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SEMI GLOBAL DOMINATION

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ABSTRACT

A subset D of vertices of a graph connected graph G is called a semi global dominating set(sgd - set) iff D is a dominating set for both G and G^{sc}, where G^{sc} is the semi complementary graph of G. The semi global domination number (sgd - number) is the minimum cardinality of a semi global dominating set of G and is denoted by γ_{sg} (G). In this paper sharp bounds for γ_{sg}, are supplied for graphs whose girth is greater than three. Exact values of this number for paths and cycles are presented as well. The characterization result for a subset of the vertex set of G to be a semi global dominating set for G is given and also characterized the graphs of order n having sgd - numbers 2, n – 1, n.

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1. INTRODUCTION & PRELIMINARIES

Domination is an active subject in graph theory, and has numerous applications to distributed computing, the web graph and adhoc networks. For a comprehensive introduction to theoretical and applied facets of domination in graphs the reader is directed to the book [2].

A set D of vertices is called a *dominating set* of G if each vertex not in D is joined to some vertex in D. The *domination number* $\gamma(G)$ is the minimum cardinality of the dominating set of G[2].

Many variants of the domination number have been studied. For instance a dominating set D is called a *global dominating set* of G if D is a dominating set of both G and its complement G^c. The *global domination number* of G, denoted by $\gamma_g(G)$ is the smallest cardinality of the global dominating set of G[5]. A dominating set D of connected graph G is called a *connected dominating set* of G if the induced sub graph $< D >$ is connected. The *connected domination number* of G, denoted by γ_c (G) is the smallest cardinality of the connected dominating set of G[6]. A dominating set D of connected graph G is called a *independent dominating set* of G if the induced sub graph $< D >$ is a null graph[2].

G be a connected graph, then the *Semi Complementary Graph* of G is denoted by G^{sc} and it has the same vertex set as that of G and edge set being $\{uv/u, v \in V(G), uv \notin E(G), \text{there is } w \in V(G) \text{ such that } uw, wv \in E(G)\}$[4].

Recently we have introduced a new type of graph known as *semi complete graph*. Let G be a connected graph, then G is said to be *semi complete* if any pair of vertices in G have a common neighbour. The necessary and sufficient condition for a connected graph to be semi complete is any pair of vertices lie on the same triangle or lie on two different triangles having a common vertex [3].

In the present paper, we introduce a new graph parameter, the *semi global domination number*, for a connected graph G. We call $D \subseteq V(G)$ a semi global dominating set (*sgd - set*) of G if D is a dominating set for both G, G^{sc}. The semi global domination number is the minimum cardinality of a semi global dominating set of G and is denoted by γ_{sg} (G).

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All graphs considered in this paper are simple, finite, undirected and connected. For all graph theoretic terminology not defined here, the reader is referred to [1].

In this paper, sharp bounds for γ_{sg} are supplied for the graphs whose girth is greater than three. Also, we have given a characterization result for a proper subset of the vertex set of G to be a s*gd - set* of G and characterized the graphs whose s*gd - numbers* are $2, n, n-1$.

Note: Unless mentioned by G we mean a connected graph.

2. MAIN RESULTS

Here, we obtain some bounds for the *sgd - numbers* of graphs whose girth is greater than three.

Theorem 2.1: If G is a triangle free graph, then

$$\frac{2e - n(n-3)}{2} \leq \gamma_{sg}(G) \leq n - \Delta(G) + 1.$$

Proof: Suppose that D be a minimum *sgd - set* of G. By our supposition each vertex in $V - D$ is non adjacent with atleast one vertex in D. Otherwise we get a contradiction to that D is a *sgd - set* for G.

$$\Rightarrow e \leq \frac{n(n-1)}{2} - [n - \gamma_{sg}(G)]$$

$$\Rightarrow \frac{2e - n(n-3)}{2} \leq \gamma_{sg}(G)] \quad (1)$$

Suppose that $d_G(v) = \Delta(G)$ for some v in V(G).

Let $v_1, v_2, \ldots, v_{\Delta(G)}$ be the neighbours of v in G. Since G is triangle free, $[V - \{v_1, v_2, \ldots v_{\Delta(G)}\}] \cup \{v_i : i$ *is one of* $1, 2, \ldots, \Delta(G)\}$ is a $sgd - set$ of G and its cardinality is $n - \Delta(G) + 1$.

$$\Rightarrow \gamma_{sg}(G) \leq n - \Delta(G) + 1 \quad (2)$$

From (1) and (2)

$$\frac{2e - n(n-3)}{2} \leq \gamma_{sg}(G) \leq n - \Delta(G) + 1$$

Furthermore the lower bound is attained in the case of C_4 and upper bound is attained in the case of P_3. Hence the bounds are sharp.

Note: The upper bound holds good for any graph G.

Proposition 2.2:

1. $\gamma_{sg}(K_n) = n, n \geq 3$
2. $\gamma_{sg}(S_n) = 2, n \geq 3$
3. $\gamma_{sg}(K_{m,n}) = 2, m + n \geq 3$
4. $\gamma_{sg}(P_n) = [n/3]$, $n = 3m+1$
   $= [n/3]+2$, $n = 3m, 3m+2$

Here $n \geq 4$.

5. $\gamma_{sg}(C_n) = [n/3]$, $n = 3m$
   $= [n/3]+1$, $n = 3m+1, 3m+2$
6. $\gamma_{sg}(C_n O K_2) = n$.

Proposition 2.3: $G = P_n (n \geq 4)$. Then there is an independent sgd – set for G iff $n = 3m+1$.

Proposition 2.4: $G = C_n (n \geq 4)$. Then there is an independent sgd – set for G iff $n = 3m$.

Proposition 2.5: $G = P_n (n \geq 3)$. Then $\gamma_{sg}(G) = n - 2$ iff $n = 4, 5$.

Proposition 2.6: $G = C_n (n \geq 4)$. Then $\gamma_{sg}(G) = n - 2$ iff $n = 4, 5$.

Proposition 2.7: If T is a tree of order $n \geq 3$, then $\gamma_{sg}(T) = 2$ iff T is obtained from P_3 or P_4 by adding zero or more leaves to the stems of the path.

Note: $2 \leq \gamma_{sg}(G) \leq n$.

Theorem 2.8: $\gamma_{sg}(G) = n$ if and only if $G \cong K_n$.

Theorem 2.9: $\gamma_{sg}(G) = n - 1$ if and only if $G \cong K_n - \{e\}$, where e is any edge in K_n.

Proof: Assume that $\gamma_{sg}(G) = n-1$. Suppose diam(G) = l, $l \geq 3$. W.l.g. assume that $d_G(u, v) = l$ for some u, v in G. Clearly u *or* v is not a cut vertex in G. Hence $D - \{u, v\}$ is a connected dominating set in G. Follows that $D - \{u, v\}$ is a *sgd − set* in G of cardinality $n-2$, which is a contradiction to our assumption. So diam(G) ≤ 2. If *diam* (G) = 1, then G = K_n.

This implies $\gamma_{sg}(G) = n$, a contrary to our assumption. Hence *diam (G)* = 2. This implies G has atleast one pair of non adjacent vertices. If G has a pendant vertex, then $\gamma_{sg}(G) = 2$.

Clearly $n \geq 4$. Hence $\gamma_{sg}(G) < n-1$, a contrary to our assumptio n. Let $u_1 v_1$, $u_2 v_2$, ..., $u_s v_s$ be distinct pairs of non adjacent vertices in G. Since diam (G) = 2, $< u_1 w_1 v_1 >$, $< u_2 w_2 v_2 >$, ..., $< u_s w_s v_s >$ are paths in G for some w_1, w_2, ..., w_s in *G*. Clearly $V - \{u_1, u_2 ... u_s\}$ or $V - \{v_1, v_2 ... v_s\}$ is a *sgd − set* in G. If $|s| \geq 2$, then we get a contradiction to our assumption. So $|s| = 1$. This implies there is exactly one pair of non adjacent vertices in G.

Hence $G \cong K_n - \{e\}$.

The converse part is clear.

Corollary 2.10: If G is a tree, then $\gamma_{sg}(G) = n - 1$ if and only if $G \cong$ P3.

Note: By Theorem.2.9
(i) $\gamma_{sg}(C_n) \neq n - 1$ for any n.
(ii) $\gamma_{sg}(P_n) = n - 1$ if and only if n = 3.

Theorem 2.11: $\gamma_{sg}(G) = 2$ if and only if
(i) There is an edge uv in G such that each vertex in $V - \{u, v\}$ is adjacent to u or v but not both.
or
(ii) There is a path P_4, each vertex in $V - V(P_4)$ lies on an edge whose end vertices are totally dominated by end vertices of P_4.

Proof: Suppose that $\gamma_{sg}(G) = 2$. W.l.g assume that $D = \{u, v\}$ be γ_{sg} – *set* in G.

Case: 1 < D > is connected in G.

Clearly *uv* is an edge in G. If any vertex w in $V - \{u, v\}$ is adjacent to both *u* and *v*, then D is not a dominating set for G^{sc}. Hence (i) holds.

Case: 2 < D > is not connected in G.

Clearly any vertex in $V - D$ cannot be adjacent to both *u* and *v*. Hence there is a path P_4 from *u* to *v* in G, say $< u v_1 v_2 v >$. Let $v_3 \in V - V(P_4)$. Since D is a *sgd − set* in G, v_3 is adjacent to *u or v(in G)* but not both. W.l.g assume that $v_3 v_1$ is in G. For v_3 to be dominated by a vertex in D, v_3, v are to be connected by a path of length two in G,say $< v_3 v_4 v >$.

Hence v_3 lies on an edge $v_3 v_4$ and v_3, v_4 are totally dominated by *u*, *v*(end vertices in P_4) respectively. Hence (ii) holds.

The converse part is clear.

Result 2.12: A sgd – set *for* G is a global dominating set for G.

Note: $\gamma_g(G) \leq \gamma_{sg}(G)$.

Result 2.13: If diam(G) = 2, then D is a sgd − set in G if and only if D is a global dominating set in G.

Corollary 2.14: G be a semi complete graph $D \subset V$. Then D is a sgd-set in G if and only if D is a global dominating set in G.

Proof: By hypothesis, *diam (G) = 2*. Hence proof follows from the above result.

Now, we give the characterization result for a non empty subset of *V* to be *sgd − set* in G

Theorem 2.15: D ⊂ V is a sgd − set in G if and only if each vertex in V-D lies on an edge whose end points are totally dominated by distinct vertices in D.

Proof: Assume that D is a *sgd−set* in G. Let $v_1 \in V - D$. By our assumption, there exists v_2, v_3 *in D* $(v_2 \neq v_3)$ such that $v_1 v_2$ is in E (G) and $v_1 v_3$ is in E (G^{sc}). Since $v_1 v_3$ is in E (G^{sc}), there is v_4 in V such that $< v_1 v_4 v_3 >$ is a path in G.

Now, we have the following cases:

Case: 1 $v_4 = v_2$.

Then $< v_1 v_2 v_3 >$ is a path in G, which implies v_1 lies on the edge $v_1 v_2$ and v_1 , v_2 are dominated by v_2 , v_3 respectively from D − {v_1 } , D − {v_1 , v_2 }.

Case: 2 $v_4 \neq v_2$.

Then $< v_2 v_1 v_4 v_3 >$ is a path in G which implies v_1 lies on the edge $v_1 v_4$ and v_1 , v_4 are dominated by v_2 , v_3 respectively from D − {v_1 , v_4 }.

Hence in either case the claimant holds.

Conversely assume that $v_1 \in V - D$. By our assumption there is an edge $v_1 v_2$ in G such that $v_1 v_3$, $v_2 v_4$ are in G and v_3, v_4 are in D ($v_3 \neq v_4$).

If $v_3 = v_2$, then $< v_1 v_2 v_4 >$ is a path in G and $v_1 v_2$ is in G, $v_1 v_4$ is in G^{sc}.

If $v_2 \neq v_3$, then $< v_3 v_1 v_2 v_4 >$ is a path in G, which implies $v_1 v_3$ is in G and $v_1 v_4$ is in G^{sc}.

Hence, in either case for v_1 in D, there are v_3, v_4 in D such that $v_1 v_3$ is in G and $v_1 v_4$ is in G^{sc}. Hence D is a *sgd − set* in G.

Theorem 2.16: G be a connected graph and D be a γ_c − set in G. Then $d_{<D \cup \{v\}>}$ (v) < n for each v in V − D if and only if D is a sgd − set in G.

Proof: Assume that $d_{<D \cup \{v\}>}(v) < n$ for each *v* in *V − D*. Let $v \in V - D$.

Then by our assumption $d_{<D \cup \{v\}>}(v) < n$. This implies there is v_1 in D such that d(*v*, v_1) ≠ 1. Since < D ∪ {v} > is connected, this implies there is a $v - v_1$ path in D ∪ {*v*} (say) P =$< v v_2 v_3 v_4 ... v_1 >$, where $v_2, v_3 ... \in D$. Since $d_{<D \cup \{v\}>}$ (v) < *n*, there is a $v_i \in D$ such that d_G (v, v_i) = 2.This implies $v v_i \in E(G^{sc})$. *D is a sgd − set in G.*

Conversely assume that $v \in V - D$. By our assumption, there is v_1 in D such that $d_G(v, v_1) = 2$. This implies $v v_1 \in G$.

Hence $d_{<D \cup \{v\}>}(v) < n$.

Theorem 2.17: G be a connected graph such that δ(G) ≥ 2 and D is an independent sgd −set for G. If D^c is independent, then D^c is a sgd – set in G.

Proof: Assume that D^c is independent. Let $v \in V - D^c = D$. This implies there is v_1 in D^c such that $v v_1$ is in *G* (since $\delta(G) \geq 2$). Since v_1 is in D^c and *D* is independent *sgd − set* in *G*, there is v_2 in *D*, v_3 in V such that $< v_1 v_3 v_2 >$ is a path in *G*. Clearly $v_3 \in D^c$. Since D^c is independent, $< v v_1 v_3 v_2 >$ is a path in *G* and $v v_3$ is not an edge in *G*. For $v \in V - D^c$, there is $v_1 \in D^c$ such that $v v_1$ is in *G* and $v v_3$ is in G^{sc} . Since *v* is arbitrary, D^c is a *sgd − set* in *G*.

Note: The converse is not true in view of *P7*.

Result 2.18: For a semi complete graph G, γ_{sg} (G) ≥ 3.

Proof: Suppose claimant does not hold. Since $\gamma_{sg}(G) \neq 1$, $\gamma_{sg}(G) = 2$. Let $D = \{v_1, v_2\}$ be a *sgd − set* in *G*.

Case: 1 < *D* > is connected in *G*.

Then $v_1\, v_2$ is an edge in G. By the nature of semi complete graph there is a v_3 in G such that $< v_1\, v_2\, v_3 >$ is a triangle in G. This implies D is not a dominating set in G^{sc}, which is a contradiction to D is a $sgd - set$ in G.

Case: 2 $< D >$ is disconnected in G.

Since G is semi complete there is v3 in G such that $< v_1\, v_3\, v_2 >$ is a path in G. Then in G^{sc} , v_3 is not dominated by vertex in D, a contradiction to D is a $sgd - set$ in G.

Hence in either case, we get a contradiction to D is a $sgd - set$ in G.

So, Our supposition is false. This implies $\gamma_{sg}(G) \geq 3$.

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