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A NEW CLASS OF NEARLY OPEN SETS

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ABSTRACT

In this paper we introduce a new class of sets, namely semi-open sets, using the generalized closure operator due to Dunham. We give a characterization of semi*-open sets. We also define semi*-interior point and the semi*-interior of a subset. Further we investigate fundamental properties of semi*-open sets.*

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1. INTRODUCTION

In 1963 Levine [5] introduced semi-open sets in topological spaces. After Levine's work, many mathematicians turned their attention to generalizing various concepts in topology by considering semi-open sets instead of open sets. Levine [6] defined and studied generalized closed sets in 1970. Das [2] defined semi-interior point and semi-limit point of a subset. Dunham [3] introduced the concept of generalized closure using Levine's generalized closed sets and defined a new topology τ^* and studied some of their properties.

In this paper, in line with Levine's semi-open sets, we define a new class of sets, namely semi*-open sets, using the generalized closure operator *Cl** due to Dunham. We further show that the class of semi*-open sets is placed between the class of semi-open sets due to Levine and the class of open sets. We give a characterization of semi*-open sets. We investigate fundamental properties of semi*-open sets. We also define semi*- interior point and semi*-interior of a subset. We also study some properties of semi*-interior.

2. PRELIMINARIES

Throughout this paper, (X, τ) will always denote a topological space on which no separation axioms are assumed, unless explicitly stated. If A is a subset of the space (X, τ), *Cl*(A) and *Int*(A) denote the closure and the interior of A respectively.

Definition 2.1: A subset A of a topological space (X, τ) is ***semi-open*** [5] if there is an open set U in X such that $U\subseteq A\subseteq Cl(U)$ or equivalently if $A\subseteq Cl(Int(A))$.
The class of all semi-open sets in (X, τ) is denoted by SO(X, τ).

Definition 2.2: A subset A of a topological space (X, τ) is ***pre-open*** [7] (resp. ***α-open*** [8]) if $A\subseteq Int(Cl(A))$ (resp. $A\subseteq Int(Cl(Int(A)))$).

Definition 2.3: If A is a subset of a space X, the ***semi-interior*** of A is defined as the union of all semi-open sets of X contained in A. It is denoted by ***sInt*(A)**.

Definition 2.4: A set A is called ***pointwise dense*** if $A=\cup\{Cl(\{x\}) : x\in A \text{ and } \{x\} \text{ is open}\}$.

Definition 2.5: A subset A of a space X is ***generalized closed*** (briefly g-closed) [6] if $Cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in X.

Definition 2.6: If A is a subset of a space X, the ***generalized closure*** [3] of A is defined as the intersection of all g-closed sets in X containing A and is denoted by *Cl**(A).

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Definition 2.7: A topological space X is $\mathbf{T_{1/2}}$ [6] if every g-closed set in X is closed.

Theorem 2.8[3]**:** *Cl** is a Kuratowski closure operator in X.

Definition 2.9[3]**:** If (X, τ) is a topological space, let τ* be the topology on X defined by the closure operator *Cl** .That is, $\tau^*= \{U\subseteq X: Cl^*(X\backslash U) =X\backslash U\}$.

Theorem 2.10[3]**:** If (X, τ) is a topological space, then (X, τ*) is $T_{1/2}$.

Definition 2.11: A space X is ***locally indiscrete*** [9] if every open set in X is closed.

Definition 2.12: The topology on the set of integers generated by the set S of all triplets of the form {2n-1, 2n, 2n+1} as sub base is called the ***Khalimsky topology*** [4] or ***digital topology*** and it is denoted by **κ**. The collection $S\cup\{\{2n+1\}: n\in\mathbb{Z}\}$ is a base for the topology κ. The digital line equipped with the Khalimsky topology is called the ***Khalimsky line or digital line***. The topological product of two Khalimsky lines $(\mathbb{Z}, \kappa)$ is called the Khalimsky ***plane*** or ***digital plane*** and is denoted by $(\mathbb{Z}^2, \kappa^2)$.

3. SEMI*-OPEN SETS

Definition 3.1: A subset A of a topological space (X, τ) is called a ***semi*-open set*** if there is an open set U in X such that $U\subseteq A\subseteq Cl^*(U)$.

Notation: The set of all semi*-open sets in (X, τ) is denoted by S*O(X, τ) or simply S*O(X).

Definition 3.2: The ***semi*-interior*** of A is defined as the union of all semi*-open sets of X contained in A. It is denoted by ***s*Int***(A).

Definition 3.3: Let A be a subset of X. A point x in X is called a ***semi*-interior point*** of A if A contains a semi*-open set containing x.

Theorem 3.4: A subset A of X is semi*-open if and only if $A\subseteq Cl^*(Int(A))$.

Proof: Necessity. If A is semi*-open, then there is an open set U such that $U\subseteq A\subseteq Cl^*(U)$. Now $U\subseteq A \Rightarrow U=Int(U)\subseteq Int(A) \Rightarrow A\subseteq Cl^*(U)\subseteq Cl^*(Int(A))$.

Sufficiency. Assume that $A\subseteq Cl^*(Int(A))$.Take $U=Int(A)$. Then U is an open set in X such that $U\subseteq A\subseteq Cl^*(U)$. Therefore A is semi*-open.

Remark 3.5:
(i) In any space (X, τ), ϕ and X are semi*-open sets. Every nonempty semi*-open set must contain at least one nonempty open set and hence cannot be nowhere dense.

(ii) In any topological space, a singleton set is semi*-open if and only if it is open and hence a subset A of X is pointwise dense if and only if $A= \cup\{Cl(\{x\}) : x\in A \text{ and } \{x\} \text{ is semi*-open}\}$.

Theorem 3.6: If $\{A_\alpha\}$ is a collection of semi*-open sets in X, then $\cup A_\alpha$ is also semi*-open in X.

Proof: Since A_α is semi*-open for each α, there is an open set U_α in X such that $U_\alpha\subseteq A_\alpha\subseteq Cl^*(U_\alpha)$.Then $\cup U_\alpha\subseteq\cup A_\alpha\subseteq\cup Cl^*(U_\alpha)\subseteq Cl^*(\cup U_\alpha)$. Since $\cup U_\alpha$ is open, $\cup A_\alpha$ is semi*-open.

Remark 3.7: The intersection of two semi*-open sets need not be semi*-open as seen from the following examples. But the intersection of a semi*-open set and an open set is semi*-open as shown in Theorem 3.10.

Example 3.8: Let X = {a, b, c, d} and τ = {ϕ, {a}, {b}, {a, b}, {a, b, c}, X}. In the space (X, τ), the subsets A = {a, d} and B = {b, d} are semi*-open but A∩B = {d} is not semi*-open.

Example 3.9: Consider the subspace (X, τ) of the digital plane where X= {1, 2} × {1, 2, 3}.
In (X, τ), the subsets A={(1,1), (2,2)} and B={(1,3), (2,2)} are semi*-open but A∩B={(2,2)} is not semi*-open.

Theorem 3.10: If A is semi*-open in X and B is open in X, then A∩B is semi*-open in X.

Proof: Since A is semi*-open in X, there is an open set U such that $U\subseteq A\subseteq Cl^*(U)$. Since B is open, we have $U\cap B\subseteq A\cap B\subseteq Cl^*(U)\cap B\subseteq Cl^*(U\cap B)$. Hence $A\cap B$ is semi*-open in X.

Theorem 3.11: A subset A of X is semi*-open if and only if A contains a semi*-open set about each of its points.

Proof: ***Necessity:*** Obvious.

Sufficiency: Let $x\in A$. Then by assumption, there is a semi*-open set U_x containing x such that $U_x \subseteq A$. Then we have $\cup\{U_x: x\in A\}=A$. By using Theorem 3.6,

A is semi*-open.

Theorem 3.12: $S^*O(X, \tau)$ forms a topology on X if and only if it is closed under finite intersection.

Proof: Follows from Remark 3.5(i) and Theorem 3.6.

Theorem 3.13: If A is any subset of X, $s^*Int(A)$ is semi*-open. In fact $s^*Int(A)$ is the largest semi*-open set contained in A.

Proof: Follows from Definition 3.2 and Theorem 3.6.

Theorem 3.14: A subset A of X is semi*-open if and only if $s^*int(A)=A$.

Proof: A is semi*-open implies $s^*Int(A)=A$ is obvious. On the other hand let $s^*Int(A)=A$.

By Theorem 3.13, $s^*Int(A)$ is semi*-open and hence A is semi*-open.

Theorem 3.15: If A is a subset of X, then $s^*Int(A)$ is the set of all semi*-interior points of A.

Proof: $x\in s^*Int(A)$ if and only if x belongs to some semi*-open subset U of A. That is, if and only if x is a semi*-interior point of A.

Corollary 3.16: A subset A of X is semi*-open if and only if every point of A is a semi*-interior point of A.

Proof: Follows from Theorem 3.14 and Theorem 3.15.

Theorem 3.17: Every open set is semi*-open.

Proof: Let U be open in X. Then $Int(U)=U$. Therefore $U\subseteq Cl^*(U)=Cl^*(Int(U))$. Hence by Theorem 3.4, U is semi*-open.

Corollary 3.18: If a subset A is semi*-open and U is open, then $A\cup U$ is semi*-open.

Proof: Follows from Theorem 3.17 and Theorem 3.6.

Remark 3.19: The converse of Theorem 3.17 is not true as shown in the following examples.

Example 3.20: Consider the topological space (X, τ) in Example 3.8. The subsets {a, d}, {b, d} and {a, b, d} are semi*-open in X but not open.

Example 3.21: Consider the subspace (X, τ) of the digital plane given in Example 3.9. In (X, τ), the subsets {(1,1),(1,3),(2,2)}, {(1,1),(1,3),(2,1),(2,2)} and {(1,1),(1,2),(1,3),(2,2),(2,3)} are semi*-open but not open.

Definition 3.22: For a topological space (X, τ), let
$\tau_{s^*}= \{U\in S^*O(X, \tau) : U\cap A\in S^*O(X, \tau) \text{ for all } A\in S^*O(X, \tau)\}$.

Theorem 3.23: If (X, τ) is a topological space, then τ_{s^*} is a topology on X finer than τ.

Proof: Clearly $\phi, X\in\tau_{s^*}$. Let $U_\alpha\in \tau_{s^*}$ and $U=\cup U_\alpha$. Since $U_\alpha\in S^*O(X, \tau)$, by using Theorem 3.6, $U\in S^*O(X, \tau)$.

Let $A\in S^*O(X, \tau)$. Then $U_\alpha\cap A\in S^*O(X, \tau)$, for each α and hence by Theorem3.6, $U\cap A=(\cup U_\alpha)\cap A =\cup(U_\alpha\cap A)\in S^*O(X, \tau)$. Therefore $U\in\tau_{s^*}$. Now let $U_1, U_2, \ldots,U_n\in\tau_{s^*}$. Then $U_1, U_2, \ldots,U_n\in S^*O(X, \tau)$ and by definition of τ_{s^*}, we get $\bigcap_{i=1}^{n} U_i \in S^*O(X, \tau)$. If $A \in S^*O(X, \tau)$, then by repeated application of the condition, we have $(\bigcap_{i=1}^{n} U_i) \cap A \in S^*O(X, \tau)$.

Hence $\bigcap_{i=1}^{n} U_i \in \tau_{s^*}$. This shows that τ_{s^*} is a topology on X. Let $V \in \tau$. By using Theorem 3.17, $V \in S^*O(X, \tau)$. Also by Theorem 3.10, $V \cap A \in S^*O(X, \tau)$ for all $A \in S^*O(X, \tau)$. Hence $V \in \tau_{s^*}$. Thus τ_{s^*} is finer than τ.

Theorem 3.24: Every semi*-open set is semi-open.

Proof: Let A be a semi*-open set. Then there is an open set U in X such that $U \subseteq A \subseteq Cl^*(U)$.

Note that $Cl^*(U) \subseteq Cl(U)$. Therefore $U \subseteq A \subseteq Cl(U)$. Hence A is semi-open.

Remark 3.25: The converse of Theorem 3.24 is not true as shown in the following examples.

Example 3.26: Consider the topological space (X, τ) given in Example 3.8. The subsets {a, c, d} and {b, c, d} are semi-open in X but not semi*-open.

Example 3.27: Consider the subspace (X, τ) of the digital plane where X= {0, 1} × {1, 2, 3}.

In (X, τ), the subsets {(1,1),(1,2)}, {(0,2),(1,1),(1,2) } and {(0,3),(1,2),(1,3)} are semi-open but not semi*-open.

Theorem 3.28: In any topological space (X, τ), $\tau \subseteq S^*O(X, \tau) \subseteq SO(X, \tau)$. That is, the class of semi*-open sets is placed between the class of open sets and the class of semi-open sets.

Proof: Follows from Theorem 3.17 and Theorem 3.24.

Remark 3.29:

(i) If (X, τ) is a locally indiscrete space, then $\tau = S^*O(X, \tau) = SO(X, \tau)$.

(ii) In the Sierpinski space (X, τ), where X = {0, 1} and τ = {ϕ, {1}, X}, $\tau = S^*O(X, \tau) = SO(X, \tau)$.

(iii) The inclusions in Theorem 3.28 may be strict and equality may also hold. This can be seen from the following examples.

Example 3.30: In the space (X, τ) where X= {a, b, c, d} and τ = {ϕ, {a}, {b, c, d}, X}

SO(X, τ) = S*O(X, τ) = {ϕ, {a}, {b, c, d}, X}.

Here $\tau = S^*O(X, \tau) = SO(X, \tau)$.

Example3.31: In the space (X, τ) where X = {a, b, c, d} and τ = {ϕ, {a}, {b}, {a, b}, {b, c}, {a, b, c}, X},

SO(X, τ) = S*O(X, τ) = {ϕ, {a}, {b}, {a, b}, {a, d}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {b, c, d}, X}.

Here $\tau \subsetneq S^*O(X, \tau) = SO(X, \tau)$.

Example 3.32: In the space (X, τ) where X= {a, b, c, d} and τ = {ϕ, {a, b}, X},

SO(X, τ) = {ϕ,{a, b},{ a, b, c }, {a, b, d}, X}; S*O(X, τ)={ ϕ, {a, b}, X }.

Here $\tau = S^*O(X, \tau) \subsetneq SO(X, \tau)$.

Example 3.33: Consider the space (X, τ) where X= {a, b, c, d} and τ = {ϕ, {a}, {a, b, c}, X}.

SO(X, τ) = {ϕ, {a}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}, X}.
S*O(X, τ) = {ϕ, {a}, {a, d}, {a, b, c}, X}. Here $\tau \subsetneq S^*O(X, \tau) \subsetneq SO(X, \tau)$.

Example 3.34: Consider the subspace (X, τ) of the digital plane where X= {1, 2, 3} × {0, 1}.

If a, b, c, d, e, f denote the points (1,0), (1,1), (2, 0), (2,1), (3,0), (3,1) respectively, then

τ={ϕ,{b},{f},{a,b},{b,f},{e,f},{a,b,f},{b,d,f},{b,e,f},{a,b,d,f},{a,b,e,f},{b,d,e,f},{a,b,d,e,f},X}.

SO(X)={ϕ,{b},{f},{a,b},{b,c},{b,d},{b,f},{c,f},{d,f},{e,f},{a,b,c},{a,b,d},{a,b,f},{b,c,d},{b,c,f},{b,d,f},{b,e,f},{c,d,f},{c,e,f},{d,e,f},{a,b,c,d},{a,b,c,f},{a,b,d,f},{a,b,e,f},{b,c,d,f},{b,c,e,f},{b,d,e,f},{c,d,e,f},{a,b,c,d,f},{a,b,c,e,f},{a,b,d,e,f},{b,c,d,e,f},X}.

S*O(X)={ϕ,{b},{f},{a,b},{b,c},{b,f},{c,f},{e,f},{a,b,c},{a,b,f},{b,c,f},{b,d,f},{b,e,f},{c,e,f},{a,b,c,f},{a,b,d,f},{a,b,e,f},{b,c,d,f},{b,c,e,f},{b,d,e,f},{a,b,c,d,f},{a,b,c,e,f},{a,b,d,e,f},{b, c, d, e, f,}, X}.

Here τ ⊊ S*O(X, τ) ⊊ SO(X, τ).

Example 3.35: Consider the subspace (X, τ) of the digital plane where X= {0, 1, 2} × {1, 2}.

If a, b, c, d, e, f denote the points (0, 1), (0, 2), (1, 1), (1, 2), (2, 1), (2, 2) respectively, then

τ={ϕ,{c},{a,c},{c,d},{c,e},{a,c,d},{a,c,e},{c,d,e},{a,b,c,d},{a,c,d,e},{c,d,e,f},{a,b,c,d,e},{a, c, d, e, f}, X}.

SO(X)= S*O(X)={ϕ,{c},{a,c},{b,c},{c,d},{c,e},{c,f},{a,b,c},{a,c,d},{a,c,e},{a,c,f},{b,c,d}, {b, c, e},{b, c, f}, {c,d,e},{c,d,f},{c,e,f},{a,b,c,d},{a,b,c,e},{a,c,d,e},{a,b,c,f},{a,c,d,e},{a,c,d,f}, {a, c, e, f}, {b, c, d, e}, {b, c, d, f}, {b,c,e,f},{c,d,e,f},{a,b,c,d,e},{a,b,c,d,f},{a,c,d,e,f},{a,b,c,e,f}, {b, c, d, e, f}, X}. Here τ ⊊ S*O(X, τ) = SO(X, τ).

Remark 3.36: If X is a $T_{1/2}$ space, the g-closed sets and the closed sets coincide and hence *Cl**(U) = *Cl*(U). Therefore the class of semi*-open sets and the class of semi-open sets coincide. In particular, in the Khalimsky line and in the real line with usual topology, the semi*-open sets and the semi-open sets coincide. But the converse is not true. That is, a space, in which the class of semi*-open sets and the class of semi-open sets coincide, need not be $T_{1/2}$ and this can be seen from the Example 3.31 and Example 3.35. In these spaces the class of semi*-open sets and the class of semi-open sets coincide but they are not $T_{1/2}$.

Theorem 3.37: If (X, τ) is any topological space, then S*O(X, τ*) =SO(X, τ*).

Proof: Follows from the fact that the space (X, τ*) is $T_{1/2}$ [Theorem 2.10] and Remark 3.36.

Lemma 3.38: If A be semi*-open, then *Cl**(A) =*Cl**(*Int*(A)).

Proof: Since A is semi*-open, A⊆*Cl**(*Int*(A)). Hence *Cl**(A)⊆*Cl**(*Int*(A)) which proves the lemma.

Theorem 3.39: Let A be semi*-open and B⊆X such that A⊆B⊆*Cl**(A).Then B is semi*-open.

Proof: Since A is semi*-open, A⊆*Cl**(*Int*(A)). Since *Int*(A)⊆*Int*(B), *Cl**(*Int*(A))⊆*Cl**(*Int*(B)). Therefore by the above lemma, B⊆*Cl**(*Int*(B)). Hence by Theorem 3.4, B is semi*-open.

Theorem 3.40: Let ß be a collection of subsets in (X, τ) satisfying (i) τ⊆ß (ii) If B∈ß and D⊆X such that B⊆D⊆*Cl**(B) implies D∈ß. Then S*O(X, τ) ⊆ß. Thus S*O(X, τ) is the smallest collection satisfying the conditions (i) and (ii).

Proof: Let A∈S*O(X, τ).Then there is an open set U in X such that U⊆A⊆*Cl**(U). By (i), U∈ß. By (ii), A∈ß. Thus S*O(X, τ) ⊆ß. Also by Theorem 3.17 and Theorem 3.39, S*O(X, τ) satisfies (i) and (ii).Thus S*O(X, τ) is the smallest collection satisfying (i) and (ii).

Theorem 3.41: If (X, τ) is a topological space, then S*O(X, τ)⊆SO(X, τ*)

That is, every semi*-open set in (X, τ) is semi-open in (X, τ*).

Proof: If A is a semi*-open set in (X, τ), then there is an open set U in (X, τ) such that U⊆A⊆*Cl**(U).Since U is open in (X, τ), U is open in (X, τ*). Thus A is semi-open in (X, τ*).

Remark 3.42: The inclusion in Theorem 3.41 can be strict and equality also holds as seen from the following examples:

Example 3.43: Consider the space (X, τ) where X={a, b, c, d} and τ ={ϕ, {a, b},{ a, b, c }, X }

S*O(X, τ) = {ϕ, {a, b},{ a, b, c }, {a, b, d}, X}. GC(X, τ) = {ϕ, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}, X}.

τ *={ϕ,{a},{b},{c},{a, b},{a, c},{b, c},{a, b, c},X}.

SO(X, τ*) ={ϕ,{a},{b},{c},{a, b},{a, c},{a, d},{b, c},{b, d},{c, d},{ a, b, c }, {a, b, d},{a, c, d}, {b, c, d}, X}
= ℘(X) \{{d}}.

Here S*O(X, τ) ⊊SO(X, τ*).

Example 3.44: Consider the space (X, τ) where X={a, b, c, d} and τ ={ ϕ,{a},{b},{c},{a, b},{a, c}, {b, c},{a, b, c },X}. S*O(X, τ) ={ ϕ,{a},{b},{c},{a, b},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, c }, {a, b, d},{a, c, d}, {b, c, d}, X }; GC(X, τ)= {ϕ,{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d}, {b, c, d}, X }.

τ *={ϕ,{a},{b},{c},{a, b},{a, c},{b, c},{a, b, c}, X }.

Here SO(X, τ*) = S*O(X, τ) = ℘(X)\{{d}}.

Remark 3.45: The concepts of semi*-open sets and α-open sets are independent as seen from the following examples:

Example 3.46: In the space (X, τ) where X= {a, b, c, d} and τ = {ϕ, {a}, {a, b, c}, X}, the subsets {a, b}, {a, c}, {a, b, d} and {a, c, d} are α-open but not semi*-open.

Example 3.47: In the space (X, τ) where X= {a, b, c, d} and τ= {ϕ, {a}, {b}, {a, b}, {a, b, c}, X}, the subsets {a, d} and {b, d} are semi*-open but not α-open.

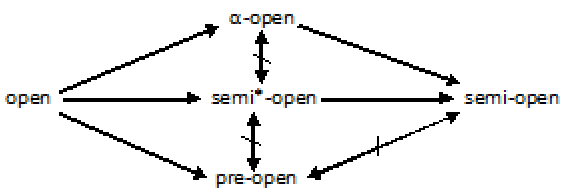
Remark 3.48: The concepts of semi*-open sets and pre-open sets are independent as seen from the following examples:

Example 3.49: In the space (X, τ) where X= {a, b, c, d} and τ= {ϕ, {a}, X}, the subsets {a, b},{a, c},{a, d},{a, b, c},{a, b, d} and {a, c, d} are pre-open but not semi*-open.

Example 3.50: In the space (X, τ) where X= {a, b, c, d} and τ= {ϕ, {a}, {b}, {a, b}, {b, c}, {a, b, c}, X}, the subsets {a, d}, {b, d} and {b, c, d}are semi*-open but not pre-open.

From the above discussions we have the following diagram:

Diagram 3.51:



Theorem 3.52: In any topological space (X, τ) the following hold:
(i) *s*Int*(ϕ)=ϕ.
(ii) *s*Int*(X)=X.

If A and B are subsets of X,

(iii) *s*Int*(A)⊆A.
(iv) A⊆B ⟹ *s*Int*(A)⊆*s*Int*(B).
(v) *s*Int*(*s*Int*(A))=*s*Int*(A). That is, the operator *s*Int* is idempotent.
(vi) *Int*(A)⊆*s*Int*(A)⊆*sInt*(A)⊆A.
(vii) *s*Int*(A∪B)⊇*s*Int*(A)∪ *s*Int*(B).
(viii) *s*Int*(A∩B)⊆*s*Int*(A)∩ *s*Int*(B).
(ix) *Int*(*s*Int*(A))=*Int*(A).
(x) *s*Int*(*Int*(A))=*Int*(A).

Proof: (i), (ii), (iii) and (iv) follow from Definition 3.2. (v) follows from Theorem 3.13 and Theorem 3.14. (vi) follows from Theorem 3.17 and Theorem 3.24. (vii) and (viii) follow from (iv) above. Since $s^*Int(A)\subseteq A$, $Int(s^*Int(A))\subseteq Int(A)$.

Also from (vi), $Int(A)\subseteq s^*Int(A)$ and so $Int(A)\subseteq Int(s^*Int(A))$.Therefore $Int(s^*Int(A))=Int(A)$. This proves (ix). (x) follows from the fact that *Int*(A) is open and hence semi*-open and by invoking Theorem 3.14, $s^*Int(Int(A))=Int(A)$.

Remark 3.53: In (vi) of Theorem 3.52, each of the inclusions may be strict and equality may also hold. This can be seen from the following examples:

Example 3.54: In the space (X, τ) where X={a, b, c, d, e, f, g} and τ={ϕ,{a},{b},{a, b}, {a, b, e},{a, c, d},{b, f, g}, {a, b, c, d},{a, b, f, g},{a, b, c, d, e},{a, b, e, f, g},{a, b, c, d, f, g}, X}. Let A= {a, b, c, d}.

Then $Int(A)=s^*Int(A)=sInt(A)$={a, b, c, d}=A.

Let B={a, e}.Then *Int*(B)={a}; $s^*Int(B)=sInt(B)$={a, e}.

Here $Int(B)\subsetneq s^*Int(B)=sInt(B)=B$.

Let C= {a, b, c, d, e, f}.Then $Int(C)=s^*Int(C)$={a, b, c, d, e}; *sInt*(C)={a, b, c, d, e, f}.

Here $Int(C) =s^*Int(C) \subsetneq sInt(C) =C$.

Let D= {b, d, f, g}.Then $Int(D)=s^*Int(D)=sInt(D)$={b, f, g}. Here $Int(D)=s^*Int(D)=sInt(D)\subsetneq D$.

Let E={a, c, e}.Then *Int*(E)={a}; $s^*Int(E)$={a, e}; *sInt*(E)={a, c, e}.

Here $Int(E)\subsetneq s^*Int(E)\subsetneq sInt(E)=E$.

Let F= {b, c, d, e}.Then *Int*(F)={b}; $s^*Int(F)=sInt(F)$={b, e}.

Here $Int(F)\subsetneq s^*Int(F)= sInt(F)\subsetneq F$.

Let G={a, d, f}.Then $Int(G)=s^*Int(G)$={a}; *sInt*(G)={a, d}.

Here $Int(G)=s^*Int(G)\subsetneq sInt(G)\subsetneq G$. Let H= {b, c, d, e, f}.

Then *Int*(H)={b}; $s^*Int(H)$={b, e}; $s^*Int(H)$={b, e, f}.

Here $Int(H) \subsetneq s^*Int(H)\subsetneq sInt(H) \subsetneq H$.

Example 3.55: Consider the subspace (X, τ) of the digital plane where X= {1, 2, 3} × {1, 2}.

If a, b, c, d, e, f denote the points (1,1),(1,2),(2,1),(2,2),(3,1),(3,2) respectively, then

τ={ϕ,{a},{e},{a,b},{a,e},{e,f},{a,b,e},{a,c,e},{a,e,f},{a,b,c,e},{a,b,e,f},{a,c,e,f},{a,b,c,e,f},X}.

SO(X)={ϕ,{a},{e},{a,b},{a,c},{a,d},{a,e},{c,e},{d,e},{e,f},{a,b,c}{a,b,d},{a,b,e},{a,c,d},{a,c,e},{a,d,e},{a,e,f},{c,d, e},{c,e,f},{d,e,f},{a,b,c,d},{a,b,c,e},{a,b,d,e}{a,b,e,f},{a,c,d,e},{a,c,e,f},{a,d,e,f},{c,d,e,f},{a,b,c,d,e},{a,b,c,e,f},{a,b, d,e,f},{a,c,d,e,f},X}.

S*O(X)={ϕ,{a},{e},{a,b},{a,d},{a,e},{d,e},{e,f},{a,b,d},{a,b,e},{a,c,e},{a,d,e},{a,e,f},{d,e,f},{a,b,c,e},{a,b,d,e}{a,b, e,f},{a,c,d,e},{a,c,e,f},{a,d,e,f},{a,b,c,d,e},{a,b,c,e,f},{a,b,d,e,f},{a,c,d,e,f}, X}.

Let A={a, b, c, d, f}.Then *Int*(A)={a, b}; $s^*Int(A)$={a, b, d}; *sInt*(A)={a, b, c, d}.

Here $Int(A)\subsetneq s^*Int(A)\subsetneq sInt(A)\subsetneq A$.

Let B={b, c, e, f}.Then $Int(B)=s^*Int(B)$={e, f}; *sInt*(B)={c, e, f}.

Here $Int(B)=s^*Int(B)\subsetneq sInt(B)\subsetneq B$.

Let C= {a, b, d, f}.Then *Int*(C)={a, b}; $s^*Int(C)= sInt(C)$={a, b, d}. Here $Int(C)\subsetneq s^*Int(C)=sInt(C)\subsetneq C$.

Let D= {a, c, d}. Then *Int*(D)={a}; *s**Int*(D)={a, d};*sInt*(D)={a, c, d}.

Here *Int*(D) ⊊ *s**Int*(D)⊊ *sInt*(D)=D.

Let E={a, b, c, d, e}.Then *Int*(E)={a, b, c, e}; *s**Int*(E)= *sInt*(E)={a, b, c, d, e}.

Here *Int*(E)⊊*s**Int*(E)=*sInt*(E)=E.

Let F= {a, c}. Then *Int*(F)= *s**Int*(F)={a} ; *sInt*(F)={a, c}. Here *Int*(F) = *s**Int*(F)⊊ *sInt*(F) = F.

Let G= {b, e, f}. Then *Int*(G)= *s**Int*(G)= *sInt*(G)={e,f}. Here *Int*(G) = *s**Int*(G)= *sInt*(G) ⊊ G.

Let H= {a, b, e, f}. Then *Int*(H)=*s**Int*(H)= *sInt*(H) = H.

Remark 3.56: The inclusions in (vii) and (viii) of Theorem 3.52 may be strict and equality may also hold. This can be seen from the following examples.

Example 3.57: Consider the space (X, τ) in Example 3.54

Let A= {b, c, e, f, g} and B={a, b, c, f, g} then A∪B={a, b, c, e, f, g} and A∩B={b, c, f, g}
*s**Int*(A)={b, e, f, g}; *s**Int*(B)={a, b, f, g}; *s**Int*(A∪B)={a, b, e, f, g}; *s**Int*(A∩B)={b, f, g}

Here *s**Int*(A∪B)= *s**Int*(A) ∪*s**Int*(B) and *s**Int*(A∩B)= *s**Int*(A) ∩*s**Int*(B)

Let C= {a, c, d, e, g} and D={b, d, e, f, g} then C∩D={d, e, g}

*s**Int*(C)={a, c, d, e}; *s**Int*(D)={ b, e, f, g}; *s**Int*(C∩D)=ϕ ; *s**Int*(C) ∩*s**Int*(D)={e}

Here *s**Int*(C∩D)⊊ *s**Int*(C) ∩*s**Int*(D)

Let E={b, c, d, f, g} and F={a, b, d, g} then E∪F={a, b, c, d, f, g};*s**Int*(E)={b, f, g};

*s**Int*(F)={a, b}; *s**Int*(E∪F)={a, b, c, d, f, g}; *s**Int*(E) ∪*s**Int*(F)={a, b, f, g};

Here s*Int*(E) ∪*s**Int*(F)⊊ *s**Int*(E∪F)

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