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QUARTER-SYMMETRIC NON -METRIC CONNECTION ON A KäHLER MANIFOLDS

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ABSTRACT

In this paper we study Kähler manifolds equipped with quarter -symmetric non -metric connection. It has been shown that a contravariant almost analytic vector field with respect to a Riemannian connection D will be contravariant almost analytic with respect to the quarter -symmetric non- metric connection ∇ . we have also shown that the Nijenhuis tensor on a Kähler manifold with respect to quarter -symmetric non- metric connection identically vanishes.

Keywords: *Quarter- symmetric connection, Kähler manifold, Nijenhuis tensor, contravariant almost analytic vector field.*

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1. INTRODUCTION:

In 1975, Golab introduced the notion of quarter symmetric connection in a Riemannian manifold with affine connection. This was further developed by Yano and Imai (1982), Rastogi (1978, 1987), Mishra and Pandey (1980), Mukhopadhyay Ray and Barua (1980), Biwas and De (1997), Sengupta and Biswas (2003), Singh and Pandey (2007) and many other geometers.

Let M^n be an even dimensional differentiable manifold of differentiability class C^{r+1}. if there exists a vector valued linear function F of differentiability class C^r such that for any vector field X

$$\bar{\bar{X}} + X = 0\,, \tag{1.1}$$

$$g(\bar{X},\bar{Y}) = g(X,Y)\,, \tag{1.2}$$

$$(D_X F)(Y) \;=\; 0, \tag{1.3}$$

where $\bar{X} = FX, g$ is non -singular metric tensor and D is a Riemannian connection, then M^n is called Kähler manifold.

A linear connection ∇ on (M^n, g) is said to be quarter-symmetric non- metric connection if the torsion tensor T of the connection ∇ and the metric tensor g of the manifold satisfy the following condition [1]:

$$(\nabla_X g)(Y,Z) = -u(Y)g(\bar{X},Z) - u(Z)g(\bar{X},Y) \tag{1.4}$$

$$T(X,Y) = u(Y)\bar{X} - u(X)\bar{Y}, \tag{1.5}$$

for arbitrary vector fields X and Y.
The symbol u stand for a 1-form associated with the vector field U on M^n by

$$u(X) = g(U,X)$$

The relation between the quarter-symmetric non- metric connections ∇ and the Riemannian connection D is given by

$$\nabla_X Y = D_X Y + u(Y)\bar{X}. \tag{1.6}$$

2. SOME THEOREMS ON A KäHLER MANIFOLD WITH RESPECT TO QUARTER – SYMMETRIC NON-METRIC CONNECTION ∇:

Let us consider a Kähler manifold M^n equipped with a quarter- symmetric non- metric connection ∇ and define

(i) $`F(Y,Z) = g(\bar{Y},Z)$ (2.1)
(ii) $`T(X,Y,Z) = g(T(X,Y),Z)$
(iii) $H(X,Y) = u(Y)\bar{X}$
(iv) $`H(X,Y,Z) = g(H(X,Y),Z)$.

In view of (2.1)(iii) , the equation (1.6) becomes

$$\nabla_X Y = D_X Y + H(X,Y).$$

Barring Y in (1.6), we have

$$(\nabla_X F)(Y) = (D_X F)(Y) + \overline{D_X Y} + u(\bar{Y})\bar{X} - \overline{\nabla_X Y}. \tag{2.2}$$

Operating F on both side of the equation (1.6) , we have

$$\overline{\nabla_X Y} = \overline{D_X Y} - u(Y)X, \tag{2.3}$$

Using (1.3) and(2.3) in (2.2), we get

$$(\nabla_X F)(Y) = u(\bar{Y})\bar{X} + u(Y)X\,, \tag{2.4}$$

Barring X and Y in (2.4), we get

$$(\nabla_{\bar{X}} F)(\bar{Y}) = u(Y)X + u(\bar{Y})\bar{X},$$

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Hence, we can state the following:

Theorem (2.1): On a Kähler manifold equipped with a quarter- symmetric non- metric connection ∇ satisfies the following relation:

(i) $(\nabla_{\bar{X}}F)\bar{Y} = (\nabla_X F)(Y)$ and
(ii) $(\nabla_X F)(Y) = 0$ if and only if $u(Y)X = u(\bar{Y})\bar{X}$.

3. NIJENHUIS TENSOR ON A KäHLER MANIFOLD WITH RESPECT TO QUARTER – SYMMETRIC NON-METRIC CONNECTION∇:

The Nijenhuis tensor with respect to the quarter symmetric non – metric connection ∇ is given by

$$N^*(X,Y) = (\nabla_{\bar{X}}F)Y - (\nabla_{\bar{Y}}F)X - \overline{(\nabla_X F)Y} + \overline{(\nabla_Y F)X}. \quad (3.1)$$

From equation (2.4) and (1.1), we have

$$(\nabla_{\bar{X}}F)Y = u(Y)\bar{X} - u(\bar{Y})X. \quad (3.2)$$

Interchanging X and Y in (3.2), we get

$$(\nabla_{\bar{Y}}F)X = u(X)\bar{Y} - u(\bar{X})Y. \quad (3.3)$$

Operating F on both side of the equation (2.4) and using (1.1), we get

$$\overline{(\nabla_X F)Y} = u(Y)\bar{X} - u(\bar{Y})X. \quad (3.4)$$

Interchanging X and Y in (3.4), we have

$$\overline{(\nabla_Y F)(X)} = u(X)\bar{Y} - u(\bar{X})Y. \quad (3.5)$$

Using (3.2), (3.3), (3.4), (3.5) in (3.1), we get.

$$N^*(X,Y) = 0.$$

Hence, we can state the following:.

Theorem 3.1: On a Kähler manifold, Nijenhuis tensor with respect to quarter- symmetric non – metric connection vanishes i.e.

$$N^*(X,Y) = 0.$$

4. CONTRAVARIANT ALMOST ANALYTIC VECTOR FIELDS ON A KäHLER MANIFOLD WITH RESPECT TO QUARTER – SYMMETRIC NON-METRIC CONNECTION ∇ :

A vector filed V is said to be contravariant almost analytic if the Lie derivative of F with respect to V vanishes identically. i.e.

$$(L_V F)X = 0,\text{for all } X. \quad (4.1)$$

The equation (4.1) is equivalent to the equation

$$[V,\bar{X}] = \overline{[V,X]} \quad (4.2)$$

In a Kähler manifold, the equation (4.2) becomes

$$(D_{\bar{X}}V) - \overline{D_X V} = 0 \text{ if and only if } \overline{(D_{\bar{X}}V)} + D_X V = 0. \quad (4.3)$$

For any vector field V, the equation (1.5) gives

$$\nabla_X V = D_X V + u(V)\bar{X} \quad (4.4)$$

On barring$X$, equation (4.4) becomes

$$\nabla_{\bar{X}}V = D_{\bar{X}}V + u(V)\bar{\bar{X}} = D_{\bar{X}}V - u(V)X$$

$$\nabla_{\bar{X}}V = D_{\bar{X}}V - u(V)X. \quad (4.5)$$

Operating F on both sides of the equation (4.4) , we have

$$\overline{\nabla_X V} = \overline{D_X V} - u(V)X \quad (4.6)$$

Subtracting the equation (4.6) from (4.5), we get

$$\nabla_{\bar{X}}V - \overline{\nabla_X V} = D_{\bar{X}}V - \overline{D_X V}.$$

Since V is contravariant almost analytic with respect to connection D, we have
$D_{\bar{X}}V - \overline{D_X V} = 0$, and then $\nabla_{\bar{X}}V - \overline{\nabla_X V} = 0$, therefore V is also contravariant almost analytic with respect to connection ∇.

Hence, we can state the following:

Theorem (4.1): On a Kähler manifold a contravariant almost analytic vector V with respect to Riemannian connection D is also contravariant almost analytic with respect to quarter-symmetric non-metric connection ∇.

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