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# Reliability Modelling of a Computer System with Independent H/W and S/W Failures Subject to Maximum Operation and Repair Times

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***ABSTRACT***

*In this paper, some reliability measures of a computer system of two cold standby identical units having hardware and software components are obtained using semi- Markov process and regenerative point technique. For this purpose, a reliability model is developed considering independent h/w and s/w failures. There is a single server who visits the system immediately to do preventive maintenance, repair and replacement of the components upon their failures. The unit undergoes for preventive maintenance after a maximum operation time at normal mode. If repair of the component is not possible up to a pre-specific time (called Maximum Repair Time), the components are replaced by new one with some replacement time. However, only replacement of the software components by new one is made after failures. The failure time of the components follow negative exponential distribution while the distributions of preventive maintenance, repair and replacement time are taken as arbitrary with different probability density functions. The graphical behaviour of the results has also been shown for a particular case to improve the importance of the study.*

***Key Words**: Computer System, H/W and S/W Failure, Maximum Operation and Repair Time, Preventive Maintenance, Reliability and Economic Measures.*

***2000 Mathematics Subject Classification:** 90B25 and 60K10.*

## 1. INTRODUCTION

As a result of the rapid advance technological advances in microelectronics and micro processors, problems of reliability have changed from those involving hardware to those involving software. Today, a large literature exists in the area of hardware reliability and software reliability. However, most of the researchers in these areas have been limited to the consideration of either the hardware subsystem alone or the software subsystem alone. A few researcher including Friedman and Tran [1992] and wilke et al. [1995] tried to establish a combined reliability model for the whole system including both H/W and S/W.

There are many complex systems such as computer systems in which hardware and software components work together to provide computer functionality. The continued operation and ageing of these systems gradually reduce their performance, reliability and safety. And, a breakdown of such systems is costly, dangerous and may create confusion in our society. It is, therefore, of great importance to operate such systems with high reliability. It is proved that / It is proved that preventive maintenance can slow the deterioration process of a repairable system and restore the system in a younger age or state. Thus, the method of preventive maintenance can be used to improve the reliability and profit of system. Recently, Malik et. al [2009] and Malik and Anand [2010] have proposed reliability models for complex systems including a computer system introducing the concept of preventive maintenance of the unit after a maximum operation time. Further, the reliability of a system can be increased by making replacement of the components by new one in case repair time is too long i.e., if it extends to a prespecific time. Singh and Agrafiotis [1995] analyzed stochastically a two-unit cold standby system subject to maximum operation and repair time.

In view of the above and considering the practical importance of computer systems in our daily lives, a reliability model for a computer system of two identical units having h/w and s/w components is developed. Initially one unit is operative and other is kept as cold standby. The hardware and software components in the unit fail independently. There is a single server who visits the system immediately for carrying out preventive maintenance, repair and replacement of the components upon their failures. The unit undergoes for preventive maintenance after a maximum operation time at normal mode. The unit fails completely directly from normal mode. If repair of the component is not possible up to a pre-specific time (called Maximum Repair Time), the components are replaced by new one with some

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replacement time. However, only replacement of the software components by new one is made after failures. The random variables are independent and uncorrelated to each other. The switch devices, preventive maintenance and repair are perfect. The failure time of the unit due to failure of h/w and s/w components are exponentially, while the distributions of preventive maintenance, repair and replacement time taken as arbitrary with different probability density functions. The expressions for various reliability measures such as mean time to system failure, availability , busy period of the server due to preventive maintenance, busy period of the server due to repair, busy period of the server due to hardware replacement , busy period of the server due to software replacement expected number of software replacements , expected number of hardware replacement and expected number of visits of the server are derived by using semi-Markov process and regenerative point technique. The graphical study of the results for a particular case has also been made.

## 2. NOTATIONS

| | | |
|---|---|---|
| E | : | The set of regenerative states |
| NO | : | The unit is operative and in normal mode |
| Cs | : | The unit is cold standby |
| a/b | : | Probability that the system has hardware / software failure |
| $\lambda_1/\lambda_2$ | : | Constant hardware / software failure rate |
| $\alpha_0$ | : | Maximum Operation Time |
| $\beta_0$ | : | Maximum Repair Time. |
| Pm/PM | : | The unit is under preventive Maintenance/ under preventive maintenance continuously from previous state |
| WPm/WPM | : | The unit is waiting for preventive Maintenance/ waiting for preventive maintenance from previous state |
| HFur/HFUR | : | The unit is failed due to hardware and is under repair / under repair continuously from previous state |
| HFurp/HFURP | : | The unit is failed due to hardware and is under replacement / under replacement continuously from previous state |
| HFwr / HFWR | : | The unit is failed due to hardware and is waiting for repair/waiting for repair continuously from previous state |
| SFurp/SFURP | : | The unit is failed due to the software and is under replacement/under replacement continuously from previous state |
| SFwrp/SFWRP | : | The unit is failed due to the software and is waiting for replacement / waiting for replacement continuously from previous state |
| h(t) / H(t) | : | pdf / cdf of replacement time of unit due to software |
| g(t) / G(t) | : | pdf / cdf of repair time of the hardware |
| m(t)/ M(t) | : | pdf / cdf of replacement time of the hardware |
| f(t) / F(t) | : | pdf / cdf of the time for PM of the unit |
| $q_{ij}(t)/Q_{ij}(t)$ | : | pdf / cdf of passage time from regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in (0, t] |
| pdf / cdf | : | Probability density function/ Cumulative density function |
| $q_{ij.kr}(t)/Q_{ij.kr}(t)$ | : | pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k, r once in (0, t] |
| $\mu_i(t)$ | : | Probability that the system up initially in state $S_i \in E$ is up at time t without visiting to any regenerative state |
| $W_i(t)$ | : | Probability that the server is busy in the state $S_i$ upto time 't'without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states. |
| $m_{ij}$ | : | Contribution to mean sojourn time ($\mu_i$) in state $S_i$ when system transit directly to state $S_j$ so that $\mu_i = \sum_j m_{ij}$ and $m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^{*\prime}(0)$ |
| Ⓢ/© | : | Symbol for Laplace-Stieltjes convolution/Laplace convolution |
| ~ / * | : | Symbol for Laplace Steiltjes Transform (LST) / Laplace Transform (LT) |
| ' (desh) | : | Used to represent alternative result |

The following are the possible transition states of the system:

## 3. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Simple probabilistic considerations yield the following expressions by taking $A = a\lambda_1 + b\lambda_2 + \alpha_0$, $B = a\lambda_1 + b\lambda_2 + \alpha_0 + \beta_0$ for the non-zero elements

 

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt \quad \text{as} \tag{1}$$

$p_{01}=\dfrac{\alpha_0}{A}$, $p_{02}=\dfrac{a\lambda_1}{A}$, $p_{03}=\dfrac{b\lambda_2}{A}$, $p_{10}=f^*(A)$, $p_{16}=\dfrac{a\lambda_1}{A}[1-f^*(A)]=p_{12.6}$, $p_{18}=\dfrac{b\lambda_2}{A}[1-f^*(A)]=p_{13.8}$,

$p_{1.13}=\dfrac{\alpha_0}{A}[1-f^*(A)]=p_{11.13}$, $p_{20}=g^*(B)$, $p_{24}=\dfrac{\beta_0}{B}[1-g^*(B)]$, $p_{25}=\dfrac{\alpha_0}{B}[1-g^*(B)]$, $p_{2.11}=\dfrac{b\lambda_2}{B}[1-g^*(B)]$,

$p_{2.12}=\dfrac{a\lambda_1}{B}[1-g^*(B)]$, $p_{30}=h^*(A)$, $p_{37}=\dfrac{a\lambda_1}{A}[1-h^*(A)]=p_{32.7}$, $p_{39}=\dfrac{\alpha_0}{A}[1-h^*(A)]=p_{3,1.9}$, $p_{40}=m^*(A)$,

$p_{3,10}=\dfrac{b\lambda_2}{A}[1-h^*(A)]=p_{33.10}$, $p_{51}=g^*(\beta_0)$, $p_{5,16}=1-g^*(\beta_0)$, $p_{4.17}=\dfrac{\alpha_0}{A}[1-m^*(A)]=p_{4,1.17}$, $p_{62}=f^*(0)$,

$p_{72}=h^*(0)$, $p_{83}=f^*(0)$, $p_{91}=h^*(0)$, $p_{10.3}=h^*(0)$, $p_{11.3}=g^*(\beta_0)$, $p_{11.14}=1-g^*(\beta_0)$, $p_{4,18}=\dfrac{b\lambda_2}{A}[1-m^*(A)]=p_{43.18}$,

$p_{12.2}=g^*(\beta_0)$, $p_{12.15}=1-g^*(\beta_0)$, $p_{13.1}=f^*(0)$, $p_{14.3}=m^*(0)$, $p_{4.19}=\dfrac{a\lambda_1}{A}[1-m^*(A)]=p_{42.19}$, $p_{15.2}=m^*(0)$,

$p_{16.1}=m^*(0)$, $p_{17.1}=m^*(0)$, $p_{18.3}=m^*(0)$, $p_{19.2}=m^*(0)$, $p_{21.5}=\dfrac{\alpha_0}{B}[1-g^*(B)]\,g^*(\beta_0)$,

$p_{21,5.16.}=\dfrac{\alpha_0}{B}[1-g^*(B)][1-g^*(\beta_0)]$, $p_{23.11}=\dfrac{b\lambda_2}{B}[1-g^*(B)][g^*(\beta_0)]$, $p_{23.11,14}=\dfrac{b\lambda_2}{B}[1-g^*(B)][1-g^*(\beta_0)]$,

$p_{22.12}=\dfrac{a\lambda_1}{B}[1-g^*(B)]\,g^*(\beta_0)$, $p_{22.12,15}=\dfrac{a\lambda_1}{B}[1-g^*(B)][1-g^*(\beta_0)]$ (2)

It can be easily verified that

$$\begin{aligned}
p_{01}+p_{02}+p_{03} &= p_{10}+p_{16}+p_{18}+p_{1.13} = p_{20}+p_{24}+p_{25}+p_{2,11}+p_{2.12}\\
&= p_{30}+p_{37}+p_{39}+p_{3,10} = p_{40}+p_{4.17}+p_{4.18}+p_{4.19} = p_{5.1}+p_{5.16} = p_{62} = p_{72} = p_{83} = p_{91} = p_{10.3}\\
&= p_{11.3}+p_{11.14} = p_{12.2}+p_{12.15} = p_{13.1} = p_{14.1} = p_{15.2} = p_{16.1} = p_{17.1} = p_{18.3} = p_{19.2}\\
&= p_{10}+p_{12.6}+p_{11.13}+p_{13.8} = p_{20}+p_{24}+p_{21.5}+p_{21.5,16}+p_{23,11}+p_{23.11,14}+p_{22,12}+p_{22.12,15}\\
&= p_{30}+p_{31.9}+p_{32.7}+p_{33.10} = p_{40}+p_{41.17}+p_{42.19}+p_{43.18} = 1
\end{aligned} \tag{3}$$

The mean sojourn times ($\mu_i$) is the state $S_i$ are

$$\mu_0=\frac{1}{a\lambda_1+b\lambda_2+\alpha_0},\quad \mu_1=\frac{1}{a\lambda_1+b\lambda_2+\alpha_0+\alpha},\quad \mu_2=\frac{1}{a\lambda_1+b\lambda_2+\alpha_0+\theta+\beta_0},$$

$$\mu_3=\frac{1}{a\lambda_1+b\lambda_2+\alpha_0+\beta},\quad \mu_4=\frac{1}{a\lambda_1+b\lambda_2+\alpha_0+\gamma}, \tag{4}$$

Also

$$\begin{aligned}
& m_{01}+m_{02}+m_{03}=\mu_0 && m_{10}+m_{16}+m_{18}+m_{1.13}=\mu_1\\
& m_{20}+m_{24}+m_{25}+m_{2.11}+m_{2.12}=\mu_2 && m_{30}+m_{37}+m_{39}+m_{3.10}=\mu_3\\
& m_{40}+m_{4.17}+m_{4.18}+m_{4.19}=\mu_4 && m_{51}+m_{5.16}=\mu_5 \quad m_{11.14}+m_{11.3}=\mu_{11}\\
& m_{12.15}+m_{12.2}=\mu_{12} \quad m_{62}=\mu_6 \quad m_{72}=\mu_7, && m_{83}=\mu_8,\ m_{91}=\mu_9,\ m_{10.3}=\mu_{10},
\end{aligned}$$

$$m_{10}+m_{12.6}+m_{13.8}+m_{11.13}=\mu_1' \text{ (say)}$$

$$m_{20}+m_{24}+m_{21.5}+m_{21.5,16}+m_{22.12}+m_{22.12,15}+m_{23.11}+m_{23.11,14}=\mu_2' \text{ (say)}$$

$$m_{30}+m_{31.9}+m_{32.7}+m_{33.10}=\mu_3' \text{ (say)},\quad m_{40}+m_{42.19}+m_{43.18}+m_{41.17}=\mu_4'(say) \tag{5}$$

## 4. RELIABILITY AND MEAN TIME TO SYSTEM FAILURE (MTSF)

Let $\phi_i(t)$ be the c.d.f of first passage time from the regenerative state i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relation for $\phi_i(t)$:

$$\phi_i(t) = \sum_j Q_{i,j}(t) \circledR \phi_j(t) + \sum_k Q_{i,k}(t) \tag{6}$$

Where *j* is an un-failed regenerative state to which the given regenerative state *i* can transit and *k* is a failed state to which the state *i* can transit directly.

Taking LT of above relation (6) and solving for $\tilde{\phi}_0(s)$

We have

$$R^*(s) = \frac{1-\tilde{\phi}_0(s)}{s} \tag{7}$$

The reliability of the system model can be obtained by taking Laplace inverse transform of (7). The mean time to system failure (MTSF) is given by

$$\text{MTSF} = \lim_{s\to o} \frac{1-\tilde{\phi}_0(s)}{s} = \frac{N_1}{D_1} \quad \text{where} \tag{8}$$

$N_1 = \mu_0 + p_{01}\mu_1 + p_{02}\mu_2 + p_{03}\mu_3 + p_{24}p_{02}\mu_4$ and $D_1 = 1 - p_{01}p_{10} - p_{02}p_{20} - p_{03}p_{30} - p_{02}p_{24}p_{40}$

## 5. STEADY STATE AVAILABILITY

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state i at t = 0. The recursive relations for $A_i(t)$ are given as

$$A_i(t) = M_i(t) + \sum_j q_{i,j}^{(n)}(t) \copyright A_j(t) \tag{9}$$

Where *j* is any successive regenerative state to which the regenerative state *i* can transit through n≥1(natural number) transitions. $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state, we have

$$M_0(t) = e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}, M_1(t) = e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}\overline{F(t)}, M_2(t) = e^{-(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)t}\overline{G(t)}$$
$$M_3(t) = e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}\overline{H(t)}, \ M_4(t) = e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}\overline{M(t)} \tag{10}$$

Taking LT of above relations (9) and solving for $A_0^*(s)$, the steady state availability is given by

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2} \text{, where} \tag{11}$$

$$M_0(t) = e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}, M_1(t) = e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}\overline{F(t)}, M_2(t) = e^{-(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)t}\overline{G(t)}$$
$$M_3(t) = e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}\overline{H(t)}, \ M_4(t) = e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}\overline{M(t)} \tag{11}$$

Taking LT of above relations (10) and solving for $A_0^*(s)$, the steady state availability is given by

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2} \text{, where} \tag{12}$$

$N_2 = (1-p_{22.12}-p_{22.12,15}-p_{24}p_{42.19})[\ \mu_0p_{10}\ (1-\ p_{33.10}) + \ \mu_0p_{13.8}p_{30} + \mu_1\ (1-\ p_{33.10}) - \ \mu_1p_{03}p_{30} + \ \mu_3p_{13.8} + \ \mu_3p_{03}p_{10}] + (p_{23.11} + p_{23.11,14} + p_{24}p_{43.18})\ [-\mu_0p_{10}p_{32.7} - p_{32.7}\ \mu_0 - \ \mu_1p_{02}p_{30} + \ p_{12.6}\ \mu_3 + \ p_{02}p_{10}\ \mu_3 + \ p_{12.6}\ \mu_0p_{30}] + (p_{20} + p_{24}\ p_{40})[\ \mu_0p_{12.6}\ (1-\ p_{33.10}) + \mu_0p_{13.8}p_{32.7}] + (\mu_4p_{24} + \mu_2)\ (p_{02})\ [p_{10}\ (1-\ p_{33.10}) + p_{13.8}p_{30}] + \ (p_{20} + p_{24}\ p_{40})[\ -\mu_1p_{02}\ (1-\ p_{33.10}) - \mu_1p_{03}p_{32.7}] - \ (\mu_4p_{24} + \mu_2)\ (p_{03})\ [p_{30}\ p_{12.6} - p_{32.7}p_{10}] + (p_{20} + p_{24}\ p_{40})[\ -\mu_3p_{02}\ p_{13.8} + \mu_3p_{03}p_{12.6}]$

 

and

$D_2$ = (1-$p_{22.12}$-$p_{22.12,15}$-$p_{24}p_{42.19}$)[ $\mu_0$ (1- $p_{11.13}$) (1- $p_{33.10}$)- $\mu_0 p_{13.8}p_{31.9}$ + $p_{01}\mu_1'$(1- $p_{33.10}$)+ $\mu_3'$ $p_{01}p_{13.8}$+ $p_{03}\mu_1'$ $p_{31.9}$+ $p_{03}\mu_3'$(1-$p_{33.10}$)] +($p_{23.11}$+ $p_{23.11,14}$+$p_{24}p_{43.18}$) [-$\mu_0$ ( 1 – $p_{11.13}$) $p_{32.7}$- $p_{31.9}p_{12.6}$ $\mu_0$- $p_{01}\mu_1'$ $p_{32.7}$ + $p_{01}\mu_3'$ $p_{12.6}$ + $p_{02}\mu_1'$ $p_{31.9}$ + $p_{02}\mu_3'$(1-$p_{11.13}$)] + ($p_{21.5}$ + $p_{21.5,16}$+$p_{24}p_{41.17}$) [-$\mu_0 p_{12.6}$ ( 1 - $p_{33.10}$) + $\mu_0 p_{13.8}p_{32.7}$ + $p_{02}\mu_1'$(1- $p_{33.10}$) + $p_{03}\mu_1'$ $p_{32.7}$ + $p_{02}\mu_3'$ $p_{13.8}$ - $p_{03}\mu_3'$ $p_{12.6}$] + $p_{01}$ ( $\mu_2'$+ $p_{24}\mu_4'$)[ $p_{12.6}$ (1- $p_{33.10}$) + $p_{13.8}p_{32.7}$] + $p_{02}$ ( $\mu_2'$+ $p_{24}\mu_4'$)[(1- $p_{11.13}$) (1- $p_{33.10}$) - $p_{13.8}p_{31.9}$] + $p_{03}$ ( $\mu_2'$+ $p_{24}\mu_4'$)[ $p_{32.7}$ (1- $p_{11.13}$) + $p_{12.6}p_{31.9}$]

## 6. BUSY PERIOD ANALYSIS FOR SERVER

### (a) Due to Preventive Maintenance (PM)

Let $B_i^P(t)$ be the probability that the server is busy in preventive maintenance of the unit at an instant ‘t’ given that the system entered state i at t = 0. The recursive relations $B_i^P(t)$ for are as follows:

$$B_i^p\left(t\right)=W_i\left(t\right)+\sum_j q_{i,j}^{(n)}\left(t\right)©B_j^p\left(t\right) \tag{12}$$

Where *j* is any successive regenerative state to which the regenerative state *i* can transit through n≥1(natural number) transitions. $W_1$(t) be the probability that the server is busy in state $S_i$ due to preventive maintenance up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_1=e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}\overline{F}(t)+(\alpha_0e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}©1\overline{F}(t)+(a\lambda_1e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}©1\overline{F}(t)+(b\lambda_2e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}©1\overline{F}(t)$$

### (b) Due to Hardware Failure

Let $B_i^R(t)$ be the probability that the server is busy in repairing the unit due to hardware failure at an instant ‘t’ given that the system entered state i at t = 0. The recursive relations $B_i^R(t)$ for are as follows:

$$B_i^R\left(t\right)=W_i\left(t\right)+\sum_j q_{i,j}^{(n)}\left(t\right)©B_j^R\left(t\right) \tag{13}$$

where *j* is any successive regenerative state to which the regenerative state *i* can transit through n≥1(natural number) transitions. $W_2$(t) be the probability that the server is busy in state $S_i$ due to hardware failure up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_2=e^{-(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)t}\overline{G}(t)+(\alpha_0e^{-(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)t}©1\overline{G}(t)+(a\lambda_1e^{-(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)t}©1\overline{G}(t)+(b\lambda_2e^{-(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)t}©1\overline{G}(t)$$

### (c) Due to replacement of the software

Let $B_i^S$ (t)be the probability that the server is busy due to replacement of the software at an instant ‘t’ given that the system entered the regenerative state i at t = 0. We have the following recursive relations for $B_i^S$ (t):

$$B_i^S\left(t\right)=W_i\left(t\right)+\sum_j q_{i,j}^{(n)}\left(t\right)©B_j^S\left(t\right) \tag{14}$$

Where *j* is any successive regenerative state to which the regenerative state *i* can transit through n≥1(natural number) transitions. $W_3$ (t) be the probability that the server is busy in state $S_i$ due to replacement of the software up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_3=e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}\overline{H}(t)+(\alpha_0e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}©1\overline{H}(t)+(a\lambda_1e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}©1\overline{H}(t)+(b\lambda_2e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}©1\overline{H}(t)$$

### (d) Due to Hardware Replacement

Let $B_i^{HRp}(t)$ be the probability that the server is busy in replacement of the unit due to hardware failure at an instant ‘t’ given that the system entered state i at t = 0. The recursive relations $B_i^{HRp}(t)$ for are as follows:

$$B_i^{HRp}\left(t\right)=W_i\left(t\right)+\sum_j q_{i,j}^{(n)}\left(t\right)©B_j^{HRp}\left(t\right) \tag{15}$$

Where *j* is any successive regenerative state to which the regenerative state *i* can transit through n≥1 (natural number) transitions. $W_4(t)$ be the probability that the server is busy in state $S_i$ due to hardware failure up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_4 = e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}\overline{M}(t)+(\alpha_0 e^{-(a\lambda_1+b\lambda_2+\alpha_0)t} © 1)\overline{M}(t)+(a\lambda_1 e^{-(a\lambda_1+b\lambda_2+\alpha_0)t} © 1)\overline{M}(t)+(b\lambda_2 e^{-(a\lambda_1+b\lambda_2+\alpha_0)t} © 1)\overline{M}(t)$$

Taking LT of above relations (12) to (15). And, solving for $B_0^{*H}$ (s) and $B_0^{*S}$ (s), the time for which server is busy due to repair and replacements respectively is given by

$$B_0^H = \lim_{s\to 0} sB_0^{*H}(s) = \frac{N_3^H}{D_2}, \quad B_0^S = \lim_{s\to 0} sB_0^{*S}(s) = \frac{N_3^S}{D_2}, \quad B_0^R = \lim_{s\to 0} sB_0^{*R}(S) = \frac{N_S^R}{D_2}$$

$$\text{and} \quad B_0^{HRp} = \lim_{s\to 0} sB_0^{*HRp}(S) = \frac{N_S^{HRp}}{D_2} \tag{16}$$

$$N_3^P = \mu_1^{'}\{(1-p_{22.12}-p_{22.12,15}p_{20}-p_{42.19}p_{24})[(1-p_{33.10})-p_{03}p_{30}]-(p_{23.11}+p_{23.11,14} + p_{43.18}p_{24})[p_{32.7}-p_{02}p_{30}]-(p_{20}+p_{24}p_{40})[(1-p_{33.10})p_{02}+(p_{03}p_{32.7}]\}$$

$$N_3^R = p_{01}(1-p_{33.10})p_{12.6}-p_{13.8}p_{32.7})+p_{02}(1-p_{33.10})(1-p_{11.13})-p_{13.8}p_{31.9}) + p_{03}(1-p_{33.10})p_{32.7}-p_{12.6}p_{31.9})$$

$$N_3^S = \mu_3\{(1-p_{22.12}-p_{22.12,15}-p_{42.19}p_{24})[(1-p_{11.13})p_{03}-p_{13.8}p_{01}]+(p_{23.11}+p_{23.11,14} + p_{43.18}p_{24})[(1-p_{11.13})p_{02}+p_{12.6}p_{01}]+(p_{02}p_{13.8}-p_{03}p_{12.6})(p_{21.5}+p_{21.5,16}+p_{41.17}p_{24})\}$$

$$N_3^{HRp} = \mu_4 p_{24}[p_{01}\{(1-p_{33.10})p_{12.6}+p_{13.8}p_{32.7})\}+p_{02}\{(1-p_{33.10})(1-p_{11.13})-p_{13.8}p_{31.9})\} + p_{03}\{(1-p_{11.13})p_{32.7}+p_{12.6}p_{31.9})\}]$$ and D2 is already mentioned.

## 7. EXPECTED NUMBER OF REPLACEMENTS OF THE UNITS

### (a) Due to Hardware Failure

Let $R_i^H(t)$ be the expected number of replacements of the failed hardware components by the server in (0, t] given that the system entered the regenerative state i at t = 0.

The recursive relations for $R_i^H(t)$ are given as

$$R_i^H(t) = \sum_j q_{i,j}^{(n)}(t)®\left[\delta_j + R_j^H(t)\right] \tag{17}$$

Where j is any regenerative state to which the given regenerative state *i* transits and δj =1, if *j* is the regenerative state where the server does job afresh, otherwise δj = 0.

### (b) Due to Software Failure

Let $R_i^S(t)$ be the expected number of replacements of the failed software by the server in (0, t]
given that the system entered the regenerative state i at t = 0. The recursive relations for $R_i^S(t)$ are given as

$$R_i^S(t) = \sum_j q_{i,j}^{(n)}(t)®\left[\delta_j + R_j^S(t)\right] \tag{18}$$

Where j is any regenerative state to which the given regenerative state *i* transits and δj =1, if *j* is the regenerative state where the server does job afresh, otherwise δj = 0.

Taking LT of relations (17) and (18). And, solving for $\tilde{R}_0^H(s)$ and $\tilde{R}_0^S(s)$. The expected numbers of replacements per unit time to the hardware and software failures are respectively of given by

$$R_0^H(\infty) = \lim_{s\to 0} s\tilde{R}_0^H(s) = \frac{N_4^H}{D_2} \quad \text{and} \quad R_0^S(\infty) = \lim_{s\to 0} s\tilde{R}_0^S(s) = \frac{N_4^S}{D_2} \tag{19}$$

 

Where $D_2$ is already mentioned.

$$N_4^H = (p_{23.11,14} + p_{21.5,16} + p_{22.12,15} + p_{24})\{p_{01}[(1-p_{33.10})p_{12.6} + p_{13.8}p_{32.7}] + p_{02}$$
$$[(1-p_{11.13})(1-p_{33.10}) - p_{31.9}p_{13.8}] + p_{03}[(1-p_{11.13})p_{32.7} + p_{31.9}p_{12.6}]\}$$
$$N_4^S = \{(1-p_{22.12} - p_{22.12,15} - p_{42.19}p_{24})[(1-p_{11.13})p_{03} + p_{13.8}p_{01}] + (p_{23.11} + p_{23.11,14} + p_{43.18}p_{24})$$
$$[(1-p_{11.13})p_{02} + p_{12.6}p_{01}] + (p_{02}p_{13.8} - p_{03}p_{12.6})(p_{21.5} + p_{21.5,16} + p_{41.17}p_{24})\}$$

## 8. EXPECTED NUMBER OF VISITS BY THE SERVER

Let $N_i(t)$ be the expected number of visits by the server in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $N_i(t)$ are given as

$$N_i\ (t) = \sum_j q_{i,j}^{(n)}(t) ® \left[\delta_j + N_j(t)\right] \tag{20}$$

Where j is any regenerative state to which the given regenerative state *i* transits and $\delta j$=1, if *j* is the regenerative state where the server does job afresh, otherwise $\delta j$= 0. Taking LT of relation (20) and solving for $\tilde{N}_0(s)$. The expected number of visit per unit time by the server are given by $N_0(\infty) = \lim_{s\to 0} s\tilde{N}_0(s) = \frac{N_5}{D_2}$, (21)

where

$N_5 = (1-p_{22.12}-p_{22.12,15}-p_{24}p_{42.19})[(1 - p_{11.13})(1- p_{33.10}) - p_{13.8}p_{31.9}] - (p_{23.11}+ p_{23.11,14}+p_{24}p_{43.18})$
$[(1 – p_{11.13}) p_{32.7} + p_{31.9} p_{12.6}] - (p_{21.5} + p_{21.5,16}+p_{24}p_{41.17})[p_{12.6} (1- p_{33.10}) + p_{13.8}p_{32.7}]$

## 9. ECONOMIC ANALYSIS

The profit incurred to the system model in steady state can be obtained as

$$P = K_0A_0 - K_1B_0^P - K_2B_0^R - K_3B_0^S - K_4B_0^{HRp} - K_5R_0^H - K_6R_0^S - K_7N_0 \tag{22}$$

$K_0$ = Revenue per unit up-time of the system
$K_1$ = Cost per unit time for which server is busy due preventive maintenance
$K_2$ = Cost per unit time for which server is busy due to hardware failure
$K_3$ = Cost per unit replacement of the failed software component
$K_4$ = Cost per unit replacement of the failed hardware component
$K_5$ = Cost per unit replacement of the failed hardware
$K_6$ =. Cost per unit replacement of the failed software
$K_7$ = Cost per unit visit by the server

## CONCLUSION

In the present study, considering a particular case by taking $g(t) = \theta e^{-\theta t}$, $h(t) = \beta e^{-\beta t}$, $f(t) = \alpha e^{-\alpha t}$ and $m(t) = \gamma e^{-\gamma t}$ the numerical results are obtained for some reliability and economic measures of a computer system of two identical units having h/w and s/w components. The graphs for mean time to system failure (MTSF), availability and profit are drawn with respect to preventive maintenance (α) rate for fixed values of parameters as shown respectively in fig. 3 and 4. Through these figures, it is revealed that MTSF, Availability and profit increases with the increase of PM rate (α) and repair rate (θ) of the hardware components. But the value of these measures decreases with the increase of maximum operation time ($\alpha_0$). Again if we increase the value of maximum repair time ($\beta_0$), the value of availability and profit are decreases are decreases while MTSF increases. Thus finally it is concluded that a system in which chances of h/w failure are high can be made reliable and economical

(i) To use reducing the repair time of the h/w components as well as conducting PM of the units after a pre-specific period of time.

(ii) Making replacement of the hardware by new one in case repair time is too long.

(iii) Makings replacement of s/w components by new one.

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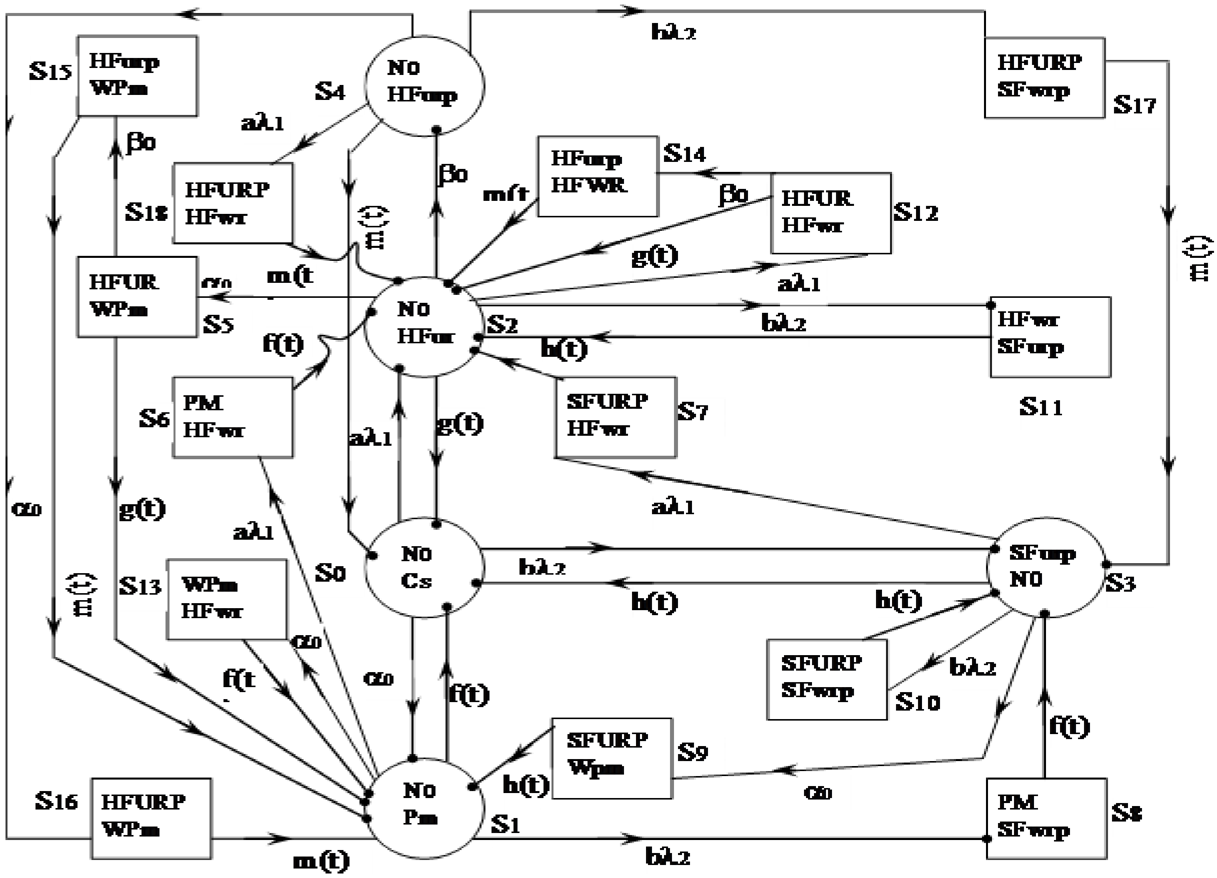



**State Transition Diagram**

○ ⇒ **Operative State**

□ ⇒ **Failed State**

• ⇒ **Regenerative Point**

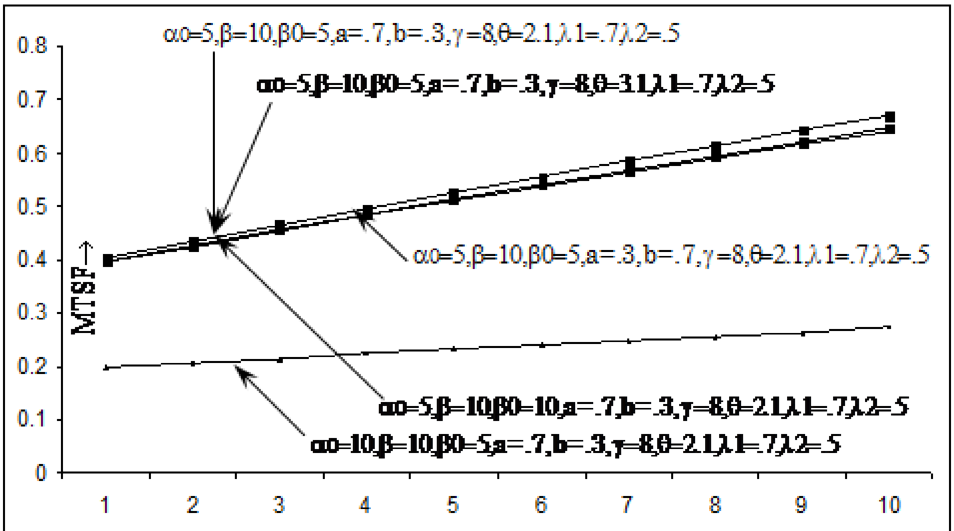



**Fig 2. MTSF Vs Preventive Maintenance Rate→**

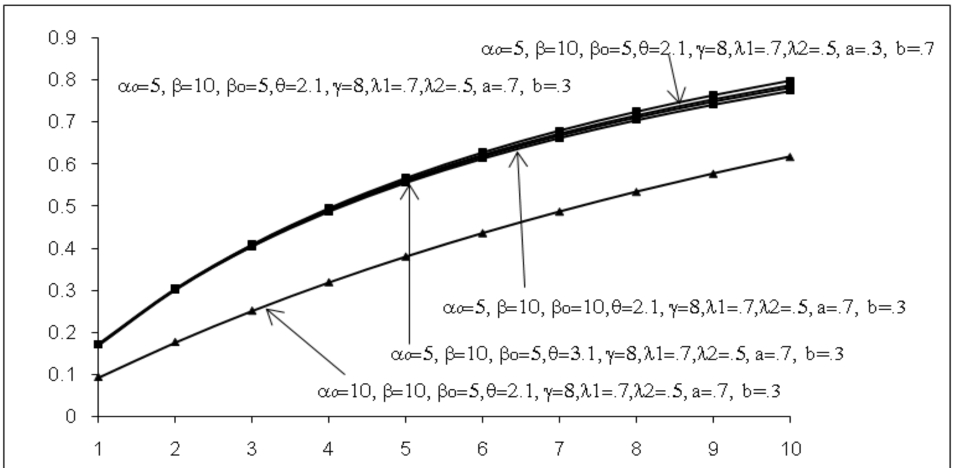
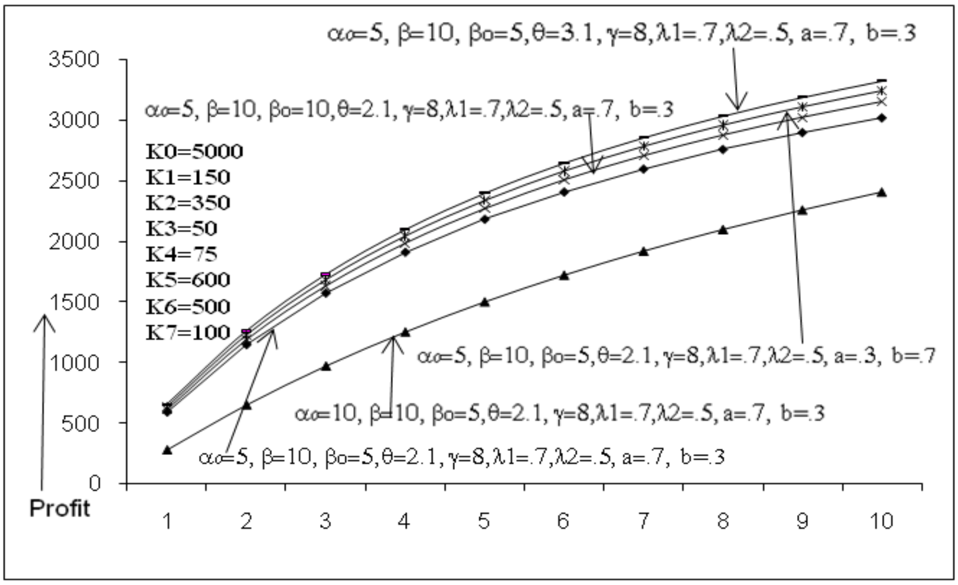



Fig.3. Availability Vs. Preventive Maintenance Rate ⟶

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