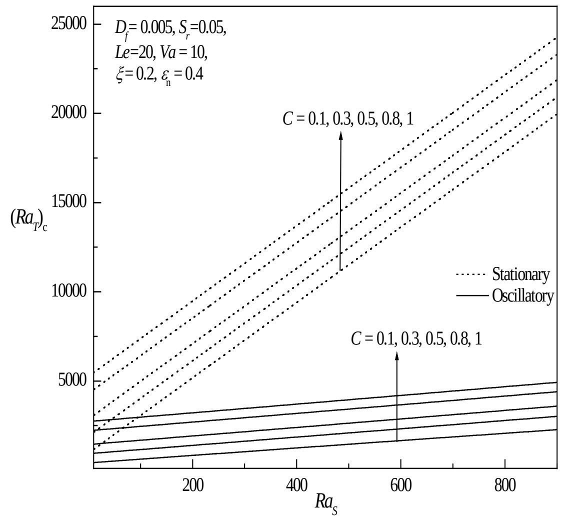

Fig.10. Variation of critical Rayleigh number with solute rayleigh number Ra_S for different values of couple stress parameter C

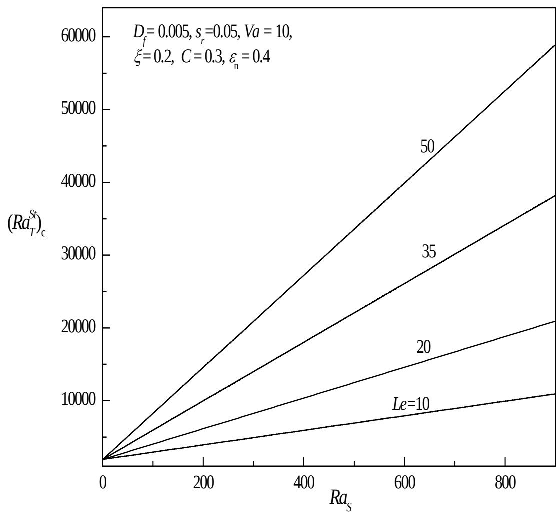

Fig.11(a). Variation of Stationary critical Rayleigh number with solute rayleigh number Ra_S for different values of Lewis number Le

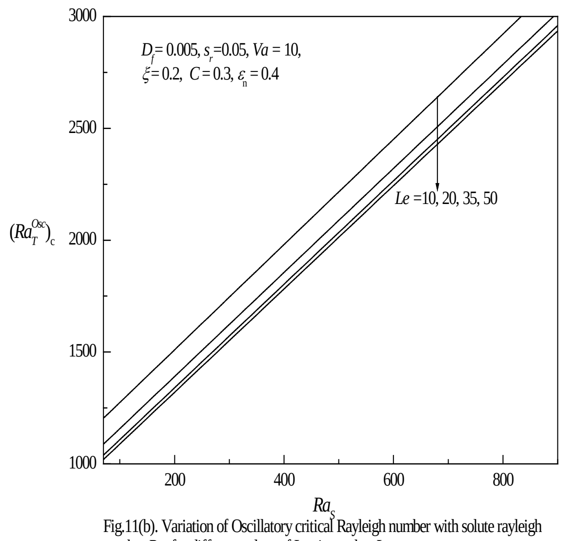

Fig.11(b). Variation of Oscillatory critical Rayleigh number with solute rayleigh number Ra_S for different values of Lewis number Le

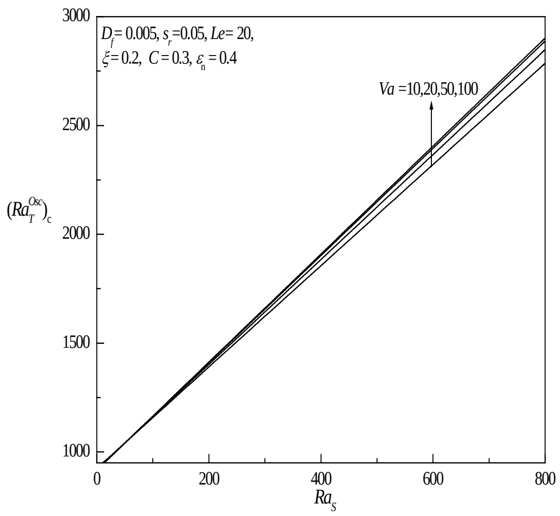

Fig.12. Variation of Oscillatory critical Rayleigh number with solute rayleigh number Ra_S for different values of Vadasz number Va

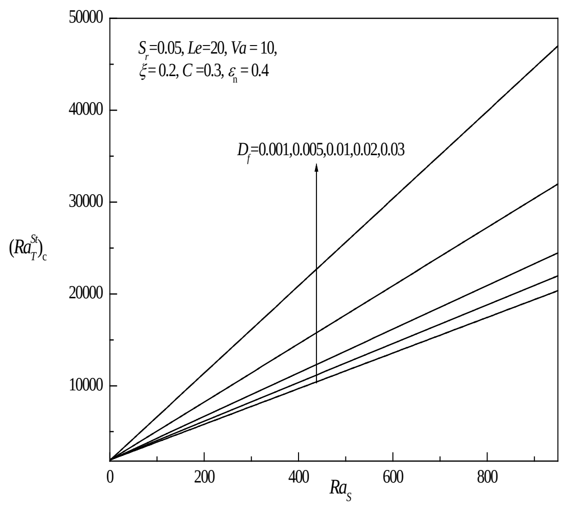

Fig.13(a). Variation of Stationary critical Rayleigh number with solute rayleigh number Ra_S for different values of Dufour parameter D_f

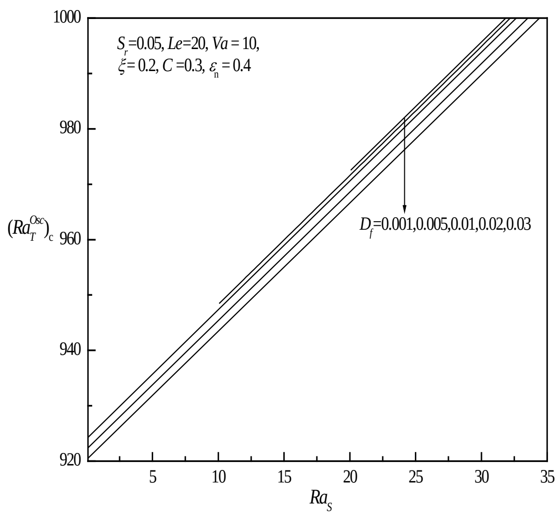

Fig.13(b).Variation of Oscillatory critical Rayleigh number with solute rayleigh number Ra_S for different values of Dufour parameter D_f

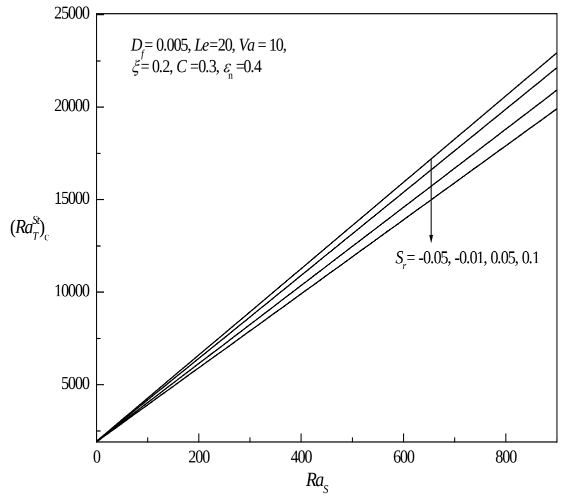

Fig.14(a).Variation of Stationary critical Rayleigh number with solute rayleigh number Ra_S for different values of Soret parameter S_r

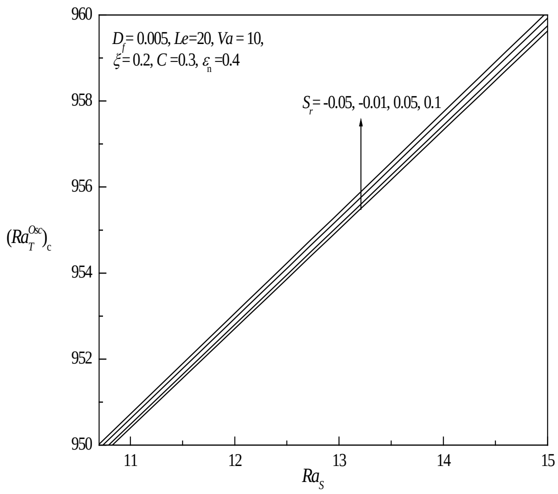

Fig.14(b). Variation of Oscillatory critical Rayleigh number with solute rayleigh number Ra_S for different values of Soret parameter S_r

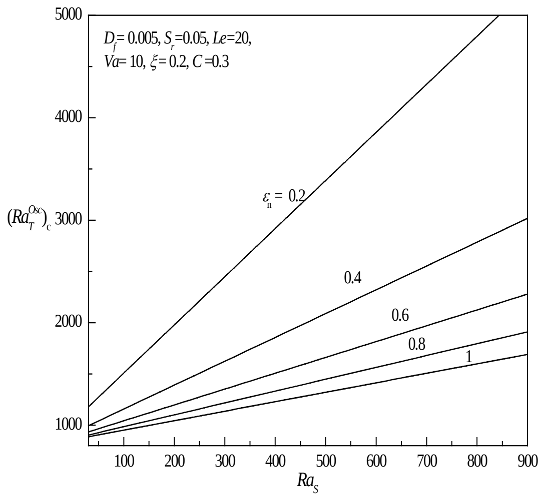

Fig.15. Variation of Oscillatory critical Rayleigh number with solute rayleigh number Ra_S for different values of normalized porosity parameter ε_n

REFERENCES

[1] Castinel, G., and Combarnous, M., Critere d' apparitition de la convection naturelle dans une couche poreuse nisotrope horizontale, C. R. Acad. Sci., B 278, 701–704, 1974.

.

[2] Eringen, A.C., Theory of micropolar fluids, J. Math. Mech., 16, 1, 1966.

[3] Gaikwad, S.N., Malashetty, M.S., and Prasad, K.R., An analytical study of linear and nonlinear double diffusive convection in a fluid saturated anisotropic porous layer with Soret effect, Appl. Math. Model, 33, 3617–3635, 2009.

[4] Gaikwad, S.N., Malashetty, M.S., and Prasad, K.R., Linear and nonlinear double diffusive convection in a fluid saturated anisotropic porous layer with cross diffusion effects, Transp. Porous Media, 80, 537–560, 2009.

[5] Govender, S., Coriolis effect on the stability of centrifugally driven convection in a rotating anisotropic porous layer subject to gravity, Transp. Porous Media, 69, 55–66, 2007.

[6] Horton, C.W., and Rogers, F.T., Convection currents in a porous medium, J. Appl. Phys., 16, 367–370, 1945.

[7] Ingham, D.B., and Pop, I., Transport Phenomena in Porous Media, Pergamon Press, Oxford, 1998.

[8] Ingham, D.B., and Pop, I., Transport Phenomena in Porous Media, Vol. III, Elsevier, Oxford, 2005.

[9] Kafoussias, N.G., and Williams, E.W., Thermal-diffusion and diffusion-thermo effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscosity, Int. J. Eng. Sci., 33, 1369–1384, 1995.

[10] Knobloch, E., Convection in binary fluids, Phys. Fluids, 23(9), 1918-1920, 1980.

[11] Lapwood, E.R., Convection of a fluid in a porous medium, Proc. Camb. Philol. Soc., 44, 508–521, 1948.

[12] McKibbin, R., Convection and heat transfer in layered and anisotropic porous media, In: Quintard, M., Todorovic, M. (eds.) Heat and Mass Transfer in Porous Media, Elsevier, Amsterdam, 327–336, 1992.

[13] Malashetty, M.S., Dulal, P., and Premila, K., Double diffusive convection in a Darcy porous medium saturated with couple stress fluid, Fluid Dyn. Res., 42, 035502–035523, 2010.

[14] Malashetty, M.S., and Swamy, M., The onset of convection in a binary fluid saturated anisotropic porous layer, Int. J. Therm. Sci., 49, 867–878, 2010.

[15] Malashetty, M.S., and Biradar, B.S., The onset of double diffusive convection in a binary Maxwell fluid saturated porous layer with cross-diffusion effects, Phys. Fluids, 23, 064109, 2011.

[16] Malashetty, M.S., and Premila Kollur, The onset of double diffusive convection in a couple stress fluid saturated anisotropic porous layer, Trans.Porous Med., 86, 465-489, 2011.

[17] Mansour, A., Amahmid, A., Hasnaoui, M., and Bourie, M., Multiplicity of solutions induced by thermosolute convection in a square porous cavity heated from below and subjected to horizontal concentration gradient in the presence of Soret effect, Numer. Heat Transf., 49, 69–94, 2006.

[18] Nield, D.A., and Bejan, A., Convection in Porous Media, third ed., Springer, New York, 2006.

[19] Rudraiah, N., and Malashetty, M.S., The influence of coupled molecular diffusion on double diffusive convection in a porous medium, ASME J. Heat Transf., 108, 872–876, 1986.

[20] Rudraiah, N., and Siddheshwar, P.G., A weak nonlinear stability analysis of double diffusive convection with cross-diffusion in a fluid saturated porous medium, Heat Mass Transf., 33, 287–293, 1998.

[21] Sharma, R.C., and Thakur, K.D., On couple stress fluid heated from below in porous medium in hydrodynamics, Czechoslov. J. Phys., 50(6), 753–758, 2000.

[22] Shivakumara, I.S., Onset of convection in a couple stress fluid saturated porous medium: effects of non uniform temperature gradients, Arch. Appl. Mech., 419, 347–355, 2009.

[23] Siddheshwar, P.G., and Pranesh, S., An analytical study of linear and non-linear convection in Boussinesq-Stokes suspensions, Int. J. Non-Linear Mech., 39, 165–172, 2004.

[24] Stokes, V.K., Couple stresses in fluids, Phys. Fluids, 9, 1709-1716, 1966.

[25] Storesletten, L., Effects of anisotropy on convective flow through porous media, In: Ingham, D.B., Pop, I. (eds.) Transport Phenomena in Porous Media, Pergamon Press, Oxford, 261–283, 1998.

[26] Storesletten, L., Effects of anisotropy on convection in horizontal and inclined porous layers, In: Ingham, D.B., et al. (eds.) Emerging Technologies and Techniques in Porous Media, Kluwer Academic Publishers, Netherlands, 285–306, 2004.

[27] Sunil, R.C., Sharma, R.S., and Chandel, effect of suspended particles on couple-stress fluid heated and soluted from below in porous medium, J. Porous Media, 7(1), 9–18, 2004.

[28] Tyvand, P.A., and Storesletten, L., Onset of convection in an anisotropic porous medium with oblique principal axis, J. Fluid Mech., 226, 371–382, 1991.

[29] Vadasz, P., Coriolis effect on gravity-driven convection in a rotating porous layer heated from below, J. Fluid Mech., 376, 351–375, 1998.

[30] Vadasz, P., Emerging Topics in Heat and Mass Transfer in Porous Media, Springer, New York, 2008.

[31] Vafai, K., Handbook of Porous Media, Marcel Dekker, New York, 2000.

[32] Vafai, K., Handbook of Porous Media, Taylor & Francis/CRC Press, London/Boca Raton, FL, 2005.

[33] Wang, S.W., and Tan, W.C., Stability analysis of Soret-driven double-diffusive convection of Maxwell fluid in a porous medium, Int. J. Heat Fluid Flow, 32, 88–94, 2011.

Source of support: UGC, Research Fellowship in Science for Meritorious Students (RFSMS) under UGC-BSR Scheme No.4-1/2010(BSR), Conflict of interest: None Declared