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A FUZZY APPROACH TO NON-PREEMPTED PRIORITY QUEUES

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ABSTRACT

This paper proposes a procedure to construct the membership functions of the performance measures for 3-priority queues when the interarrival time and service time are fuzzy numbers with the application of fuzzy set theory. The basic idea is to reduce a fuzzy queue in to a family of crisp queues by applying α-cut approach and Zadeh's extension principle. A pair of parametric programs is formulated to describe the family of crisp queues, via which the membership functions of the performance measures are derived. Trapezoidal fuzzy numbers are used to demonstrate the validity of the proposal. An illustration is given to establish the performance measures of the characteristics of 3-priority queues.

Keywords: *Fuzzy sets, Membership functions, Priority Queues, Mathematical Programming, Trapezoidal Fuzzy numbers, Perfomance measures and Non-Preemption.*

AMS Subject classification code: *68M20.*

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1. INTRODUCTION

The problem of fuzzy queues has been analysed by Zadeh, L.A [4] and Li and Lee [2] through the use of the extension principle. However, these approaches are fairly complicated and are generally unsuited for computational purposes.

Recent developments on fuzzy numbers by random variables can be used to analyse the queueing system. For example, Zadeh,L.A[4] has introduced the concept of fuzzy probabilities and the properties of fuzzy probability Markov chains were discussed.

Fuzzy queueing model have been described by such researchers like Negi and Lee [3], Kao et al [1] and Chen [1] have analysed fuzzy queues using Zadeh's extension principle. Koa et al has constructed the membership functions of the system characteristics for the fuzzy queues using parametric linear programming.

Although Poisson arrival in a queueing system is a fairly reasonable assumption, the service rate is really more possibilistic than probabilistic. Zadeh's extension principle forms the basic approach for this investigation of these fuzzified stochastic processes.

2. FUZZY SET THEORY

Definition2.1: Let Z denote a universe of discourse. Then a fuzzy set $\widetilde{A}$ in Z is characterized by a membership function as follows, $\mu_{\widetilde{A}}: Z \rightarrow [0,1]$, which assigns to each element x in Z, a real number $\mu_{\widetilde{A}}(x)$ is in the interval [0,1].Thus, the function value of $\mu_{\widetilde{A}}(x)$ is termed as the grade of membership of x in $\widetilde{A}$.

Definition 2.2: If a fuzzy set $\tilde{A}$ is defined on X, for any $\alpha \in [0,1]$, the α-cuts of the fuzzy set $\tilde{A}$ is represented by $\tilde{A}_\alpha = \{x/\mu_{\widetilde{A}}(x) \geq \alpha, x \in Z\} = \{l_{\widetilde{A}}(\alpha), u_{\widetilde{A}}(\alpha)\}$, $\tilde{A}_\alpha$ is a non-empty bounded interval contained in Z, $l_{\widetilde{A}}(\alpha)$ and $u_{\widetilde{A}}(\alpha)$ represent the lower bound and upper bound of the α-cut of $\widetilde{A}$ respectively.

Definition 2.3: A fuzzy set $\widetilde{A}$ is convex subset of Z if and only if $\mu_{\widetilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2))$ for all $x_1, x_2 \in X$ and $\lambda \in [0,1]$.

3. TRAPEZOIDAL FUZZY NUMBER

The trapezoidal fuzzy number is usually defined as $\tilde{A}$ = [a_2-d_1, a_2, a_3, a_3+d_2]. The membership function for the trapezoidal number $\tilde{A}$ = [a_1, a_2, a_3, a_4] is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ \frac{a_4-x}{a_4-a_3}, & a_3 \leq x \leq a_4 \end{cases}$$

4. NON-PREEMPTIVE SYSTEMS WITH MANY CLASSES

Suppose that customers of the k^{th} priority (the smaller the number, the higher the priority) arrive at a single channel queue according to a poisson process with λ_k (k = 1,2,3,…r) and that these customers wait on a first-come, first-served within their respective priorities.Let the service distribution for the k^{th} –priority be exponential with mean $\frac{1}{\mu_k}$. A unit that begins service and completes its service before another item is admitted, regardless of priorities.

We begin $\rho_k = \frac{\lambda_k}{\mu_k}$ $(1\leq k \leq r)$, $\sigma_k = \sum_{i=1}^{k} \rho_i$ $(\sigma_o \equiv 0, \sigma_r \equiv \rho)$ (1)

The system is stationary for $\sigma_r = \rho < 1$.

Let $\mu_{\tilde{\lambda}}(x)$ and $\mu_{\tilde{\mu}}(y)$ are membership functions of arrival rate and service rate respectively. We have $\tilde{\lambda}$ = {x , $\mu_{\tilde{\lambda}}(x)$ /x∈ S($\tilde{\lambda}$)} and $\tilde{\mu}$ = {y , $\mu_{\tilde{\mu}}(y)$ /y ∈ S($\tilde{\mu}$)} where S($\tilde{\lambda}$) and S($\tilde{\mu}$) are the supports of $\tilde{\lambda}$ and $\tilde{\mu}$ which denote the universal set of arrival rate and service rate respectively.

Clearly when $\tilde{\lambda}$ and $\tilde{\mu}$ are fuzzy numbers then the performance measure $\tilde{\rho}(\tilde{\lambda}, \tilde{\mu})$ are also fuzzy as well. On the basis of Zadeh's extension principle [4] ,the membership function of the performance measure is defined as

$$\tilde{\mu}_{\tilde{\rho}(\tilde{\lambda},\tilde{\mu})}(Z) = sup_{x\in X, y\in Y} \min\{\mu_{\tilde{\lambda}}(x), \mu_{\tilde{\mu}}(y) / Z = \rho(x,y)\} \tag{2}$$

Without loss of generality let us assume that the performance measures of interest for 3-priority queues.From the knowledge of traditional queueing theory [5] under the steady-state conditions $\rho k = \frac{\lambda_k}{\mu_k} < 1$, the expected queue size is

$$\sum_{i=1}^{r} L_q^{(i)} = \sum_{i=1}^{r} \frac{\lambda_i \sum_{k=1}^{r} \frac{\rho_k}{\mu_k}}{(1-\sigma_{i-1})(1-\sigma_i)} \tag{3}$$

And by using Little's formula, the waiting time in queue

$$W_q = \sum_{i=1}^{r} \frac{\lambda_i W_q^{(i)}}{\lambda} \tag{4}$$

$$\text{where } W_q^{(i)} = \sum_{i=1}^{k} \frac{\sum_{k=1}^{r} \frac{\rho_k}{\mu_k}}{(1-\sigma_{i-1})(1-\sigma_i)} \tag{5}$$

5. MATHEMATICAL FORMULATION

Consider a single server FM/FM/I queueing system with 3-priority queues. The interarrival times $\widetilde{A_i}$, i=1,2,3 of units in the first, second and third priority queues and service time $\tilde{S}$ are approximately known and are represented by the following fuzzy sets.

$$\widetilde{A_i} = \{(x, \mu_{\widetilde{A_i}}(x)) / x \in X\}, i = 1,2,3 \tag{6}$$

$$\tilde{S} = \{(y, \mu_{\tilde{S}}(y)) / y \in Y\} \tag{7}$$

where X and Y are crisp universal sets of the interarrival times and $\mu_{\widetilde{A_i}}(x)$, i=1,2,3 and $\mu_{\tilde{S}}(y)$) are the respective membership functions. The α-cut of $\widetilde{A}_i$, i=1,2,3 and $\tilde{S}$ are

$$A_i(\alpha) = \{x \in X / \mu_{\tilde{A}_i}(x) \geq \alpha\}, i=1,2,3 \tag{8}$$

$$S(\alpha) = \{y \in Y / \mu_{\tilde{S}}(y) \geq \alpha\} \tag{9}$$

where 0<α≤1. All $A_i(\alpha)$, i=1,2,3 and S(α) are the crisp sets. Using α-cut, the interarrival times and service times can be represented by different levels of confidence intervals [0,1]. Hence a fuzzy queue can be reduced to a family of crisp queues with different α-cuts $\{A_i(\alpha)/0<\alpha\leq 1\}$,i=1,2,3 and $\{S(\alpha)/0<\alpha\leq 1\}$. These two sets represent sets of movable boundaries and they form nested structure [Zimmermann] for expressing the relationship between the crisp sets and fuzzy sets. Let the confidence intervals of the fuzzy sets $\tilde{A}_i$,i=1,2,3 and $\tilde{S}$ be $[l_{A_i(\alpha)}, u_{A_i(\alpha)}]$, i =1,2,3 and $[l_{S(\alpha)}, u_{S(\alpha)}]$ respectively. Since all the interarrival times $\tilde{A}_i$, i=1, 2,3 and $\tilde{S}$ are fuzzy numbers, using Zadeh's extension principle [1,4], the membership function of the performance measure $p(\tilde{A}_i,\tilde{S})$,i=1,2,3 is defined as

$$\mu_{p(\tilde{A}_i,\tilde{S})}(z) = \sup_{x\in X, y\in Y} \min\{\mu_{\tilde{A}_i}(x), \mu_{\tilde{S}}(y) / z=p(x,y)\}, i=1,2,3 \tag{10}$$

Construction of the membership function $\mu_{p(\tilde{A}_i,\tilde{S})}(z)$, i=1,2,3 is equivalent to say that the derivation of α-cuts of $\mu_{p(\tilde{A}_i,\tilde{S})}$. From equation (10), the equation $\mu_{p(\tilde{A}_i,\tilde{S})}(z)$ =α,i=1,2,3 is true only when either $\mu_{\tilde{A}_i}(x)$ =α, $\mu_{\tilde{S}}(y)\geq\alpha$ (or) $\mu_{\tilde{A}_i}(x))\geq\alpha$, $\mu_{\tilde{S}}(y)=\alpha$ is true.

The parametric programming problems have the following form,

$$l_{p(\alpha)} = \min p(x,y) \tag{11}$$
$$\text{such that} \quad l_{A_i(\alpha)} \leq x \leq u_{A_i(\alpha)}, i=1,2,3$$
$$l_{S(\alpha)} \leq y \leq u_{S(\alpha)}$$

and

$$u_{p(\alpha)} = \max p(x,y) \tag{12}$$
$$\text{such that} \quad l_{A_i(\alpha)} \leq x \leq u_{A_i(\alpha)}, i=1,2,3$$
$$l_{S(\alpha)} \leq y \leq u_{S(\alpha)}$$

If both $l_{p(\alpha)}$ and $u_{p(\alpha)}$are invertible with respect to α, then the left shape function $L(z)=(l_{p(\alpha)})^{-1}$and the right shape function $R(z) = (u_{p(\alpha)})^{-1}$can be obtained, from which the membership function $\mu_{p(\tilde{A}_i,\tilde{S})}(z)$, i=1,2,3 is constructed as

$$\mu_{p(\tilde{A}_i,\tilde{S})}(z) = \begin{cases} L(z), & for\ z_1 \leq z \leq z_2 \\ 1 , & for\ z_2 \leq z \leq z_3 \\ R(z), & for\ z_3 \leq z \leq z_4 \end{cases} \tag{13}$$

where $z_1 \leq z_2 \leq z_3 \leq z_4$ and $L(z_1) = R(z_4) =0$ for the trapezoidal fuzzy number.

Using the concept of α-cut the FM/FM/I queue with 3-priority queues can be reduced as M/M/I queue with 3-priority queues with service rates ,ie , $\mu_1 = \mu_2 = \mu_3 = \mu$.Further $\rho_1 = \frac{\lambda_1}{\mu_1}$, $\rho_2 = \frac{\lambda_2}{\mu_2}$, $\rho_3 = \frac{\lambda_3}{\mu_3}$

$$\left.\begin{array}{r} \text{since } \rho = \rho_1+\rho_2+\rho_3,\ \rho = \frac{\lambda}{\mu},\ \lambda = \lambda_1+\lambda_2+\lambda_3,\ \rho = \frac{\lambda_1+\lambda_2+\lambda_3}{\mu} \\ \sigma_k = \sum_{i=1}^{k} \rho_i\ ,\ \sigma_o = 0 \end{array}\right\} \tag{14}$$

From (5),
$$W_q^{(i)} = \frac{\frac{(\rho_1+\rho_2+\rho_3)}{\mu}}{(1-\sigma_{i-1})(1-\sigma_i)} \tag{15}$$

From which, we can deduce that

$$W_q^{(1)} = \frac{\lambda}{\mu(\mu-\lambda_1)} \tag{16}$$

$$W_q^{(2)} = \frac{\lambda}{(\mu-\lambda_1)[\mu-(\lambda_1+\lambda_2)]} \tag{17}$$

$$W_q^{(3)} = \frac{\lambda}{[\mu-(\lambda_1+\lambda_2)](\mu-\lambda)} \tag{18}$$

$$L_q^{(1)} = \frac{\left(\frac{\lambda_1+\lambda_2+\lambda_3}{\mu}\right)\left(\frac{\lambda_1}{\mu}\right)}{\left(1-\frac{\lambda_1}{\mu}\right)} \tag{19}$$

$$L_q^{(2)} = \frac{\left(\frac{\lambda_1+\lambda_2+\lambda_3}{\mu}\right)\left(\frac{\lambda_2}{\mu}\right)}{\left(1-\frac{\lambda_1}{\mu}\right)\left(1-\frac{\lambda_1+\lambda_2}{\mu}\right)} \tag{20}$$

$$L_q^{(3)} = \frac{\left(\frac{\lambda_1+\lambda_2+\lambda_3}{\mu}\right)\left(\frac{\lambda_3}{\mu}\right)}{\left(1-\frac{\lambda_1+\lambda_2}{\mu}\right)\left(1-\frac{\lambda_1+\lambda_2+\lambda_3}{\mu}\right)} \tag{21}$$

where λ_1,λ_2,and λ_3 are the arrival rates of first,second and third priority units respectively and μ is the service rate.

If the functions $l_{p(\alpha)}$ and $u_{p(\alpha)}$ are not invertible with respect to α, the membership functions $\mu_{\mathrm{p}(\tilde{A}_i,\tilde{S})}(z)$ cannot be derived. But we can trace the graph of $\mu_{\mathrm{p}(\tilde{A}_i,\tilde{S})}$(z) from the α-cuts of $[l_{p(\alpha)}, u_{p(\alpha)}]$. This procedure can be applied and the membership functions $\mu_{\mathrm{p}(\tilde{A}_i,\tilde{S})}$(z) for the queueing model with 3-priority queues can be obtained.

6. NUMERICAL EXAMPLE

Expected waiting time and expected number of customer in the queue for FM/FM/I queue with 3-priority classes.

Suppose that the rates of first, second and third priority with the same service rates are fuzzy numbers represented by

$\widetilde{A_1}$ = [2,3,5,6], $\widetilde{A_2}$ = [3,4,6,7], $\widetilde{A_3}$ = [4,5,7,8] and $\tilde{S}$ = [22,23,25,26] per hour respectively. The α-cut of the membership functions $\mu_{\widetilde{A_1}}(\alpha)$, $\mu_{\widetilde{A_2}}(\alpha)$, $\mu_{\widetilde{A_3}}(\alpha)$ and $\mu_{\tilde{S}}(\alpha)$ are [2+α ,6-α], [3+α ,7-α], [4+α ,8-α] and [22+α ,26-α] respectively. From equations (11) and (12) the parametric programming problems are formulated to derive the membership functions $\bar{L}_{q_1}$, $\bar{L}_{q_2}$, $\bar{L}_{q_3}$, $\bar{W}_{q_1}$, $\bar{W}_{q_2}$ and $\bar{W}_{q_3}$. They are calculated as follows.

The performance functions of

(i). $\bar{L}_{q_1}$ - average queue length of first priority
(ii). $\bar{L}_{q_2}$ -average queue length of second priority
(iii). $\bar{L}_{q_3}$- average queue length of third priority
(iv) $\bar{W}_{q_1}$ -average waiting time of units of first priority in the queue.
(v) . $\bar{W}_{q_2}$ -average waiting time of units of second priority in the queue.
(vi) $\bar{W}_{q_3}$ - average waiting time of units of third priority in the queue.

are derived from the respective parametric programs. These differ only in their objective functions and are listed below.

$$\left.\begin{array}{l} \text{(i)} \quad l_{L_{q1}(\alpha)} = \min\left\{ \frac{\left(\frac{\lambda_1+\lambda_2+\lambda_3}{\mu}\right)\left(\frac{\lambda_1}{\mu}\right)}{\left(1-\frac{\lambda_1}{\mu}\right)} \right\} \\ \text{such that} \quad 2+\alpha \le \lambda_1 \le 6-\alpha \\ \qquad 3+\alpha \le \lambda_2 \le 7-\alpha \\ \qquad 4+\alpha \le \lambda_3 \le 8-\alpha \end{array}\right\} \tag{22}$$

and

$$\left.\begin{array}{l} \text{(ii)} \quad u_{L_{q1}(\alpha)} = \max\left\{ \frac{\left(\frac{\lambda_1+\lambda_2+\lambda_3}{\mu}\right)\left(\frac{\lambda_1}{\mu}\right)}{\left(1-\frac{\lambda_1}{\mu}\right)} \right\} \\ \text{such that} \quad 2+\alpha \le \lambda_1 \le 6-\alpha \\ \qquad 3+\alpha \le \lambda_2 \le 7-\alpha \\ \qquad 4+\alpha \le \lambda_3 \le 8-\alpha \end{array}\right\} \tag{23}$$

where $0 < \alpha \le 1$.

$l_{L_{q1}(\alpha)}$ is found when λ_1, λ_2 and λ_3 approach their lower bounds and μ approaches its upper bound . Consequently the optimal solution for (22) is

$$l_{L_{q1}(\alpha)} = \frac{18+15\alpha+3\alpha^2}{624-76\alpha+2\alpha^2} \tag{24}$$

also $u_{L_{q1}(\alpha)}$ is found when λ_1, λ_2 and λ_3 approach their upper bounds and μ approaches its lower bound .In this case, the optimal solution for (23) is

$$u_{L_{q1}(\alpha)} = \frac{126-39\alpha+3\alpha^2}{352+60\alpha+2\alpha^2} \tag{25}$$

The membership function $\mu_{\bar{L}_{q1}(z)} = \begin{cases} L(z), & [l_{L_{q1}(\alpha)}]_{\alpha=0} \leq z \leq [l_{L_{q1}(\alpha)}]_{\alpha=1} \\ 1, & [l_{L_{q1}(\alpha)}]_{\alpha=1} \leq z \leq [u_{L_{q1}(\alpha)}]_{\alpha=1} \\ R(z), & [u_{L_{q1}(\alpha)}]_{\alpha=1} \leq z \leq [u_{L_{q1}(\alpha)}]_{\alpha=0} \end{cases}$

which is estimated as

$$\mu_{\bar{L}_{q1}(z)} = \begin{cases} \dfrac{(76z+15)-(784z^2+9912z+9)^{\frac{1}{2}}}{2(2z-3)}, & .028846 \leq z \leq .065455 \\ 1, & .065455 \leq z \leq .217391 \\ \dfrac{-(60z+39)+(784z^2+9912z+9)^{\frac{1}{2}}}{2(2z-3)}, & .217391 \leq z \leq .357955 \end{cases} \tag{26}$$

Similarly the performance functions of $\bar{L}_{q2}$, $\bar{L}_{q3}$, $\bar{W}_{q1}$, $\bar{W}_{q2}$ and $\bar{W}_{q3}$ are derived from the respective parametric programs.These differ only in their objective functions and are listed in (20)-(21) and(16)-(18) with the constraints given with the equations (22) and (23) yield the following results:

$$l_{L_{q2}(\alpha)} = \frac{27+18\alpha+3\alpha^2}{504-114\alpha+6\alpha^2}\,;\; u_{L_{q2}(\alpha)} = \frac{147-42\alpha+3\alpha^2}{144+66\alpha+6\alpha^2} \tag{27}$$

$$\mu_{\bar{L}_{q2}(z)} = \begin{cases} \dfrac{(114z+18)-(900z^2+10800z)^{\frac{1}{2}}}{2(6z-3)}, & .053571 \leq z \leq .121212 \\ 1, & .121212 \leq z \leq 0.5 \\ \dfrac{-(66z+42)+(900z^2+10800z)^{\frac{1}{2}}}{2(6z-3)}, & 0.5 \leq z \leq 1.020833 \end{cases} \tag{28}$$

$$l_{L_{q3}(\alpha)} = \frac{36+21\alpha+3\alpha^2}{357-135\alpha+12\alpha^2}\,;\; u_{L_{q3}(\alpha)} = \frac{168-45\alpha+3\alpha^2}{9+39\alpha+12\alpha^2} \tag{29}$$

$$\mu_{\bar{L}_{q3}(z)} = \begin{cases} \dfrac{(135z+21)-(1089z^2+11682z+9)^{\frac{1}{2}}}{2(12z-3)}, & .10084 \leq z \leq .25641 \\ 1, & .25641 \leq z \leq 2.1 \\ \dfrac{-(39z+45)+(1089z^2+11682z+9)^{\frac{1}{2}}}{2(12z-3)}, & 2.1 \leq z \leq 18.66667 \end{cases} \tag{30}$$

$$l_{W_{q1}(\alpha)} = \frac{9+3\alpha}{624-76\alpha+2\alpha^2}\,;\; u_{W_{q1}(\alpha)} = \frac{21-3\alpha}{352+60\alpha+2\alpha^2} \tag{31}$$

$$\mu_{\bar{W}_{q1}(z)} = \begin{cases} \dfrac{(76z+3)-(784z^2+528z+9)^{\frac{1}{2}}}{4z}, & .014 \leq z \leq .022 \\ 1, & .022 \leq z \leq .043 \\ \dfrac{-(60z+3)+(784z^2+528z+9)^{\frac{1}{2}}}{4z}, & .043 \leq z \leq .060 \end{cases} \tag{32}$$

$$l_{W_{q2}(\alpha)} = \frac{9+3\alpha}{504-114\alpha+6\alpha^2}\,;\; u_{W_{q2}(\alpha)} = \frac{21-3\alpha}{144+66\alpha+6\alpha^2} \tag{33}$$

$$\mu_{\overline{W}_{q2}(z)} = \begin{cases} \dfrac{(114z+3)-(900z^2+900z+9)^{\frac{1}{2}}}{12z}, & .018 \le z \le .030 \\ 1, & .030 \le z \le .083 \\ \dfrac{-(66z+3)+(900z^2+900z+9)^{\frac{1}{2}}}{12z}, & .083 \le z \le .146 \end{cases} \tag{34}$$

$$l_{W_{q3}(\alpha)} = \frac{9+3\alpha}{357-135\alpha+12\alpha^2}\,;\; u_{W_{q3}(\alpha)} = \frac{21-3\alpha}{9+39\alpha+12\alpha^2} \tag{35}$$

$$\mu_{\overline{W}_{q3}(z)} = \begin{cases} \dfrac{(135z+3)-(1089z^2+1242z+9)^{\frac{1}{2}}}{24z}, & .025 \le z \le .051 \\ 1, & .051 \le z \le 0.3 \\ \dfrac{-(39z+3)+(1089z^2+1242z+9)^{\frac{1}{2}}}{24z}, & 0.3 \le z \le 2.33 \end{cases} \tag{36}$$

7. CONCLUSION

The α-cut approach is simple and straight forward. The biggest advantage of this approach is that it retains all the original fuzzy information for the designer to use. However, when there are more than one mutually independent fuzzy numbers, the family of problems increases exponentially and thus a large number of problems must be solved even for a relatively small number of α-cuts.

Fuzzy set theory has been applied to 3-priority queues. When the interarrival and service time are fuzzy variables, according to Zadeh's extension principle, the performance measures such as the system length, the waiting time will be fuzzy as well. This paper applies the concept of α-cut to reduce a fuzzy queue in to family of crisp queues which can be described by a pair of parametric programs to find the α-cuts of the membership functions of the performance measures. With the α-cuts, the membership functions are derived and the graphs to the corresponding measures are obtained.

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α-cuts of arrival rates,service rate, queue length and waiting time in queue of first, second and third priority

α	$l_{x_1(\alpha)}$	$u_{x_1(\alpha)}$	$l_{x_2(\alpha)}$	$u_{x_2(\alpha)}$	$l_{x_3(\alpha)}$	$u_{x_3(\alpha)}$	$l_{y(\alpha)}$	$u_{y(\alpha)}$	$l_{L_{q_1}(\alpha)}$	$u_{L_{q_1}(\alpha)}$	$l_{L_{q_2}(\alpha)}$	$u_{L_{q_2}(\alpha)}$	$l_{L_{q_3}(\alpha)}$	$u_{L_{q_3}(\alpha)}$	$l_{W_{q_1}(\alpha)}$	$u_{W_{q_1}(\alpha)}$	$l_{W_{q_2}(\alpha)}$	$u_{W_{q_2}(\alpha)}$	$l_{W_{q_3}(\alpha)}$	$u_{W_{q_3}(\alpha)}$
0	2	6	3	7	4	8	22	26	0.0288	0.3580	0.0536	1.0208	0.1008	18.6667	0.0144	0.0597	0.0179	0.1458	0.0252	2.3333
0.1	2.1	5.9	3.1	6.9	4.1	7.9	22.1	25.9	0.0317	0.3411	0.0585	0.9480	0.1110	12.5599	0.0151	0.0578	0.0189	0.1374	0.0271	1.5899
0.2	2.2	5.8	3.2	6.8	4.2	7.8	22.2	25.8	0.0347	0.3250	0.0638	0.8811	0.1220	9.2083	0.0158	0.0560	0.0199	0.1296	0.0290	1.1806
0.3	2.3	5.7	3.3	6.7	4.3	7.7	22.3	25.7	0.0379	0.3095	0.0695	0.8195	0.1340	7.1061	0.0165	0.0543	0.0210	0.1223	0.0312	0.9229
0.4	2.4	5.6	3.4	6.6	4.4	7.6	22.4	25.6	0.0412	0.2946	0.0755	0.7626	0.1472	5.6742	0.0172	0.0526	0.0222	0.1155	0.0335	0.7466
0.5	2.5	5.5	3.5	6.5	4.5	7.5	22.5	25.5	0.0448	0.2804	0.0819	0.7101	0.1615	4.6429	0.0179	0.0510	0.0234	0.1092	0.0359	0.6190
0.6	2.6	5.4	3.6	6.4	4.6	7.4	22.6	25.4	0.0485	0.2667	0.0888	0.6615	0.1772	3.8693	0.0186	0.0494	0.0247	0.1034	0.0385	0.5229
0.7	2.7	5.3	3.7	6.3	4.7	7.3	22.7	25.3	0.0524	0.2536	0.0962	0.6165	0.1944	3.2710	0.0194	0.0479	0.0260	0.0979	0.0414	0.4481
0.8	2.8	5.2	3.8	6.2	4.8	7.2	22.8	25.2	0.0565	0.2410	0.1040	0.5748	0.2132	2.7970	0.0202	0.0464	0.0274	0.0927	0.0444	0.3885
0.9	2.9	5.1	3.9	6.1	4.9	7.1	22.9	25.1	0.0609	0.2290	0.1123	0.5360	0.2338	2.4142	0.0210	0.0449	0.0288	0.0879	0.0477	0.3400
1	3	5	4	6	5	7	23	25	0.0655	0.2174	0.1212	0.5000	0.2564	2.1000	0.0218	0.0435	0.0303	0.0833	0.0513	0.3000

Table 6.1

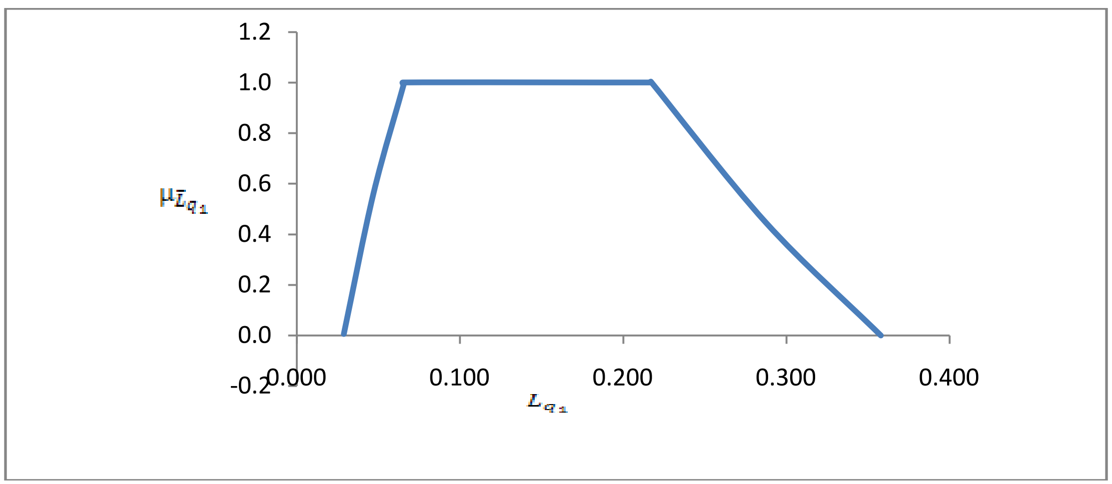

Figure 6.1: Performance measure of the average queue length of first priority

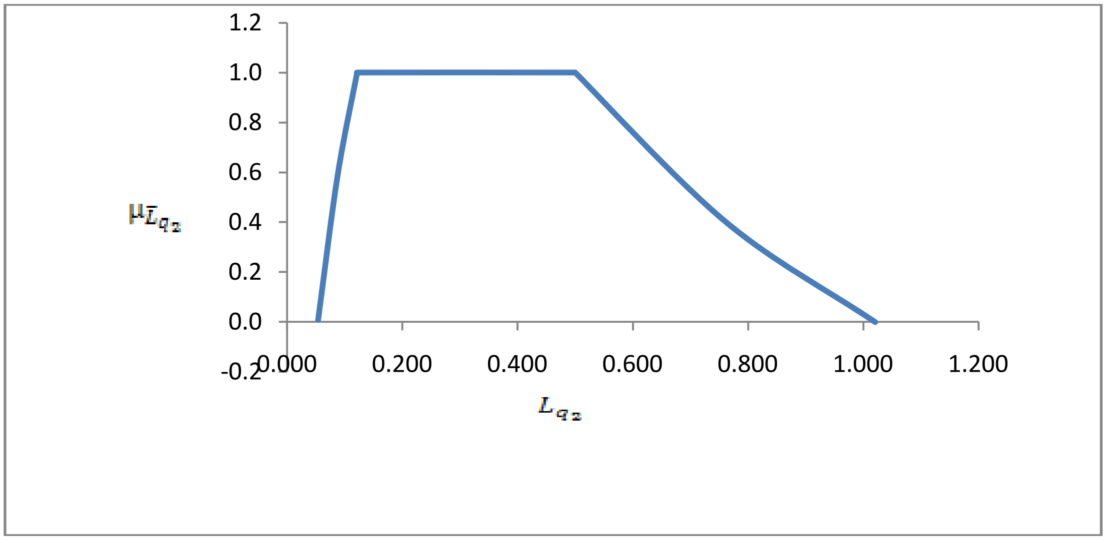

Figure 6.2: Performance measure of the average queue length of second priority

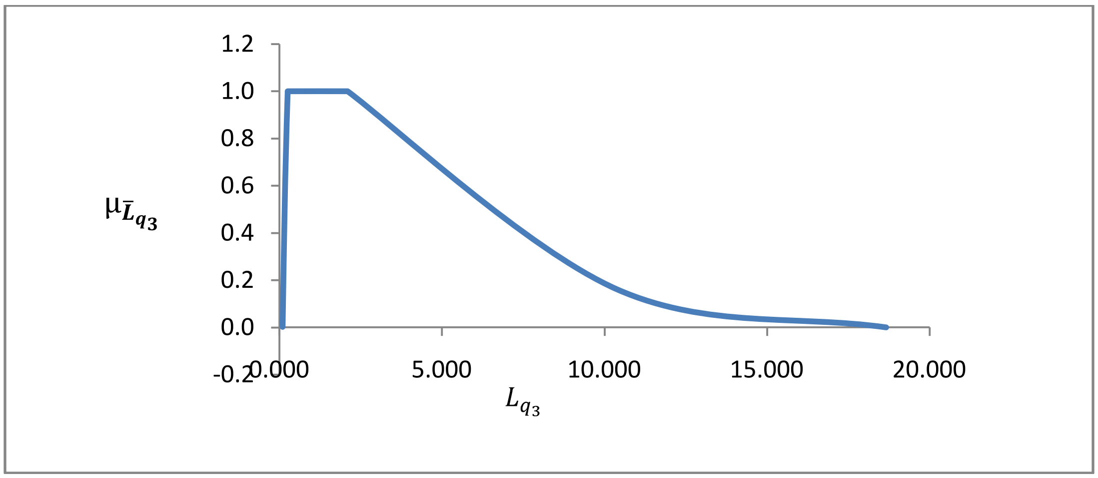

Figure 6.3: Performance measure of the average queue length of third priority

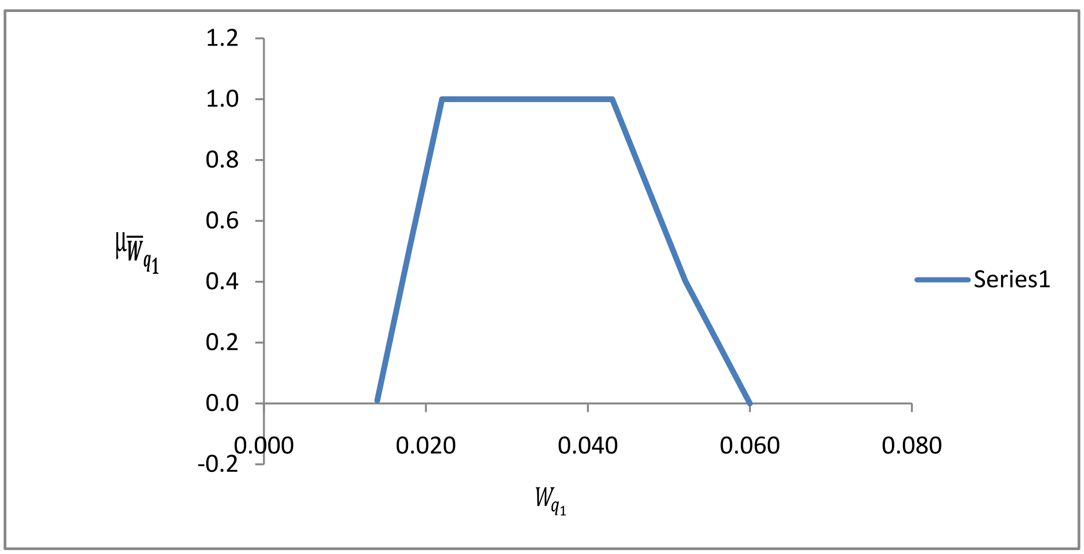

Figure 6.4: Performance measure of the average waiting time of units of first priority in the queue

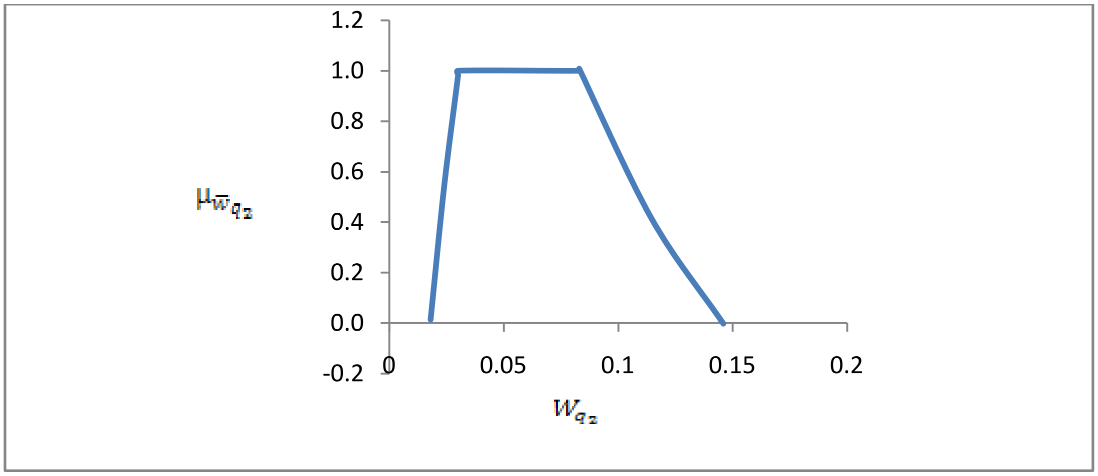

Figure 6.5: Performance measure of the average waiting time of units of second priority in the queue

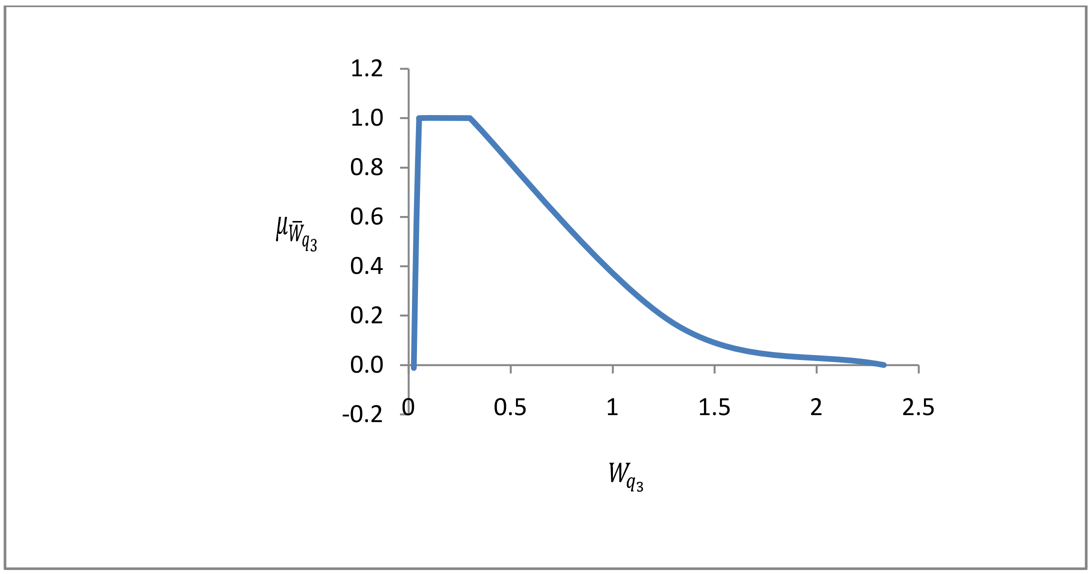

Figure 6.6: Performance measure of the average waiting time of units of third priority in the queue

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