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INVENTORY POLICY UNDER TRADE CREDIT WHEN TIME OF PAYMENT IS UNCERTAIN

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ABSTRACT

This paper discusses the optimum order quantity of the EOQ model that is not only dependent on the inventory policy but also on firm's credit policy. In practice, the supplier may simultaneously offer the retailer a permissible delay in payments to attract new customers and increases his/her sales and also a cash discount to motivate immediate payment and reduce credit expenses. However, not all the time retailer is able to pay within the trade credit period. Here we take into account the possibility of all situations like making the payment before and after the trade credit limit. We assume that the points could be random but with a trend. Models incorporate this through probability distribution functions. Since all cash outflows related to inventory control that occur at different points of time have different values, we use discount cash flow approach to establish an optimal ordering policies to the problem. The model is illustrated through numerical examples.

Keywords: *Inventory model, discounted cash flow, trade credit, probability distribution.*

1. INTRODUCTION

Numerous authors have discussed delay in payment for goods already delivered in literature. This delay period is known as a trade credit period, for paying the amount for the purchased items. In addition, the supplier offers a cash discount to encourage the retailer to pay for his purchases earlier. In practice, the supplier offers a cash discount to the retailer who can avail this cash discount if he makes the payment against delivery. It is assumed that otherwise the retailer will pay full payment within the trade credit period. Thus the supplier often makes use of this trade credit policy to promote his /her commodities, also use the cash discount policy to attract retailer to make the payment to shorten the collection period. The credit term that contains cash discount is very realistic and prevailing in real life business as an incentive for an early payment. Other realistic fact is that rarely retailer is able to pay at an earmarked point of time. Even when the optimal time points are known, it may not be always possible to make the payments at the specified time because of several reasons. This problem needs to be addressed.

Several papers on trade credit have appeared in the literatures that investigate inventory problems under varying conditions. Some of the prominent papers are discussed here. A single item inventory model for determining the economic ordering quantity in the case that the supplier offers the retailer the opportunity to delay his payment within a fixed time period is established in [9]. To provide more practical features of the real inventory systems, model [12] is extended in [9] that include the situation of cash discount offered by the supplier for immediate payment.

Goyal's model was amended in [17] by considering the difference between unit price and unit cost, and found that it makes economic sense for a well established buyer to order less quantity and take the benefits of the permissible delay more frequently. In [15] an inventory model for stock dependent consumption rate when delay in payment is permissible is developed. Pros and cons of price discount versus trade credit are analyzed in [2]. A model for retailer's optimal replenishment decisions with credit- linked demand under permissible delay in payments is developed in [13]. It also incorporates the concept of credit linked demand and developed a new inventory model under two levels of trade credit policy to reflect the real-life situation. An EOQ model under conditionally permissible delay in payments was developed in [11] and obtained the retailer's optimal replenishment policy under permissible delay in payments. Similar work can also be found in [5, 3, 7, 16, 18 and 19].

As the transactions that occur at different points of time will have different values and that can not be compared with one another, so the face value of amounts paid at different time points can not be considered as such. Therefore, it is

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necessary to take the effect of the time-value of money on the inventory system. These are discussed though discount cash flow (DCF) or net present value (NPV) approach. Certain authors discussed inventory models taking DCF approach. The need to explore the inventory problems by using the present value concept or DCF approach is recognised by [21,14]. [8] presented the discounted cash flows approach for the analysis of an optimal inventory policy in the presence of trade credit. [16] adopted discount discount cash flow approach to establish the optimal order policies when suppliers offer both a cash discount and permits delay in payment. [6] incorporated all these concepts of a DCF approach and trade credits linked to ordering quantity and developed new model for deteriorating items. An inventory model is developed on integrated inventory system with the effect of inflation and credit period [1]. An EOQ model credit financing in economic ordering policies for non- deteriorating items with time dependent demand rate in the presence of trade credit using a DCF approach is developed in [20]. Some of authors have discussed inventory models under trade credit which consider time value corrections, for instance [4, 10].

In the literature of trade credit policies two types of incentives are discussed. They are cash discount and delayed payment. That is, if a retailer pays against delivery then he can avail cash discount otherwise, the full purchasing price must be paid before the trade credit period. In this paper, we discuss the possibility of uncertainty in payment behavior. In practice, it is not always possible to pay at a specified point of time, even if it is optimal. Though retailer is intended to follow a particular pattern of payment, more often this may not be adhered because of lack of fund on hand at that moment. Here we consider situations where a retailer can pay against delivery sometimes, or makes payment within trade credit. However in extreme situations the retailer may not able to pay within the trade credit period. Though it may not happen quit regularly but is not unlikely. All these situations are captured in the model through an appropriate probability distribution. Hence, we propose one more payment interval with a penalty rate.

Mainly this paper is intended to include and develop the model that incorporates the payment pattern of the retailer. Model is developed by including the possibility of payment after trade credit limit that occurs with certain probability. According to money value it is optimal to pay at the end of the trade credit period if the purchase cost remains same through this period. Hence retailer may have a tendency of making payment on the last day of the period rather than with in the trade credit period. We also consider in another model, payment within the trade credit period as otherwise retailer may face the possibility of not having cash at the end of the period. We model retailers' inventory system as a cost minimization problem to determine the optimum inventory cycle time and optimal order quantity. Our proposed research topic is an important to retailer when the supplier provides cash – discount and a permissible delay. Furthermore, we also provide an approximate easy-to-use closed form optimal solution to the problem for any given price. Numerical examples are given to investigate the effect of changes in some parameter values on the optimal solution. We also conduct a sensitivity analysis and summary of the paper.

2.1 NOTATIONS:

D = Demand rate per year
A = Ordering cost per replenishment
C = Purchasing cost per item
h = Unit stock holding cost per item per year excluding interest charges
∂ = Cash discount rate for early payment ($0 < \partial < 1$)
M = Trade credit period
T = Cycle time in years
r = interest rate or opportunity cost
r_p = Penalty rate, where $r_p > r$
T^* = the optimal cycle time
Q^* = the optimal order quantity = DT^*

2.2 ASSUMPTIONS:

1. Demand rate D is known and constant.
2. Shortages are not allowed.
3. Planning horizon is infinite.
4. Replenishment occurs instantaneously on ordering, that is, lead time is zero.
5. Two possible trade credit combinations are considered here.

(A) There is a discount for payment made against delivery(at rate ∂). Any payment made with in or at trade credit will be charged with regular purchase price. If payment is not made with in trade credit limit a penalty of r_p is imposed on the purchase cost

(B) This is similar to (A) except for the situation where retailer can make payment before the trade credit limit rather than waiting till last day of the trade credit period. Here two cases are considered as supplier may not give incentive or may give incentive through a time dependent discount.

3.1 MODEL A: No payments done during (0, *M*)

In this model we discuss the situation given in assumption (A). Generally retailer may not able to follow the same pattern of payment, that is, consistency of payment schedule is not adhered because of uncertainty of cash in hand at various time points. However, the retailer's payment pattern can be modeled through a probability distribution of various time points of the payments made. According to the past payment habit we assume that retailer makes payment in the beginning of the trade credit period and takes up the discount price with probability p_1, the probability that retailer pays at trade credit by paying regular price is p_2 and retailer makes payment after the trade credit period by paying penalty rate with probability p_3, where $p_3=1- p_1- p_2$. Let g (.) denote the conditional density function of the random duration of the payment which is made after the trade credit period.

Hence the cumulative probability function of the payment made is given by,

$$F(t)=\begin{cases} 0 & for \quad -\infty < t < 0 \\ p_1 & for \quad 0 \le t < M \\ p_1+p_2+p_3\int_M^t g(y-M)dy & for \quad M \le t < \infty \end{cases}$$

Where $p_1 + p_2 + p_3 =1$

The present value of the total cost consists of the following elements:

- Present value of the ordering cost
- Present value of the inventory carrying Cost
- Present value of the purchasing cost.

PV_1 (T) = Present value of all future cash flow when payment made against delivery.
PV_2 (T) = Present value of all future cash flow when payment made at trade credit period M
PV_3 (T) = Present value of all future cash flow when payment made after M.

Present value of the ordering cost:

$$A+Ae^{-rT}+Ae^{-2rT}+.........$$

$$A\left[1+e^{-rT}+e^{-2rT}+........\right]$$

$$\frac{A}{(1-e^{-rT})}$$

Present value of the inventory carrying cost:

$$h\left[\int_0^T D(T-t)\quad e^{-rt}dt+\int_T^{2T} D(2T-t)\quad e^{-rt}dt+\int_{2T}^{3T} D(3T-t)\quad e^{-rt}dt+...\right]$$

$$h\left[\int_0^T D(T-t)e^{-rt}dt+\int_0^T D(T-t)e^{-r(T+t)}+\int_0^T D(T-t)e^{-r(2T+t)}+...\right]$$

$$\frac{hDT}{r\left(1-e^{-rT}\right)}-\frac{hD}{r^2}$$

The purchase cost per unit item at various time points of payment are:

$$\begin{cases} C(1-\partial) & if \quad t=0 \\ C & if \quad t=M \\ Ce^{r_p(t-M)} & if \quad M<t<\infty \end{cases}$$

Expected net present value of the purchase cost for various time points of payment are:

Case (i): when payment is made against delivery

$$CD(1-\partial)T + CD(1-\partial)Te^{-rT} + CD(1-\partial)Te^{-2rT} + ... = CDT(1-\partial)(1+e^{-rT}+e^{-2rT}+....)$$
$$= \frac{CD(1-\partial)T}{(1-e^{-rT})}$$

Case (ii): when payment is made at Trade credit period *M*

$$CDTe^{-rM} + CDTe^{-r(T+M)} + CDTe^{-r(2T+M)}........ = CDTe^{-rM}\left[1+e^{-rT}+e^{-2rT}+........\right]$$
$$= \frac{CDTe^{-rM}}{(1-e^{-rT})}$$

Case (iii): when payment is made after trade credit period *M* with penalty rate r_p.

If payment is made at time *t* after the trade credit period *M* then payment towards the purchasing cost is, $CDTe^{r_p(t-M)}$

Hence conditional expectation of the this payment is $\int_M^\infty CDTe^{r_p(t-M)} g(t-M)dt$

Expected net present value of the above cost with interest rate r is $DT\int_M^\infty Ce^{r_p(t-M)}e^{-rt} g(t-M)\,dt$

Taking $y = t-M$, the above expression is

$$= CDT\int_0^\infty e^{r_p y} e^{-r(y+M)} g(y)\,dy$$

$$= CDT\int_0^\infty e^{y(r_p-r)} \; e^{-rM} g(y)\,dy$$

$$= CDTe^{-rM}\int_0^\infty e^{(r_p-r)y} \; g(y)\,dy$$

$$= CDTe^{-rM} M_X(r_p - r)$$

Where $M_X(r_p - r)$ represents Moment generating function of distribution function X.

Any distribution which has limit zero to infinity can be considered for payment time beyond trade credit period. One of the appropriate distributions for the delay in payment beyond trade credit period is gamma distribution. In our discussion here, we assume this distribution and derive the cost function

$$= CDT\; e^{-rM} \frac{1}{\Gamma(\alpha)}\int_0^\infty e^{(r_p-r)y}\; \theta^\alpha \; e^{-\theta y}\; y^{\alpha-1}dy$$

where α and θ are parameters of gamma distribution.

$$= \frac{CDT\theta^\alpha}{\Gamma(\alpha)} e^{-rM} \int_0^\infty \mathrm{e}^{-(\theta-(r_p-r))y}\; y^{\;\alpha-1}dy$$

$$= \frac{CDT\theta^\alpha}{\Gamma(\alpha)} e^{-rM} \int_0^\infty \mathrm{e}^{-(\theta+r-r_p)y}\; y^{\alpha-1}dy$$

$$= \frac{CDT\theta^{\alpha} e^{-rM}}{\Gamma(\alpha)} \frac{\Gamma(\alpha)}{(\theta + r - r_p)^{\alpha}}$$, (by definition of gamma function)

$$= \left(\frac{CDT\theta^{\alpha} e^{-rM}}{(\theta + r - r_p)^{\alpha}} \right)$$

Expected present value of the purchasing cost is $\left(\frac{CDT\theta^{\alpha} e^{-rM}}{(\theta + r - r_p)^{\alpha}} \quad \frac{1}{1 - e^{-rT}} \right)$.

3.2 Present value of total cost

Present value of all future cash flow when payment made against delivery with probability p_1 + Present value of all future cash flow when payment made at trade credit period M with probability p_2 + Present value of all future cash flow when payment made after M with penalty rate r_p with probability p_3

$$PV(T) = p_1 PV_1(T) + p_2 PV_2(T) + p_3 PV_3(T)$$

$$= p_1 \left[\frac{A}{(1-e^{-rT})} + \frac{hDT}{r(1-e^{-rT})} - \frac{hD}{r^2} + \frac{CD(1-\partial)T}{(1-e^{-rT})} \right] + p_2 \left[\frac{A}{(1-e^{-rT})} + \frac{hDT}{r(1-e^{-rT})} - \frac{hD}{r^2} + \frac{C\,DTe^{-rM}}{(1-e^{-rT})} \right]$$

$$+ \ p_3 \left[\frac{A}{(1-e^{-rT})} + \frac{hDT}{r(1-e^{-rT})} - \frac{hD}{r^2} + \left\{ \frac{C\,DT\theta^{\alpha} e^{-rM}}{(\theta + r - r_p)^{\alpha}} \quad \frac{1}{(1-e^{-rT})} \right\} \right]$$

$$= \frac{A}{1-e^{-rT}} - \frac{h\,D}{r^2} + \frac{CDT}{r(1-e^{-rT})} \left[\frac{h}{C} + p_1 r\,(1-\partial) + p_2\, re^{-rM} + p_3 \frac{r\theta^{\alpha}\, e^{-rM}}{(\theta + r - r_p)^{\alpha}} \right]$$

Thus $PV(T) = \frac{A}{1-e^{-rT}} - \frac{hD}{r^2} + \frac{CDTk}{r(1-e^{-rT})}$

Where $k = \left[\frac{h}{C} + p_1 r(1-\partial) + p_2 re^{-rM} + p_3 \frac{r\theta^{\alpha}\, e^{-rM}}{(\theta + r - r_p)^{\alpha}} \right]$

To obtain optimal replenishment time T*, we need to minimize PV (T) with respect to T and we get,

$$\frac{dPV}{dT} = \frac{d}{dT} \left\{ \frac{A}{1-e^{-rT}} - \frac{hD}{r^2} + \frac{CDTk}{(1-e^{-rT})r} \right\}$$

Differentiating above equation partially with respect T two times, we get $\frac{d^2PV}{dT^2} < 0$

Then equating $\frac{dPV}{dT} = 0$ which implies,

$$r^2 A e^{-rT^*} = CDk(1 - e^{-rT^*} - rT^* e^{-rT^*})$$

$$r^2 A = CDk(e^{rT^*} - 1 - rT^*)$$

The optimal T* is obtained by solving the above equation

However, for ready use an approximate closed form solution can be obtained by using Taylor's series approximation of e^{rT}

i.e e^{rT} can be approximated as $1 + rT + \frac{(rT)^2}{2}$

Then above equation can be written as $r^2 A = \quad CDk\frac{(rT^*)^2}{2}$

After simplification, we get $T^* = \sqrt{\frac{2A}{CDk}}$

Using T* optimal cycle time, the optimal order quantity Q* is obtained as Q*= DT*

We get $Q^* = \sqrt{\frac{2DA}{Ck}}$

Since $Ck = h$ when r=0 the above expression is the EOQ.

3.3 Model B: Payment are also made within (0, *M*)

Case (i): Supplier will charge the regular price for the payment made any time during the trade credit period except at t=0 Optimum order quantity can also be studied for the situation when payment is also made during the period (*0, M*). Retailer chooses to pay during this period rather than waiting till last day of trade credit period without any incentive from the supplier for an early payment. Retailer will have to pay same amount that he would have to pay at *M*. Retailer may have this habit in order to avoid paucity of fund at *M*, as cash could be diverted to some other transaction. If the payment is made at the time of purchasing an item, then discount rate ∂ is attached. If the payment is made any time before the trade credit period as explained above, then regular/ actual price need to be paid. Payment during this period could occur at any random time point and we assume the payment time during this interval follow uniform random variable. We assume that he makes payment against delivery with probability p_1, payment made within the trade credit period with probability p_4. Payment made at the trade credit period with probability p_2 and finally payment made after the trade credit period with probability p_3. We define,

$PV_1\ (T)$= Present value of all future cash flow when payment made in the beginning.
$PV_2\ (T)$ = Present value of all future cash flow when payment made at trade credit period M
$PV_3\ (T)$ = Present value of all future cash flow when payment made after M, with penalty rate r_p.
$PV_4\ (T)$ = Present value of all future cash flow when payment made within the trade credit period.

The expected present value of the payment made during (0, *M*) in the first cycle is

$$C\int_0^M \frac{e^{-rt}}{M}\, dt = \frac{C}{rM}[1 - e^{-rM}]$$

The net expected present value of the purchase cost at different set of time points are:

$$= \begin{cases} \dfrac{CDT(1-\partial)}{(1-e^{-rT})} & if\ t = 0 \\ \dfrac{CDT}{rM}\dfrac{(1-e^{-rM})}{(1-e^{-rT})} & \text{for the interval } 0 < t < M \\ \dfrac{CDT \quad e^{-rM}}{(1-e^{-rT})} & if\ t = M \\ \dfrac{CDTe^{-rM}\theta^{\alpha}}{\left(\theta + r - r_p\right)^{\alpha}}\dfrac{1}{(1-e^{-rT})} & \text{for the interval } M < t < \infty \end{cases}$$

3.4 Present value of total cost

Present value of all future cash flow when payment made against delivery with probability p_1 + Present value of all future cash flow when payment made at trade credit period *M* with probability p_2 + Present value of all future cash flow when payment made after *M*, with penalty rate r_p with probability p_3+ Present value of all future cash flow when payment made within the trade credit period with probability p_4

$$PV(T) = \quad p_1 PV_1(T) + p_2 PV_2(T) + p_3 PV_3 + p_4 PV_4(T)$$

$$PV(T) \quad = \frac{A}{1-e^{-rT}} - \frac{hD}{r^2} + \frac{CDT}{r(1-e^{-rT})}\left[\frac{h}{C} + p_1 r(1-\partial) + p_2 re^{-rM} + p_3 \frac{r\theta^{\alpha}\ \mathrm{e}^{-\mathrm{rM}}}{(\theta + \mathrm{r} - \mathrm{r_p})^{\alpha}} + \frac{p_4(1-e^{-rM})}{M}\right]$$

$$PV(T) = \frac{A}{1-e^{-rT}} - \frac{hD}{r^2} + \frac{CDT}{r(1-e^{-rT})}(k)$$

Where k= $\dfrac{h}{C} + p_1 r(1-\partial) + p_2 re^{-rM} + p_3 \dfrac{r\theta^{\alpha}\ \mathrm{e}^{-\mathrm{rM}}}{(\theta + \mathrm{r} - \mathrm{r_p})^{\alpha}} + \dfrac{p_4(1-e^{-rM})}{M}$

To find an optimal replenishment cycle time T* we need to minimize PV (T) with respect to ‘T’ and we get $\dfrac{dPV}{dT} = 0$ which implies,

$$r^2 Ae^{-rT^*} = CDk(1-e^{-rT^*} - rT^* e^{-rT^*})$$

$$r^2 A = CDk(e^{rT^*} - 1 - rT^*)$$

Note that e^{rT} can be approximated as $e^{rT} \approx 1 + rT + \dfrac{(rT)^2}{2}$

Then above equation can be written as $r^2 A = \quad CDk\dfrac{(rT^*)^2}{2}$

After simplification, we get $T^* = \quad \sqrt{\dfrac{2A}{DCk}}$

Using T* optimal cycle time the optimal order quantity Q* is obtained as Q*= DT*

We get $Q^* = \sqrt{\dfrac{2DA}{Ck}}$

Since C*k* =*h* when *r*=0 the above expression is the EOQ.

Case(ii): Suppose supplier gives a discount for payment made earlier to the credit period

In addition to above assumptions when retailer makes payment within trade credit limit if supplier gives an incentive for this early payment through a discount which is time dependent. For instance if the payment is made at *t* the purchase cost will be $C\,e^{-(M-t)r_1}$. Accordingly if payment is made at $t = 0$ then it costs $C(1-\partial) = C\,e^{-M\,r_1}$. In such cases the optimal order size will be effected by discounted cash flow. Normally $0 < r_1 \leq r$. Note that probabilities p_1, p_2, p_3, and p_4 means same as explained above, but could change because of incentive given.

If discount factor r_1 is applied to the time point $t=0$ then ∂ should satisfy $\partial = 1 - e^{-Mr_1}$. Purchase cost payment function per unit item when made at time point *t* is

$$\begin{cases} Ce^{-(M-t)r_1} & if \quad 0 \leq t \leq M \\ C\,e^{r_p(t-M)} & if\ M < t < \infty \end{cases}$$

The expected present value of the payment made during (0, *M*) in the first cycle is

$$C\int_0^M \frac{e^{-(M-t)r_1}}{M} e^{-rt}\, dt = \frac{C}{M(r-r_1)}[e^{-r_1M} - e^{-rM}]$$

The net expected **value** of the purchase cost at different set of time points are:

$$=\begin{cases} \dfrac{CDTe^{-r_1M}}{(1-e^{-rT})} & if\ t=0 \\ \dfrac{CDT}{M(r-r_1)}\dfrac{[e^{-r_1M}-e^{-rM}]}{(1-e^{-rT})} & \text{for the interval } 0<t<M \\ \dfrac{CDT\mathrm{e}^{-\mathrm{rM}}}{(1-e^{-rT})} & if\ t=M \\ \dfrac{CDTe^{-rM}\theta^{\alpha}}{\left(\theta+r-r_p\right)^{\alpha}}\dfrac{1}{(1-e^{-rT})} & \text{for the interval } M<t<\infty \end{cases}$$

Present value of total cost

$$PV(T) = \quad p_1PV_1(T) + p_2PV_2(T) + p_3PV_3 + p_4PV_4(T)$$

$$PV(T) = \frac{A}{1-e^{-rT}} - \frac{hD}{r^2} + \frac{CDT}{r(1-e^{-rT})}\left[\frac{h}{C} + p_1re^{-r_1M} + p_2\mathrm{re}^{-\mathrm{rM}} + p_3\frac{r\theta^{\alpha}\ \mathrm{e}^{-\mathrm{rM}}}{\left(\theta+\mathrm{r}\text{-}\mathrm{r_p}\right)^{\alpha}} + \frac{rp_4}{M(r-r_1)}\frac{[e^{-r_1M}-e^{-rM}]}{}\right]$$

$$PV(T) = \frac{A}{1-e^{-rT}} - \frac{hD}{r^2} + \frac{CDT}{r(1-e^{-rT})}(k)$$

Where $k = \left[\dfrac{h}{C} + p_1re^{-r_1M} + p_2\mathrm{re}^{-\mathrm{rM}} + p_3\dfrac{r\theta^{\alpha}\ \mathrm{e}^{-\mathrm{rM}}}{\left(\theta+\mathrm{r}\text{-}\mathrm{r_p}\right)^{\alpha}} + \dfrac{rp_4}{M(r-r_1)}\dfrac{[e^{-r_1M}-e^{-rM}]}{}\right]$

To find an optimal replenishment cycle time T* we need to minimize PV (T) with respect to 'T' and we get

$\dfrac{dPV}{dT} = 0$ which implies,

$$r^2Ae^{-rT^*} = CDk(1-e^{-rT^*} - rT^*e^{-rT^*})$$

$$r^2A = CDk(e^{rT^*} - 1 - rT^*)$$

With approximation $e^{rT} \approx 1 + rT + \dfrac{(rT)^2}{2}$,

we get $T^* = \sqrt{\dfrac{2A}{CDk}}$ and $Q^* = \sqrt{\dfrac{2DA}{Ck}}$

Since *Ck* =*h* when *r*=0 the above expression is the EOQ.

4 NUMERICAL EXAMPLES

To illustrate and verify the above theoretical results, we consider the following examples.

4.1 Examples for Model A

For the purpose of illustration, consider demand rate per year D =1000 units; r=0.06; M=15 days=15/365= 0.04 years; h=6/unit/year; A=100/order ; ∂ =0.1; r_p=0.2; α =0.1; θ =2.

Table 1 shows the optimum order quantity, optimum replenishment time points and optimal total cost for various values of purchase price under different possible probability values for payment pattern. By table.1.

The following inference is made based on the Table.1

(i) Higher the value of p_1 compared to p_2 and p_3 will result the lower values of total cost
(ii) Higher the value of p_2 compared to p_1 will results higher the values of total cost compare to (i)
(iii) Higher the value of p_3 compared to p_2 and p_1 will results higher the values of total cost compare to case (i) and case (ii)
(iv) As seen in basic EOQ model we can notice that as purchase cost increases there is significant decrease in value of optimal quantity as well as the value of optimal cycle time. But there is significant increase in total relevant cost.

4.2 Examples for Model B:

Demand rate per year D =1000 units; r=0.06; M=15 days=15/365= 0.04 years; h=6/unit/year; A=100/order; ∂=0.1; r_p=0.2; α =0.1; r1=0.02; θ =2.

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From Table 2 it is clear that there is significant difference between the total optimal costs when paying habits change, especially after the trade credit period. Hence when a retailer is not making payment then actual cost will be much different and higher.

From both tables it is clear that there is significant difference in the present value of the costs under different payment habits. Hence assuming a constant payment habit if not really true gives misleading answers and hence policies used are not optimal.

SUMMARY

Most of the inventory models with trade credit assumed that retailer pays either before the trade credit period with in cash discount or at credit period every time. Thus models allow making the payment every time at one of these two possible points. However in real market place it is common that the retailer is not able to pay consistently at the similar time point every time. Sometimes the retailer pays before the trade credit and sometimes at the trade credit period. In extreme cases he/she pays after trade credit period. In order to model this and possibly not very punctual payment habit, the model incorporates possibility of payment even after the trade credit period of course with a penalty rate that will occur with certain probability and retailer's payment time is also considered as a random point which is modeled through a probability distribution. Further under the condition of trade credit it is beneficial to pay only at the trade credit limit point rather than before. But retailer may find it convenient to pay wherever cash is available and hence further situations arise. From this study it can be seen that if retailer is not able to adhere to same payment pattern then the total cost differs very much. Hence, assuming models without considering various probabilities will not only mislead the total cost but also the solutions obtained are suboptimal. As can be seen from both the tables that the total costs differ when probabilities are different. All models discussed earlier can be taken care as special cases by assuming appropriate probabilities as zero in the present model. Hence the present model is a generalization by taking various possibilities into the present model. In addition, the computation results on the two models discussed in the paper reveal that a lower value of purchasing cost results in higher values for the optimal replenishment cycle time T* and also the optimal order quantity Q* and vice versa. For different time points the present value of total cost is also calculated.

REFERENCES:

[1] Aggrawal R, Rajput D, and Varshney N. Inventory system with the effect of inflation and credit period, *International Journal of Applied Engineering Research*, 2009;4(11):2337–2348.

[2] Arcelus FJ, Shah NH, and Srinivasan G. Retailer's response to special sales: price discount vs. trade credit, OMEGA, 2001; 29: 417-428.

[3] Chang CT and Teng JT. Retailer's optimal ordering policy under supplier Credits, *Math Meth Oper Res*, 2004;60:471–483.

[4] Chung KJ and Huang CK. The convexities of the present value function of all future cash outflows in inventory and credit, *Prod. Plan. & Control,* 2000; 11(2): 133- 140.

[5] Chung KJ, Huang YF, and Huang CK. The replenishment decision for EOQ inventory model under permissible delay in payments, *Opsearch,* 2002; 39: 327–340.

[6] Chung KJ and Liao JJ. The optimal ordering policy in a DCF analysis for deteriorating items when trade credit depends on the order quantity, *International Journal of Production Economics*, 2006; 100: 116–130.

[7] Chung KJ and Liao JJ. Lot-sizing decisions under trade credit depending on the ordering quantity, *Computers and Operations Research,* 2004; 31: 909-928.

[8] Chung KH. Inventory control & Trade credit Revisited, *Journal of the Operational Research society,* 1989; 40:495-498.

[9] Goyal SK. EOQ under conditions of permissible delay in payments, *Journal of the Operational Research Society*, 1985; 36: 335-338.

[10] Haley CW and Higgins RC. Inventory Policy and Trade Credit Financing, *Management Science* 1973; 20(4):464-471.

[11] Huang YF. Economic order quantity under conditionally permissible delay in payments, *European Journal of Operational Research*, 2007; 176: 911-924.

[12] Huang YF and Chung KJ. Optimal replenishment and payment policies in the EOQ model under cash discount and trade credit, *Asia –pacific J. operations research*, 2003; 20:177-190.

[13] Jaggi CK, Goyal SK, and Goel SK. Retailer's optimal replenishment decisions with credit-linked demand under permissible delay in payments, *European Journal of Operational Research*, 2008; 190: 130-135.

[14] Kim YH and Chung KH. An integrated evaluation of investment in inventory and trade credit: A cash flow approach, *J. of Business Finance and Accounting,* 1990;17: 381-389.

[15] Liao HC, Tsai CH, Su CT. An inventory model with deteriorating items under inflation when a delay in payment is permissible, *International Journal of Production Economics,* 2000; 63: 207–214.

[16] Teng JT. Discount Cash – Flow Analysis on Inventory control under various supplier's trade credits, *International Journal of Operations Research,* 2006; 3: 23-29.

[17] Teng JT. On the economic order quantity under conditions of permissible delay in payments, *Journal of the Operational Research Society,* 2002; 53: 915–918.

[18] Teng JT, Chang CT, and Chern MS. Retailer's Optimal Ordering Policies in the EOQ Models withTrade Credit Financing, *International service of systems science*, 2007; 38; 269-278.

[19] Teng JT and Goyal SK. Optimal ordering policies for a retailer in a supply chain with up-stream and down-stream trade credits. *J. Oper. Res. Soc.* 2007; 58: 1252–1255.

[20] Tripathi RP and Misra SS. Credit financing in economic ordering policies of non- deteriorating items with time dependent demand rate , *International Review of Business and Finance*, 2010; 2:47-55.

[21] Trippi RR and Lewin DE. A present value formulation of the classical EOQ problem", *Decision Science*, :30–35.

Table 1: optimal solutions with various values of probabilities for payment pattern and purchase cost with following input values according to Model A.

Probabilities (Chance of making Payment)	C=30		C=50		C=80	
	Q^*	PV(T)	Q^*	PV(T)	Q^*	PV(T)
p_1=0.8 p_2=0.1 p_3=0.1	162	4.8081×10^5	151	7.89×10^5	139	1.2511×10^6
p_1=0.1 p_2=0.1 p_3=0.8	160	5.177×10^5	149	8.5045×10^5	136	1.3494×10^6
p_1=0.2 p_2=0.4 p_3=0.4	161	5.1133×10^5	150	8.39850×10^5	136	1.3324×10^6
p_1=0.2 p_2=0.6 p_3=0.2	161	5.106×10^5	150	8.3863×10^5	137	1.3305×10^6
p_1=0.5 p_2=0.3 p_3=0.2	161	4.9589×10^5	150	8.1412×10^5	138	1.2913×10^6
p_1=0.2 p_2=0.3 p_3=0.5	161	5.117×10^5	150	8.4046×10^5	137	1.3644×10^6
p_1=1 p_2=0 p_3=0	162	4.7064×10^5	152	7.7205×10^5	139	1.2240×10^6
p_1=0 p_2=1 p_3=0	160	5.1968×10^5	149	8.5375×10^5	136	1.3547×10^6
p_1=0 p_2=0 p_3=1	160	5.2333×10^5	149	8.5984×10^5	136	1.3644×10^6

Table 2: optimal solutions with various values of probabilities for payment pattern and purchase cost with following input values according to Model B.

Probabilities (Chance of making Payment)	C= 30				C= 50				C= 80			
	Model B case(i)		**Model B case(ii)**		**Model B case(i)**		**Model B case(ii)**		**Model B case(i)**		**Model B case(ii)**	
	Q^*	PV(T)	Q^*	PV(T)	Q^*	PV(T)	Q^*	PV(T)	Q^*	PV(T)	Q^*	PV(T)
p_1=0.5 p_2=0.3 p_3=0.1 p_4=0.1	163	4.454×10^5	162	4.7037×10^5	153	7.30×10^5	152	7.716×10^5	141	11.568×10^5	139	12.232×10^5
p_1=0.1 p_2=0.2 p_3=0.5 p_4=0.2	164	4.1636×10^5	164	4.2134×10^5	154	6.816×10^5	154	6.8995×10^5	142	10.793×10^5	142	10.926×10^5
p_1=0.1 p_2=0.2 p_3=0.2 p_4=0.5	171	2.6487×10^5	170	2.6987×10^5	164	4.2919×10^5	163	4.3752×10^5	155	6.756×10^5	154	6.8896×10^5
p_1=0.2 p_2=0.4 p_3=0.2 p_4=0.2	164	4.1036×10^5	164	4.2034×10^5	155	6.716×10^5	154	6.8826×10^5	143	10.634×10^5	143	10.9×10^5
p_1=0.3 p_2=0.3 p_3=0.2 p_4=0.2	165	4.0545×10^5	164	4.2044×10^5	155	6.6343×10^5	154	6.8841×10^5	144	10.503×10^5	143	10.901×10^5
p_1=0.4 p_2=0.4 p_3=0.1 p_4=0.1	163	4.5031×10^5	162	4.7025×10^5	153	7.3817×10^5	152	7.7144×10^5	141	11.698×10^5	139	12.23×10^5