

**FINITE ELEMENT ANALYSIS OF COMBINED INFLUENCE OF SORET AND DUFOUR EFFECTS ON NON DARCY CONVECTIVE HEAT AND MASS TRANSFER FLOW OF A VISCOUS FLUID THROUGH A CYLINDRICAL ANNULUS WITH HEAT GENERATING SOURCES AND CONSTANT MASS FLUX**

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**ABSTRACT**

*We discuss the combined influence of thermo diffusion and diffusion thermo on non darcy convective heat and mass transfer flow of a viscous fluid in a cylindrical annulus with outer cylinder maintained at constant heat flux. By employing galarkin finite element analysis the coupled non liner equations energy, momentum and diffusion are solved. All the flow characteristics are analyzed for different variations of parameters.*

*Key Words: Heat and Mass Transfer, Soret and Deffer Effect, Heat sources, Finite element Analysis,*

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**1. INTRODUCTION**

Convection through annulus region under steady state conditions has also been discussed with two cylindrical surface kept at different temperatures [9]. This work has been extended in temperature dependent convection flow [9] as well as convection flows through horizontal porous channel whose inner surface in maintained at constant temperature while the other surface is maintained at circumferentially varying sinusoidal temperature [14, 19].

Convection heat transfer in magneto hydrodynamic case is important in nuclear and space technology [9, 13]. Nanda and Purushotham [9] have discussed the convection heat transfer of a thermal conducting viscous fluid induced by traveling thermal waves on the circumference of a long vertical circular cylindrical pipe. Whithead [19] have made a study of the fluid flow and heat transfer in a viscous incompressible fluid confined in an annulus bounded by two rigid cylinders. The flow is generated by periodical traveling waves imposed on the outer cylinder and the inner cylinder is maintained at constant temperature.

Chen and Yih [4] have investigated the heat and mass transfer characteristics of natural convection flow along a vertical cylinder under the combined buoyancy effects of thermal and species diffusion. Sivanjaneya Prasad [16] has investigated the free convection flow of an incompressible, viscous fluid through a porous medium in the annulus between the porous concentric cylinders under the influence of a radial magnetic field. Antonio[3] has investigated the laminar flow, heat transfer in a vertical cylindrical duct by taking into account both viscous dissipation and the effect of buoyancy, the limiting case of fully developed natural convection in porous annuli is solved analytically for steady and transient cases by E. Sharawi and Al-Nimir [15]. Philip [13] has obtained solutions for the annular porous media valid for low modified Reynolds number. Sreevani [18] has studied the convective heat and mass transfer through a porous medium in a cylindrical annulus under radial magnetic field with soret effect. Reddy [17] has discussed the soret effect on mixed convective heat and mass transfer through a porous cylindrical annulus. For natural convection, the existence of large temperature differences between the surfaces is important The Soret and Dufour effects have garnered considerable interest in both Newtonian and non-Newtonian convective heat and mass transfer. Such effects are significant when density differences exist in the flow regime. Soret and Dufour effects are important for intermediate molecular weight gases in coupled heat and mass transfer in binary systems, often encountered in chemical process engineering and also in high-speed aerodynamics. Soret and Dufour effects are also critical in various porous flow regimes occurring in chemical and geophysical systems. The Soret effect for instance has been utilized for isotope separation and in mixtures between gases with very low molecular weight (H<sub>2</sub>, He) and the medium molecular weight (N<sub>2</sub>, air) the Soret effect was found to be of a magnitude just it can not be neglected. Most of the research activity in the field of convection in binary fluid

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mixtures has been focused on these binary mixtures. The importance of the Dufour effect in gas mixtures has two causes: (i) The Lewis number  $L = D/k$ , i.e., the ratio of concentration diffusion constant  $D$  and thermal diffusivity being of order 1 in gas mixtures is about 100 times larger than in liquid mixtures. (ii) the Dufour number 'Du' measuring the contribution to the generalized thermodynamic forces in the linear Onsager relations from gradients of chemical potential that are caused by concentration gradients can be estimated to be  $Du \sim 20-40$  in gas mixtures. There are few studies about the Soret and Dufour effects in a Darcy or non-Darcy porous medium. Angel et al [2] has examined the composite Soret and Dufour effects on free convective heat and mass transfer in a Darcian porous medium with Soret and Dufour effects. Postelnicu [12] has studied the heat and mass transfer characteristics of natural convection about vertical surface embedded in a saturated porous medium subjected to magnetic field by considering the Soret and Dufour effects. Partha et al. [11] have examined the Soret and Dufour effects in a non-Darcy porous medium. Mansour et al. [8] have studied the multiplicity of solutions induced by thermosolutal convection in a square porous cavity heated from below and submitted to horizontal concentration gradient in the presence of Soret effect. Lakshmi Narayana et al. [6] have studied the Soret and Dufour effects in a doubly stratified Darcy porous medium. Lakshmi Narayana and Murthy [7] have examined the Soret and Dufour effects on free convective heat and mass transfer from a horizontal flat plate in a Darcy porous medium. Very recently Robert Harris [5] have discussed Thermal-diffusion and diffusion thermo effects on combined heat and mass transfer of a steady MHD convective and slip flow due to a rotating disk with viscous dissipation and ohmic heating.

In this paper we discuss the free and forced convective heat and mass transfer of a viscous fluid flow through a porous medium in a circular cylindrical annulus with Thermal-Diffusion and Diffusion-Thermo effects in the presence of constant heat source, where the inner wall is maintained constant temperature while the outer wall is maintained at constant heat flux and the concentration is constant on the both walls. The Brinkman-Forchhimer extended Darcy equations which take into account the boundary and inertia effects are used in the governing linear momentum equations. The effect of density variation is confined to the buoyancy term under Boussinesque - approximation. The momentum, energy and diffusion equations are coupled equations. In order to obtain a better insight into this complex problem, we make use of Galerkin finite element analysis with quadratic polynomial approximations. The Galerkin finite element analysis has two important features. The first is that the approximation solution is written directly as a linear combination of approximation functions with unknown nodal values as coefficients. Secondly, the approximation polynomials are chosen exclusively from the lower order piecewise polynomials restricted to contiguous elements. The behavior of velocity, temperature and concentration are analyzed for different parameters. The shear stress and the rate of heat and mass transfer have also obtained for variations in the governing parameters.

## 2. FORMULATION OF THE PROBLEM

We consider free and force convective flow of a viscous through a porous medium in a circular cylindrical annulus with Thermal-Diffusion and Diffusion-Thermo effects in the presence of constant heat source, whose inner wall is maintained at a constant temperature and the outer wall is maintained constant heat flux also the concentration is constant on the both walls. The flow, temperature and concentration in the fluid are assumed to be fully developed. Both the fluid and porous region have constant physical properties and the flow is a mixed convection flow taking place under thermal and molecular buoyancies and uniform axial pressure gradient. The boussenisque approximation is invoked so that the density variation is confined to the thermal and molecular buoyancy forces. The Brinkman-Forchhimer-Extended Darcy model which accounts for the inertia and boundary effects has been used for the momentum equation in the porous region. In the momentum, energy and diffusion are coupled and non-linear. Also the flow in is unidirectional along the axial cylindrical annulus. Making use of the above assumptions the governing equations are

$$-\frac{\partial p}{\partial z} + \frac{\mu}{\delta} \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) - \frac{\mu}{k} u - \frac{\rho \delta F}{\sqrt{k}} u^2 + \rho g \beta (T - T_0) + \rho g \beta^* (C - C_0) = 0 \quad (1)$$

$$\rho c_p u \frac{\partial T}{\partial z} = \lambda \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) - Q(T - T_0) + \frac{D_m K_t}{c_s c_p} \left( \frac{\partial^2 c}{\partial r^2} + \frac{1}{r} \frac{\partial c}{\partial r} \right) \quad (2)$$

$$u \frac{\partial C}{\partial z} = D_1 \left( \frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} \right) + \frac{D_m K_t}{T_m} \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) \quad (3)$$

Where  $u$  is the axial velocity in the porous region,  $T$  &  $C$  are the temperature and concentrations of the fluid,  $k$  is the permeability of porous medium,  $F$  is a function that depends on Reynolds number and the microstructure of the porous medium and  $D_1$  is the Molecular diffusivity,  $D_m$  is the coefficient of mass diffusivity,  $T_m$  is the mean fluid

temperature,  $K_t$  is the thermal diffusion,  $C_s$  is the concentration susceptibility,  $C_p$  is the specific heat,  $\rho$  is density,  $g$  is gravity,  $\beta$  is the coefficient of thermal expansion,  $\beta^*$  is the coefficient of volume expansion. The boundary conditions relevant to

$$u = 0 \quad \& \quad T=T_i, \quad C = C_i \quad \text{at } r=a \quad (4)$$

$$u = 0 \quad \& \quad \frac{\partial T}{\partial r} = Q_1, \quad C=C_0 \quad \text{at } r=a+s \quad (5)$$

The axial temperature gradient  $\frac{\partial T}{\partial z}$  and concentration gradient  $\frac{\partial C}{\partial z}$  are assumed to be constant say A and B respectively.

we now define the following non-dimensional variables

$$z^* = \frac{z}{a}, \quad r^* = \frac{r}{a}, \quad u^* = \frac{a}{\gamma} u, \quad p^* = \frac{pa\delta}{\rho\gamma^2}, \quad \theta^* = \frac{T-T_i}{Aa}, \quad s^* = \frac{s}{a}, \quad C^* = \frac{C-C_i}{C_i-C_0}$$

Introducing these non-dimensional variables, the governing equations in the non-dimensional form are (on removing the stars)

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} = p + \delta(D^{-1})u + \delta^2 \Lambda u^2 - \delta G(\theta + N C) \quad (6)$$

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} - \alpha \theta = P_r N_t u - Du N_t \left( C_{rr} + \frac{1}{r} C_r \right) \quad (7)$$

$$\frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} = Sc N_c u - Sc Sr \left( \theta_{rr} + \frac{1}{r} \theta_r \right) \quad (8)$$

Where

$$\Lambda = FD^{-1} \quad (\text{Inertia parameter}), \quad G = \frac{g\beta(T_1-T_0)a^3}{\gamma^2} \quad (\text{Grashof number})$$

$$D^{-1} = \frac{a^2}{k} \quad (\text{Inverse Darcy parameter}), \quad P_r = \frac{\rho c_p \gamma}{\lambda} \quad (\text{Prandtl number})$$

$$N_t = \frac{Aa}{T_1-T_0} \quad (\text{Non-dimensional temperature gradient})$$

$$N_c = \frac{Ba}{C_1-C_0} \quad (\text{Non-dimensional concentration gradient})$$

$$Du = \left( \frac{D_m K_t \Delta c a^2}{C_s C_p \Delta T \lambda} \right) \quad (\text{Dufour Number}), \quad Sr = \left( \frac{D_m K_t \Delta T}{\nu T_m \Delta C} \right) \quad (\text{Soret number})$$

$$Sc = \frac{\nu}{D_m} \quad (\text{Schmidt number}), \quad \alpha = \frac{QL^2}{\lambda} \quad (\text{Heat source parameter})$$

With the corresponding boundary conditions are;

$$u = 0, \quad \theta = 0, \quad C=1 \quad \text{at } r=1 \quad (9)$$

$$u = 0, \quad \frac{\partial \theta}{\partial r} = Q_1, \quad C = 0 \quad \text{at } r=1+s \quad (10)$$

### 3. FINITE ELEMENT ANALYSIS

The finite element analysis with quadratic polynomial approximation functions is carried out along the radial distance across the circular cylindrical annulus. The behavior of the velocity, temperature and concentration profiles has been discussed computationally for different variations in governing parameters. The Galerkin method have been adopted in the variational formulation in each element to obtain the global coupled matrices for the velocity, temperature and concentration in course of the finite element analysis.

Choose an arbitrary element  $e_k$  and let  $u^k$ ,  $\theta^k$  and  $C^k$  be the values of  $u, \theta$  and  $C$  in the element  $e_k$ .

We define the error residuals as

$$E_u^k = \frac{d}{dr} \left( r \frac{du^k}{dr} \right) + \delta G(\theta^k + NC^k) - \delta(D^{-1})ru^k - \delta^2 \Lambda r(u^k)^2 \quad (11)$$

$$E_\theta^k = \frac{d}{dr} \left( r \frac{d\theta^k}{dr} \right) - \alpha \theta^k - r P_r N_i u^k + Du N_i \frac{d}{dr} \left( r \frac{d^k}{dr} \right) \zeta \quad (12)$$

$$E_c^k = \frac{d}{dr} \left( r \frac{d^k}{dr} \right) C - r S_c N_c u^k + S_c S_r \frac{d}{dr} \left( r \frac{d\theta^k}{dr} \right) \quad (13)$$

Where  $u^k$ ,  $\theta^k$  &  $C^k$  are values of  $u, \theta$  &  $C$  in the arbitrary element  $e_k$ . These are expressed has linear combinations in terms of respective local nodal values.

#### 3.2 Variational Formulation:

The variational formulation associated with (11) – (13) over a typical three noded quadratic element is given by

$$\int_{rA}^{rb} w_1 E_u^k dr = 0 \quad (14)$$

$$\int_{rA}^{rb} w_2 E_\theta^k dr = 0 \quad (15)$$

$$\int_{rA}^{rb} w_3 E_c^k dr = 0 \quad (16)$$

Where  $w_1, w_2, w_3$  are arbitrary test functions and may be viewed as the variations in the functions  $u, \theta$  and  $C$  respectively.

#### 3.3 Finite element formulation

The finite element model may be obtained from equations (14)-(16) by substituting finite element approximations of the form

$$\begin{aligned} u^k &= u_1^k \psi_1^k + u_2^k \psi_2^k + u_3^k \psi_3^k \\ \theta^k &= \theta_1^k \psi_1^k + \theta_2^k \psi_2^k + \theta_3^k \psi_3^k \\ C^k &= C_1^k \psi_1^k + C_2^k \psi_2^k + C_3^k \psi_3^k \end{aligned} \quad (17)$$

With  $w_1=w_2=w_3=\psi_I$  (1,2,3)

where  $\psi_1^k, \psi_2^k, \dots$  etc are Lagrange's quadratic polynomials given by

$$\begin{aligned} \psi_1 &= \frac{(r - (\frac{2k-1}{n})(s_1-1))(r - s_1(\frac{2k}{n}-1))}{s_1(\frac{2k-2}{n} - \frac{2k-1}{n})(s_1(\frac{2k-2}{n} - \frac{2k}{n}))} \\ \psi_2 &= \frac{(r - s_1(\frac{2k-2}{n}-1))(r - s_1(\frac{2k}{n}-1))}{s_1(\frac{2k-2}{n} - \frac{2k-2}{n})(s_1(\frac{2k-1}{n} - \frac{2k}{n}))} \\ \psi_3 &= \frac{(r - s_1(\frac{2k-1}{n}-1))(r - s_1(\frac{2k-2}{n}-1))}{s_1(\frac{2k}{n} - \frac{2k-2}{n})(s_1(\frac{2k}{n} - \frac{2k-1}{n}))} \end{aligned} \quad (18)$$

Where k = 1, 2, -5.

Following the Gelarkin weighted residual method and integrating by parts (11) - (13) we obtain

$$\int_{r_{A_1}}^{r_{B_1}} r \frac{du^k}{dr} \frac{d\psi_j^k}{dr} dr - \delta G \int_{r_{A_1}}^{r_{B_1}} r(\theta^k + NC^k)\psi_j^k dr + \delta(D^{-1}) \int_{r_{A_1}}^{r_{B_1}} ru^k\psi_j^k dr + \delta^2 \Lambda \int_{r_{A_1}}^{r_{B_1}} r(u^k)^2\psi_j^k dr = Q_{2j}^k + Q_{1j}^k - Q_{1j}^k = \left[ \left( \frac{du^k}{dr} \right) (r\psi_j^k) \right]_{r_{A_1}}, -Q_{2j}^k = \left[ \left( \frac{du^k}{dr} \right) (r\psi_j^k) \right]_{r_{B_1}} \quad (19)$$

$$\int_{r_{A_1}}^{r_{B_1}} r \frac{d\theta^k}{dr} \frac{d\psi_j^k}{dr} dr - \alpha \int_{r_{A_1}}^{r_{B_1}} \theta^k \psi_j^i \psi_j^k r dr = N_i P_r \int_{r_{A_1}}^{r_{B_1}} ru^k\psi_j^k dr + R_{2j}^k + R_{1j}^k + DuN_i \int_{r_{A_1}}^{r_{B_1}} r \frac{dC^k}{dr} \frac{d\psi_j^k}{dr} dr - R_{1j}^k = \left[ \left( \frac{d\theta^k}{dr} \right) (r\psi_j^k) \right]_{r_{A_1}}, R_{2j}^k = \left[ \left( \frac{d\theta^k}{dr} \right) (r\psi_j^k) \right]_{r_{B_1}} \quad (20)$$

$$\int_{r_{A_1}}^{r_{B_1}} r \frac{dC^k}{dr} \frac{d\psi_j^k}{dr} dr = N_c Sc \int_{r_{A_1}}^{r_{B_1}} ru^k\psi_j^k dr + S_{2j}^k + S_{1j}^k + ScSr \int_{r_{A_1}}^{r_{B_1}} r \frac{d\theta^k}{dr} \frac{d\psi_j^k}{dr} dr - S_{1j}^k = \left[ \left( \frac{dC^k}{dr} \right) (r\psi_j^k) \right]_{r_{A_1}}, S_{2j}^k = \left[ \left( \frac{dC^k}{dr} \right) (r\psi_j^k) \right]_{r_{B_1}} \quad (21)$$

Expressing  $u^k, \theta^k, C^k$  in terms of local nodal values in (19) - (21) we obtain

$$\sum_{i=1}^3 u_i^k \int_{r_{A_1}}^{r_{B_1}} r \frac{d\psi_i^k}{dr} \frac{d\psi_j^k}{dr} dr - \delta G \sum_{i=1}^3 (\theta_i^k + NC_i^k) \int_{r_{A_1}}^{r_{B_1}} r\psi_i^k\psi_j^k dr + \delta D^{-1} \sum_{i=1}^3 \int_{r_{A_1}}^{r_{B_1}} r\psi_i^k\psi_j^k dr + \delta^2 \Lambda \sum_{i=1}^3 u_i^k \int_{r_{A_1}}^{r_{B_1}} rU_i^k\psi_i^k\psi_j^k dr = Q_{2j}^k + Q_{1j}^k \quad (22)$$

$$\sum_{i=1}^3 \theta_i^k \int_{r_{A_1}}^{r_{B_1}} r \frac{d\psi_i^k}{dr} \frac{d\psi_j^k}{dr} dr - \alpha \sum_{i=1}^3 \theta_i^k \int_{r_{A_1}}^{r_{B_1}} r\psi_i^k\psi_j^k dr - N_i P_r \sum_{i=1}^3 u_i^k \int_{r_{A_1}}^{r_{B_1}} r\psi_i^k\psi_j^k dr - \alpha \int_{r_{A_1}}^{r_{B_1}} dr + DuN_i \sum_{i=1}^3 C_i^k \int_{r_{A_1}}^{r_{B_1}} r \frac{d\psi_i^k}{dr} \frac{d\psi_j^k}{dr} dr = R_{2j}^k + R_{1j}^k \quad (23)$$

$$\sum_{i=1}^3 C_i^k \int_{r_{A_1}}^{r_{B_1}} r \frac{d\psi_i^k}{dr} \frac{d\psi_j^k}{dr} dr - N_c S_c \sum_{i=1}^3 u_i^k \int_{r_{A_1}}^{r_{B_1}} r \psi_i^k \psi_j^k dr + S_c S_r \sum_{i=1}^3 \theta_i^k \int_{r_{A_1}}^{r_{B_1}} r \frac{d\psi_i^k}{dr} \frac{d\psi_j^k}{dr} dr = S_{2j}^k + S_{1j}^k \quad (24)$$

choosing different  $\psi_j^k$ 's corresponding to each element  $e_k$  in the equation (22) yields a local stiffness matrix of order  $3 \times 3$  in the form

$$(f_{ij}^k)(u_i^k) - \delta G(g_{ij}^k)(\theta_i^k + NC_i^k) + \delta D^{-1}(m_{ij}^k)(u_i^k) + \delta^2 \Lambda(n_{ij}^k)(u_i^k) = (Q_{2j}^k) + (Q_{1j}^k) + (v_j^k) \quad (25)$$

Likewise the equation (23) & (24) gives rise to a Stiffness matrices

$$(e_{ij}^k)(\theta_i^k) - N_i P_r(t_{ij}^k)(u_i^k) + Du N_i(C_i^k) = R_{2j}^k + R_{1j}^k \quad (26)$$

$$(l_{ij}^k)(C_i^k) - N_2 c(t_{ij}^k)(u_i^k) + S_c S_r(\theta_i^k) = S_{2j}^k + S_{1j}^k \quad (27)$$

Where  $(f_{ij}^k), (g_{ij}^k), (m_{ij}^k), (n_{ij}^k), (e_{ij}^k), (l_{ij}^k)$  and  $(t_{ij}^k)$  are  $3 \times 3$  matrices and

$$v_j^k = -P_1 \int_{r_{A_1}}^{r_{B_1}} r \psi_i^k \psi_j^k dr$$

and,  $(Q_{2j}^k), (Q_{1j}^k), (R_{2j}^k) \& (R_{1j}^k), (S_{2j}^k) \& (S_{1j}^k)$  are  $3 \times 1$  column matrices and such stiffness matrices (25)- (27) in terms of local nodes an each elements are assembled using inter element continuity and equilibrium conditions to obtained the coupled global matrices in terms of the global nodal values of u,  $\theta$  and C in the region.

In fact, the non linear terms arises in the modified brinkman linear momentum equation (19) of the porous medium. The iteration procedure in taking the global matrices is as follows. We split the square term into a product term and keeping one of them say  $U_i$ 's under integration, the other is expanded in terms of local nodal values as in (22), resulting in corresponding coefficient matrices  $n_{ij}^k$  in (25), whose co efficient involve the  $u_i$ 's. To evaluate (25), to begin with, choose coefficients involve the unknown  $U_i$ 's as zeros in the zeroth approximation. We evaluate  $u_i$ 's,  $\theta_i$ 's and  $C_i$ 's in the usual procedure mentioned earlier. Later choosing these values of  $u_i$ 's as first order approximation calculate  $\theta_i$ 's and  $C_i$ 's. In the second iteration, we substitute for  $u_i$ 's the first order approximation of  $u_i$ 's and  $U_i$ 's and the first approximation of  $\theta_i$ 's and  $C_i$ 's and obtain second order approximation. This procedure is repeated till the consecutive values of  $U_i$ 's,  $\theta_i$ 's and  $C_i$ 's differ by a preassigned percentage.

The rate of heat transfer (Nusselt Number) is evaluated using the formula

$$Nu = -\left(\frac{d\theta}{dr}\right)_{r=1}$$

The rate of mass transfer (Sherwood Number) is evaluated using the formula

$$Sh = -\left(\frac{dC}{dr}\right)_{r=1,1+s}$$

In the absence of mass transfer and the results are in good agreement with padmavathi[10].

## 5. RESULTS and DISCUSSIONS

In this analysis we investigate non-darcy mixed convective heat and mass transfer flow of a viscous incompressible electrically conducting fluid through a porous medium in a cylindrical annulus in the presence of heat generating sources. The velocity, temperature and concentration have been discussed for variation of the governing parameters Viz, Schmidt number  $S_c$ , Soret parameter  $S_0$ , Dofour effect  $Du$  and buoyancy ratio  $N$ .

With respect to variation of  $w$  with  $S_c$  we find that lesser the molecular diffusivity smaller  $w$  in the flow field (fig-1). Fig-2 represents the variation of  $w$  with soret parameter  $S_0$ . An increase in  $S_0$  leads to a depreciation in  $w$

everywhere in the flow region. Fig-3 represents the variation of  $w$  with Dufour parameter  $Du$ . It is found that for an increase in  $Du \leq 0.3$ , we find an enhancement in the axial velocity  $w$  and for higher  $Du$ , we notice a depreciation in  $w$ . The variation of  $w$  with buoyancy ratio  $N$  exhibits that when the molecular buoyancy force dominates over the thermal buoyancy force the velocity experiences an enhancements irrespective of the directions of the buoyancy forces (fig-4).

With respect to variation of  $\theta$  with Schmidt number  $Sc$  we find that lesser the molecular diffusivity, lesser the temperature in the flow region (fig-5). An increase in the sorlet parameter  $S_0$  depreciates the temperature everywhere in the region (fig-6). The variation of  $\theta$  with Dufour Parameter  $Du$  reveals that higher the Duffer effect larger the temperature in the flow region and for further increase in  $Du$  lesser the temperature in the flow region (fig-7). When the Molecular buoyancy force dominates over the thermal buoyancy force, the temperature enhances when the buoyancy forces act in the same direction and for forces acting in opposite directions the temperature experiences a depreciation in the flow region (fig-8).

Fig-9 shows that lesser the molecular diffusivity smaller the concentration. Also an increase in  $S_0$  leads to a depreciation in 'C' with maximum at  $r=2$  (fig-10). With respect to variation of  $C$  with Dufour effect  $Du$ , we find that, for  $Du \leq 0.15$ , the concentration experiences depreciation and for further higher  $Du$ , the concentration enhances in flow field (fig-11). Also when the molecular buoyancy force dominates over the thermal buoyancy force the concentration enhances when the buoyancy forces act in the same direction and for forces acting in opposite directions the concentration reduces in the flow field (fig-12).

The Nusselt number ( $Nu$ ) which measures the rate of heat transfer at  $r=1$  is shown in table 1-4 for different values of the governing parameters. The rate of heat transfer at  $r=1$  enhances with increase in  $G$ . The variation of  $Nu$  with  $D^{-1}$  and  $M$  exhibits that lesser the permeability porous medium / higher the Lorentz force, larger the rate of heat transfer and for further increase in Lorentz force smaller  $|Nu|$  (table 1). An increase in the strength of heat generating source depreciates the rate of heat transfer at the inner cylinder. The variation of  $Nu$  with Schmidt number  $Sc$  shows that lesser the molecular diffusivity smaller  $|Nu|$ . Also it reduces with increase in the sorlet parameter  $S_0$  (table 3). From table 4, we notice that the rate of heat transfer increases with increase in the Dufour parameter  $Du$ . The variation of  $Nu$  with buoyancy ration  $N$  shows that when the molecular buoyancy force dominates over the thermal buoyancy force the rate of heat transfer enhances when the buoyancy force act in the same direction and for the forces acting in opposite directions it reduces at the inner cylinder.

The Sherwood number. ( $Sh$ ) which represents the rate of mass transfer at the inner and outer cylinder is shown in tables 5-12 for the different values of the parameters. It is noticed that an increase in the Grashof number  $G$  reduces the rate of mass transfer at  $r=1$  and enhances it  $r=2$ . Lesser the permeability of the porous medium higher  $|Sh|$  and for further lowering of permeability smaller  $|Sh|$  at  $r=1$ , while at  $r=2$  larger  $|Sh|$ . Also higher the Lorentz force smaller  $|Sh|$  and for further increase in the Lorentz force larger  $|Sh|$  at  $r=1$ , while at  $r=2$  larger  $|Sh|$  fixing the other parameters (Table 5&9). From the variation of  $Sh$  with  $\alpha$ , we notice that  $|Sh|$  decreases with  $\alpha \leq 4$  and enhances with  $\alpha \geq 6$  at  $r=1$  while at  $r=2$  a reversed effect is observed (table 6&10). With respect to the behavior of  $Sh$  with  $Sc$  we find that the lesser the molecular diffusivity smaller  $|Sh|$  at  $r=1$  and larger  $|Sh|$  at  $r=2$ . Also an increase in  $S_0$  reduces  $|Sh|$  at  $r=1$  and enhances it  $r=2$  (table 7&11). From table 8 & 12 we observe that an increase in the Dufour parameter  $Du \leq 0.15$  reduces  $|Sh|$  and for further higher  $Du \geq 0.3$  enhances  $|Sh|$  at  $r=1$  while at  $r=2$  larger  $|Sh|$ . The behavior of  $Sh$  with buoyancy ratio  $N$  shows that the rate of mass transfer enhances at  $r=1$  and reduces at  $r=2$  when the buoyancy forces act in the same direction while for the force acting in opposite directions  $Sh$  decrease at  $r=1$  and enhances at  $r=2$ .

**Figures:**

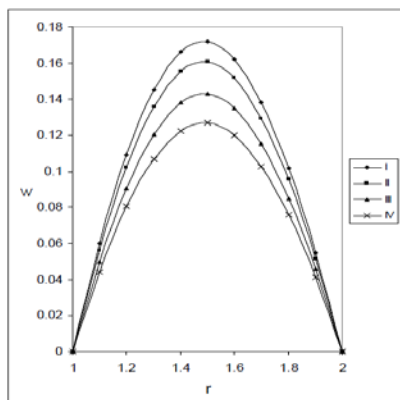


Fig-1: Variation of  $w$  with  $Sc$   
 $N=1, M=2, G=10^3, \alpha=2$   

|           |     |     |    |
|-----------|-----|-----|----|
| I         | II  | III | IV |
| $Sc$ 0.22 | 0.6 | 1.3 | 2  |

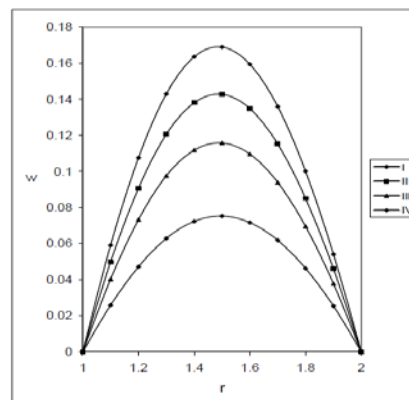


Fig-2: Variation of  $w$  with  $S_0$   
 $N=1, M=2, Sc=1.3, \alpha=2$   

|           |     |     |    |
|-----------|-----|-----|----|
| I         | II  | III | IV |
| $S_0$ 0.1 | 0.5 | 1   | 2  |

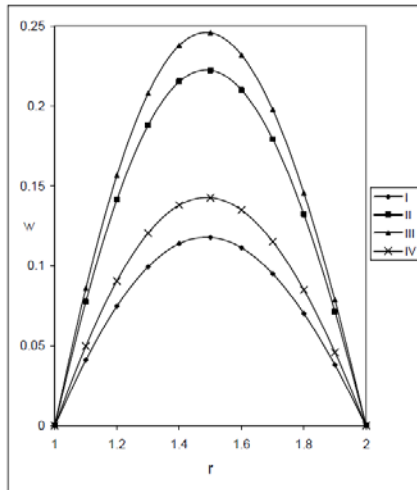


Fig-3: Variation of w with Du  
 $N=1, M=2, Sc=1.3, \alpha=2$   

|    |      |      |     |      |
|----|------|------|-----|------|
| I  | II   | III  | IV  |      |
| Du | 0.03 | 0.15 | 0.3 | 0.05 |

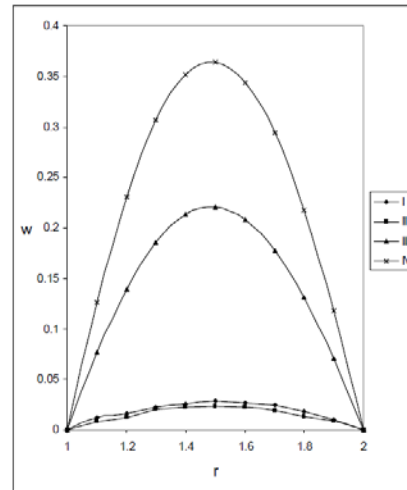


Fig-4: Variation of w with N  
 $G=10^3, M=2, Sc=1.3, \alpha=2$   

|   |      |      |    |   |
|---|------|------|----|---|
| I | II   | III  | IV |   |
| N | -0.8 | -0.5 | 1  | 2 |

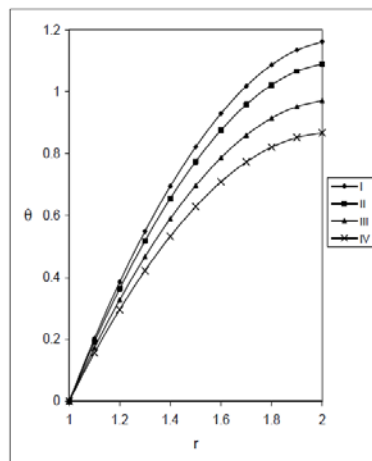


Fig-5: Variation of  $\theta$  with Sc  
 $N=1, M=2, G=10^3, \alpha=2$   

|    |      |     |     |   |
|----|------|-----|-----|---|
| I  | II   | III | IV  |   |
| Sc | 0.22 | 0.6 | 1.3 | 2 |

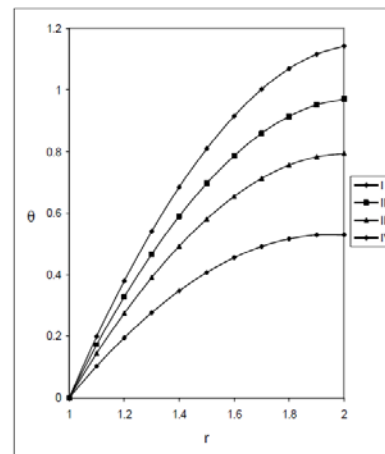


Fig-6: Variation of  $\theta$  with  $S_0$   
 $N=1, M=2, Sc=1.3, \alpha=2$   

|       |     |     |    |   |
|-------|-----|-----|----|---|
| I     | II  | III | IV |   |
| $S_0$ | 0.1 | 0.5 | 1  | 2 |

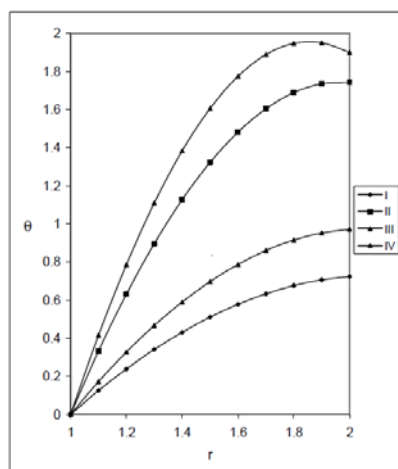


Fig-7: Variation of  $\theta$  with Du  
 $N=1, M=2, Sc=1.3, \alpha=2$   

|    |      |      |     |      |
|----|------|------|-----|------|
| I  | II   | III  | IV  |      |
| Du | 0.03 | 0.15 | 0.3 | 0.05 |

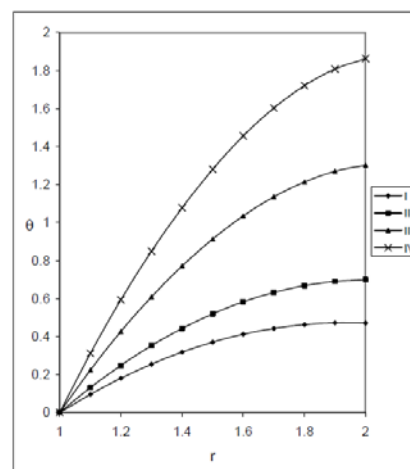


Fig-8: Variation of  $\theta$  with N  
 $G=10^3, M=2, Sc=1.3, \alpha=2$   

|   |      |      |    |   |
|---|------|------|----|---|
| I | II   | III  | IV |   |
| N | -0.8 | -0.5 | 1  | 2 |



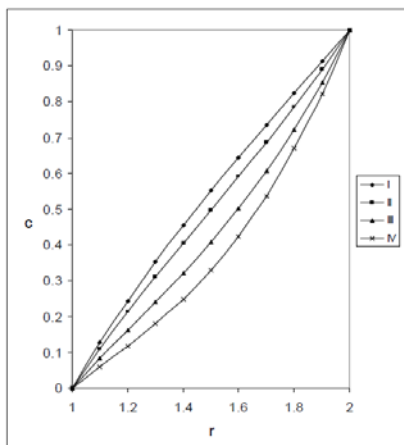


Fig-9: Variation of C with Sc  
 $N=1, M=2, G=10^3, \alpha=2$   
 I II III IV  
 $Sc$  0.22 0.6 1.3 2

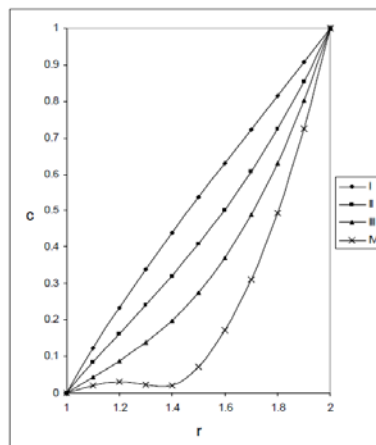


Fig-10: Variation of C with So  
 $N=1, M=2, Sc=1.3, \alpha=2$   
 I II III IV  
 $So$  0.1 0.5 1 2

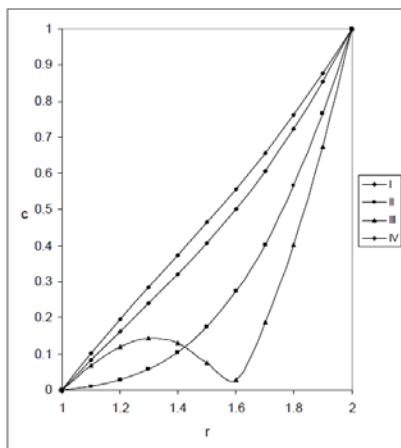


Fig-11: Variation of C with Du  
 $N=1, M=2, Sc=1.3, \alpha=2$   
 I II III IV  
 $Du$  0.03 0.15 0.3 0.05

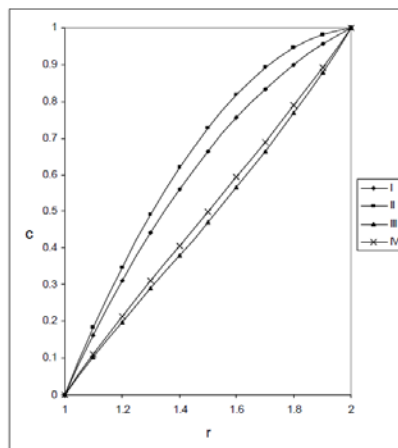


Fig-12: Variation of C with N  
 $G=10^3, M=2, Sc=1.3, \alpha=2$   
 I II III IV  
 $N$  -0.8 -0.5 1 2

**Tables:**

**Table-1**  
**Nusselt Number (Nu) at r=1**  
 $P=0.71, \alpha=2, N=0.5, Du=0.05$

| $D^{-1}$        | I       | II      | III     | IV       | V       | VI       |
|-----------------|---------|---------|---------|----------|---------|----------|
| $2 \times 10^3$ | 1.14589 | 1.34444 | 1.59305 | 0.672836 | 1.3126  | 0.688053 |
| $4 \times 10^3$ | 1.22481 | 1.58056 | 2.08636 | 0.807601 | 1.41212 | 0.770059 |
| G               | 200     | 400     | 600     | 500      | 500     | 500      |
| M               | 2       | 2       | 2       | 5        | 8       | 10       |

**Table-2**  
**Nusselt Number (Nu) at r=1**  
 $P=0.71, G=500, M=2, N=0.5, Du=0.05$

| $D^{-1}$        | I         | II        | III       |
|-----------------|-----------|-----------|-----------|
| $2 \times 10^3$ | -0.116159 | 0.0765303 | 0.0293443 |
| $4 \times 10^3$ | -0.12993  | 0.06695   | 0.0155975 |
| $6 \times 10^3$ | -0.194639 | 0.0237254 | -0.048843 |
| $\alpha$        | 0         | 4         | 6         |

**Table-3**  
Nusselt Number (Nu) at r=1  
P=0.71, G=500, M=2,  $\alpha=2, N=0.5, Du=0.05$

| D <sup>-1</sup>   | I       | II      | III     | IV      | V       | VI       |
|-------------------|---------|---------|---------|---------|---------|----------|
| 2x10 <sup>3</sup> | 1.66509 | 1.58967 | 1.34523 | 1.64972 | 1.25957 | 0.939433 |
| 4x10 <sup>3</sup> | 2.12127 | 2.00302 | 1.63569 | 2.09091 | 1.51663 | 1.08158  |
| 6x10 <sup>3</sup> | 6.37661 | 5.57539 | 3.60995 | 6.03591 | 3.17397 | 1.81244  |
| Sc                | 0.22    | 0.6     | 2       | 1.3     | 1.3     | 1.3      |
| So                | 0.5     | 0.5     | 0.5     | 0.1     | 1       | 2        |

**Table-4**  
Nusselt Number (Nu) at r=1  
P=0.71, G=500, M=2,  $\alpha=2$

| D <sup>-1</sup>   | I       | II      | III     | IV       | V       | VI      | VII     |
|-------------------|---------|---------|---------|----------|---------|---------|---------|
| 2x10 <sup>3</sup> | 1.01882 | 3.03863 | 3.98197 | 1.07906  | 1.30942 | 1.79475 | 2.33781 |
| 4x10 <sup>3</sup> | 1.31437 | 3.49447 | 4.37695 | 1.00435  | 1.37639 | 2.347   | 3.26111 |
| 6x10 <sup>3</sup> | 3.65698 | 6.41041 | 6.49674 | 0.231285 | 1.94802 | 6.63971 | 10.293  |
| Du                | 0.03    | 0.15    | 0.3     | 0.08     | 0.05    | 0.05    | 0.05    |
| N                 | 0.5     | 0.5     | 0.5     | -0.8     | -0.5    | 1       | 2       |

**Table-5**  
Sherwood Number (Sh) at r=1  
P=0.71,  $\alpha=2, N=0.5, Du=0.05$

| D <sup>-1</sup>   | I        | II       | III       | IV       | V        | VI      |
|-------------------|----------|----------|-----------|----------|----------|---------|
| 2x10 <sup>3</sup> | 0.983091 | 0.942269 | 0.891106  | 0.995113 | 0.693369 | 1.24229 |
| 4x10 <sup>3</sup> | 1.31293  | 0.902537 | 0.805895  | 0.987088 | 0.623739 | 1.1797  |
| 6x10 <sup>3</sup> | 0.913365 | 0.682087 | -0.144382 | 0.978458 | 0.268951 | 1.16018 |
| G                 | 200      | 400      | 600       | 500      | 500      | 500     |
| M                 | 2        | 2        | 2         | 5        | 8        | 10      |

**Table-6**  
Sherwood Number (Sh) at r=1  
P=0.71, Sr=0.5, G=500, M=2, N=0.5, Sc=1.3, Du=0.05

| D <sup>-1</sup>   | I       | II      | III     |
|-------------------|---------|---------|---------|
| 2x10 <sup>3</sup> | 1.48404 | 1.42136 | 1.44094 |
| 4x10 <sup>3</sup> | 1.49966 | 1.44244 | 1.46543 |
| 6x10 <sup>3</sup> | 1.57261 | 1.54631 | 1.58502 |
| $\alpha$          | 0       | 4       | 6       |

**Table-7**  
Sherwood Number (Sh) at r=1  
P=0.71, G=500, M=2,  $\alpha=2, N=0.5, Du=0.05$

| D <sup>-1</sup>   | I       | II       | III      | IV      | V         | VI       |
|-------------------|---------|----------|----------|---------|-----------|----------|
| 2x10 <sup>3</sup> | 1.34066 | 1.18492  | 0.674305 | 1.30477 | 0.496494  | -0.20291 |
| 4x10 <sup>3</sup> | 1.3271  | 1.15173  | 0.599842 | 1.2732  | 0.425982  | -0.25827 |
| 6x10 <sup>3</sup> | 1.20249 | 0.869434 | 0.104471 | 0.9921  | -0.014753 | -0.51998 |
| Sc                | 0.22    | 0.6      | 2        | 1.3     | 1.3       | 1.3      |
| So                | 0.5     | 0.5      | 0.5      | 0.1     | 1         | 2        |

**Table-8**  
Sherwood Number (Sh) at r=1  
P=0.71, G=500, M=2,  $\alpha = 2$

| D <sup>-1</sup>   | I       | II       | III      | IV      | V       | VI       | VII      |
|-------------------|---------|----------|----------|---------|---------|----------|----------|
| 2x10 <sup>3</sup> | 1.09586 | 0.151796 | -0.73749 | 1.71183 | 1.90858 | 1.12625  | 1.22052  |
| 4x10 <sup>3</sup> | 1.0447  | 0.081887 | -0.77864 | 1.70862 | 1.91505 | 1.06543  | 1.1484   |
| 6x10 <sup>3</sup> | 0.64604 | -0.35297 | -0.98362 | 1.67726 | 1.96842 | 0.598241 | 0.602521 |
| Du                | 0.03    | 0.15     | 0.3      | 0.05    | 0.05    | 0.05     | 0.05     |
| N                 | 0.5     | 0.5      | 0.5      | -0.8    | -0.5    | 1        | 2        |

**Table-9**  
Sherwood Number (Sh) at r=2  
P=0.71, Sr=0.5,  $\alpha = 2$ , Sc=1.3, N=0.5, Du=0.05

| D <sup>-1</sup>   | I        | II      | III     | IV      | V       | VI      |
|-------------------|----------|---------|---------|---------|---------|---------|
| 2x10 <sup>3</sup> | 1.28815  | 1.35395 | 1.43625 | 1.02982 | 1.1537  | 1.18404 |
| 4x10 <sup>3</sup> | 1.369794 | 1.44912 | 1.63086 | 1.0975  | 1.16152 | 1.19229 |
| 6x10 <sup>3</sup> | 1.46858  | 1.99316 | 3.85898 | 1.1202  | 1.14007 | 1.19245 |
| G                 | 200      | 400     | 600     | 500     | 500     | 500     |
| M                 | 2        | 2       | 2       | 5       | 8       | 10      |

**Table-10**  
Sherwood Number (Sh) at r=2  
P=0.71, Sr=0.5, G=500, M=2, Sc=1.3, N=0.5, Du=0.05

| D <sup>-1</sup>   | I        | II       | III      |
|-------------------|----------|----------|----------|
| 2x10 <sup>3</sup> | 0.664981 | 0.753701 | 0.733803 |
| 4x10 <sup>3</sup> | 0.657664 | 0.749735 | 0.729186 |
| 6x10 <sup>3</sup> | 0.608862 | 0.712612 | 0.68958  |
| $\alpha$          | 0        | 4        | 6        |

**Table-11**  
Sherwood Number (Sh) at r=2  
P=0.71, G=500, M=2,  $\alpha = 2$ , N=0.5, Du=0.05

| D <sup>-1</sup>   | I        | II      | III     | IV       | V       | VI      |
|-------------------|----------|---------|---------|----------|---------|---------|
| 2x10 <sup>3</sup> | 0.844062 | 1.04805 | 1.70141 | 0.880402 | 1.93168 | 2.74768 |
| 4x10 <sup>3</sup> | 0.873743 | 1.12232 | 1.88416 | 0.925964 | 2.13723 | 2.99602 |
| 6x10 <sup>3</sup> | 1.15298  | 1.76969 | 3.13881 | 1.33616  | 3.47756 | 4.29737 |
| Sc                | 0.22     | 0.6     | 2       | 1.3      | 1.3     | 1.3     |
| So                | 0.5      | 0.5     | 0.5     | 0.1      | 1       | 2       |

**Table-12**  
Sherwood Number (Sh) at r=2  
P=0.71, G=500, M=2,  $\alpha = 2$

| D <sup>-1</sup>   | I       | II      | III     | IV       | V         | VI      | VII      |
|-------------------|---------|---------|---------|----------|-----------|---------|----------|
| 2x10 <sup>3</sup> | 1.16753 | 2.34012 | 3.33846 | 0.38047  | 0.119096  | 1.12031 | 0.987642 |
| 4x10 <sup>3</sup> | 1.2833  | 2.5414  | 3.55429 | 0.394086 | 0.0985303 | 1.23725 | 1.09949  |
| 6x10 <sup>3</sup> | 2.209   | 3.84321 | 4.73026 | 0.53633  | -0.07959  | 2.15381 | 1.95773  |
| Du                | 0.03    | 0.15    | 0.3     | 0.05     | 0.05      | 0.05    | 0.05     |
| N                 | 0.5     | 0.5     | 0.5     | -0.8     | -0.5      | 1       | 2        |

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