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Anti Fuzzy Ideals of CI- algebras and its lower level cuts

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ABSTRACT

In this paper, we introduce the concept of Anti fuzzy ideals of CI-algebras, lower level cuts of a fuzzy set and prove some results . We show that a fuzzy set of a CI-algebra is a fuzzy ideal if and only if the complement of this fuzzy set is an anti fuzzy ideal.We discussed few results of antifuzzy ideal of CI-algebra under transitive and self-distributive. Also we discussed few results of antifuzzy ideal with lower level cuts.

Keywords: *CI-algebra, Transitive, Self-Distributive, fuzzy ideal, Anti fuzzy ideal, lower level cuts.*

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1. INTRODUCTION

Y.Imai and K.Iseki introduced two classes of abstract algebras: BCK-algebras and BCI –algebras **[6,7]**. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. In **[4,5]**Q.P.Hu and X .Li introduced a wide class of abstract BCH-algebras. They have shown that the class of BCI-algebras. J.Neggers, S.S.Ahn and H.S.Kim introduced Q-algebras which is generalization of BCK / BCI algebras and obtained several results. In **[9]** K.Megalai and A.Tamilarasi introduced a class of abstract algebras: TM-algebras , which is a generalisation of Q / BCK / BCI / BCH algebras. In **[10]** B.L.Meng introduced the notion of a CI-algebra as a generation of a BE-algebra. The concept of fuzzification of ideals in CI-algebra have introduced by Samy.M.Mostafa**[13].**R.Biswas introduced the concept of Anti fuzzy subgroups of groups**[2].** Modifying his idea, in this paper we apply the idea of CI-algebras . We introduce the notion of Anti fuzzy ideals of CI-algebras and investigate some of its properties.

2. PRELIMINARIES

In this section we site the fundamental definitions that will be used in the sequel.

Definition 2.1: [10] An algebraic system (X,*,1) of type (2,0) is called a CI -algebra if it satisfies the following axioms.

1. $x * x = 1$ (2.1)

2. $1 * x = x$, (2.2)

3. $x * (y * z) = y * (x * z)$, for all $x, y, z \in X$ (2.3)

In X we can define a binary operation $\leq$ by $x \leq y$ if and only if $x * y = 1$ for all $x, y \in X$. (2.4)

Example 2.1: Let X = {1, 2, 3, 4 } be a set with a binary operation * defined by the following table

*	1	2	3	4
1	1	2	3	4
2	1	1	2	4
3	1	1	1	4
4	1	2	3	1

Then (X , * , 1) is a CI-algebra.

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Definition 2.2:[8] A CI - algebra (X ,* ,1) is said to be transitive if it satisfies:

$$(y * z) * ((x * y) * (x * z)) = 1 \quad \text{for all } x, y, z \in X \tag{2.5}$$

Definition 2.3: [8] A CI - algebra (X ,* ,1) is said to be self- distributive if it satisfies:

$$x * (y * z) = (x * y) * (x * z) \quad \text{for all } x, y, z \in X \tag{2.6}$$

Note that every self-distributive is transitive.

In an CI- algebra, the following identities are true:

4. $y * ((y * x) * x) = 1.$ (2.7)

5. $(x * 1) * (y * 1) = (x * y) * 1.$ (2.8)

Definition 2.4: [14] Let (X , * , 1) be a CI -algebra. A non empty subset I of X is called an ideal of X if it satisfies the following conditions

(i) If $x \in X$ and $a \in I$, then $x * a \in I$, (i.e) $X * I \subseteq I$, (2.9)

(ii)

(iii) If $x \in X$ and $a,b \in I$, then $(a * (b * x)) * x \in I$. (2.10)

Let X be a CI –algebra. Then (i) Every ideal of X contains 1.(ii) If I is an ideal of X, then $(a * x) * x \in I$ for all $a \in I$ and $x \in X$.

Definition 2.5: Let X be a non-empty set**.** A fuzzy subset μ of the set X is a mapping $\mu : X \to [0, 1]$.

Definition 2.6: [14]Let X be a CI -algebra. A fuzzy set μ in X is called a fuzzy ideal of X if

(i) $\mu(x * y) \geq \mu(y)$, for all $x, y \in X$. (2.11)

(ii) $\mu((x * (y * z)) * z) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y, z \in X$. (2.12)

Definition 2.7: A fuzzy set μ of a CI-algebra X is called an anti fuzzy ideal of X, if

(i) $\mu(x * y) \leq \mu(y)$, for all $x, y \in X$. (2.13)

(ii) $\mu((x * (y * z)) * z) \leq \max\{\mu(x), \mu(y)\}$, for all $x, y, z \in X$. (2.14)

Theorem 2.1: Every anti fuzzy ideal μ of a CI-algebra X satisfies the inequality $\mu(1) \leq \mu(x)$, for any $x \in X$. (2.15)

Proof: Using (2.1) and (2.13), we have

$$\begin{aligned}\mu(1) &= \mu(x * x)\\ &\leq \mu(x).\end{aligned}$$

Theorem 2.2: If μ is an anti fuzzy ideal of a CI – algebra X, then for all $x,y \in X$, $\mu((x * y) * y) \leq \mu(x)$ (2.16)

Proof: Taking y = 1 and z = y in (2.14) and Using (2.2) and (2.15), we have

$$\begin{aligned}\mu((x * y) * y) &= \mu[(x * (1 * y)) * y] && [\text{by } (2.2)]\\ &\leq \text{Max}\{\mu(x), \mu(1)\} && [\text{by } (2.14)]\\ &= \mu(x) && [\text{by } (2.15)]\end{aligned}$$

Theorem 2.3: Every Anti fuzzy ideal μ of a CI- algebra X is order reversing.That is , If $x \leq y$, then $\mu(x) \geq \mu(y)$, for all $x, y \in X$. (2.17)

Proof: Let μ be an anti fuzzy ideal of a CI- algebra X and let $x,y \in X$ be such that $x \leq y$, then $x * y = 1$

$$\begin{aligned}\text{Now } \mu(y) &= \mu(1 * y) && [\text{by } (2.2)]\\ &= \mu((x * y) * y)\\ &\leq \mu(x) && [\text{by } (2.16)]\end{aligned}$$

Hence $\mu(x) \geq \mu(y)$.

Theorem 2.4: Let μ be a fuzzy set of a CI- algebra X which satisfies

$$\mu(1) \leq \mu(x) \text{ and } \mu(x * z) \leq \text{Max}\{\mu(x * (y * z)), \mu(y)\}, \qquad (2.18)$$

for all x , y, z ∈ X.Then μ is order reversing.

Proof: Let x,y ∈ X be such that $x \leq y$, then $x * y = 1$.

Using (2.2),(2.18) and (2.15),we get

$$\begin{aligned} \text{Now } \mu(y) &= \mu(1 * y) \\ &\leq \text{Max}\{\mu(1 * (x * y)), \mu(x)\} \\ &= \text{Max}\{\mu(1 * 1), \mu(x)\} \\ &= \text{Max}\{\mu(1), \mu(x)\} \\ &= \mu(x) \end{aligned}$$

Therefore $\mu(x) \geq \mu(y)$.

Hence μ is order reversing.

Theorem 2.5: Let X be a transitive CI- algebra.If a fuzzy set μ in X is an antifuzzy ideal of X then it satisfies condition (2.15) and (2.18).

Proof : Let μ be an antifuzzy ideal of X. By theorem 2.2 $\mu(1) \leq \mu(x)$. Since X is transitive,We have

$$((y * z) * z) * ((x * (y * z)) * (x * z)) = 1 \text{for all } x, y, z \in X \qquad (2.19)$$

It follows from (2.2),(2.19),(2.14) and (2.16) that

$$\begin{aligned} \mu(x * z) &= \mu(1 * (x * z)) \\ &= \mu(\{(y * z) * z) * ((x * (y * z)) * (x * z))\} * (x * z)) \\ &\leq \text{Max}\{\mu((y * z) * z), \mu(x * (y * z))\} \\ &\leq \text{Max}\{\mu(y), \mu(x * (y * z))\} \end{aligned}$$

Hence μ satisfies (2.18).

Theorem 2.6: Let X be a self-distributive CI- algebra.If a fuzzy set μ in X is an antifuzzy ideal of X then it satisfies condition (2.15) and (2.18).

Proof: Straightforward.

Theorem 2.7: μ is a fuzzy ideal of a CI-algebra X if and only if μ^c is an anti fuzzy ideal of X.

Proof: Let μ be a fuzzy ideal of X. Let x,y,z ∈ X.

$$\begin{aligned} \text{Then (i) } \mu^c(x * y) &= 1 - \mu(x * y) \\ &\leq 1 - \mu(y) \qquad [\text{ by } (2.11)] \\ &= \mu^c(y) \end{aligned}$$

(i.e) $\mu^c(x * y) \leq \mu^c(y)$

$$\begin{aligned} \text{(ii) } \mu^c((x * (y * z)) * z) &= 1 - \mu((x * (y * z)) * z) \\ &\leq 1 - \min\{\mu(x), \mu(y)\} \qquad [\text{ by } (2.12)] \\ &= 1 - \min\{1 - \mu^c(x), 1 - \mu^c(y)\} \\ &= \max\{\mu^c(x), \mu^c(y)\} \end{aligned}$$

That is , $\mu^c((x * (y * z)) * z) \leq \max\{\mu^c(x), \mu^c(y)\}$

Thus, μ^c is an anti fuzzy ideal of X. The converse also can be proved similarly.

Definition 2.8: [13] Let μ be a fuzzy set of X. For a fixed t ∈ [0, 1], the set $\mu^t = \{x \in X \mid \mu(x) \le t\}$ is called the lower level subset of μ.

Clearly $\mu^t \cup \mu_t = X$ for $t \in [0,1]$ if $t_1 < t_2$, then $\mu^{t1} \subseteq \mu^{t2}$.

Theorem 2.8: If μ is an antifuzzy ideal of CI-algebra X,then μ^t is an ideal of X for every t∈ [0,1]

Proof: Let μ be an antifuzzy ideal of CI-algebra X.

(i) Let x ∈ X and $y \in \mu^t \Rightarrow \mu(y) \le t$.

$\mu(x * y) \le \mu(y) \le t$.

$\Rightarrow x * y \in \mu^t$.

(ii) Let x ∈ X and $a,b \in \mu^t$.

$\Rightarrow \mu(a) \le t$ and $\mu(b) \le t$.

$\mu((a * (b * x)) * x) \le \text{Max}\{\mu(a), \mu(b)\} \le \text{Max}\{t,t\} = t$.

$\Rightarrow (a * (b * x)) * x \in \mu^t$.

Hence μ^t is an ideal of X.

Theorem 2.9: Let μ be a fuzzy set of CI- algebra X.If for each t ∈ [0,1], the lower level cut μ^t is an ideal of X, then μ is an antifuzzy ideal of X.

Proof: Let μ^t be an idea l of X.

If $\mu(x * y) > \mu(y)$ for some x,y ∈ X.Then

$\mu(x * y) > t_0 > \mu(y)$ by taking $t_0 = \frac{1}{2}\{\mu(x * y) + \mu(y)\}$

Hence $x * y \notin \mu^{t0}$ and $y \in \mu^{t0}$, which is a contradiction.

Therefore, $\mu(x * y) \le \mu(y)$.

Let x,y,z ∈ X be such that $\mu((x * (y * z)) * z) > \text{Max}\{\mu(x), \mu(y)\}$

Taking $t_1 = \frac{1}{2}\{\mu((x * (y * z)) * z) + \text{Max}\{\mu(x), \mu(y)\}\}$ and $\mu((x * (y * z)) * z) > t_1 > \text{Max}\{\mu(x), \mu(y)\}$.

It follows that $x,y \in \mu^{t1}$ and $(x * (y * z)) * z \notin \mu^{t1}$. This is a contradiction.

Hence $\mu((x * (y * z)) * z) \le \text{Max}\{\mu(x), \mu(y)\}$

Therefore μ is an antifuzzy ideal of X.

Theorem 2.10 [14]: A nonempty subset I of a CI – algebra X is an ideal of X if it satisfies

(i) 1∈ I (2.20)

(ii) $x * (y * z) \in I \Rightarrow x * z \in I$, for all x,z ∈ X and y ∈ I (2.21)

Definition 2.9[8]: Let X be an CI- algebra and a,b ∈ X.We can define an upper set A(a,b) by $A(a,b) = \{x \in X / a * (b * x) = 1\}$. It is easy to see that 1,a,b ∈ A(a,b) for all a,b ∈ X.

Theorem 2.11: Let μ be a fuzzy set in CI-algebra X.Then μ is an antifuzzy ideal of X iff μ satisfies the following condition.

$$(\forall a, b \in X), (\forall t \in [0,1])\ (a, b) \in \mu^t \Rightarrow A(a,b) \subseteq \mu^t \quad (2.22)$$

Proof: Assume that μ is an antifuzzy ideal of X.

Let $a,b \in \mu^t$. Then $\mu(a) \le t$ and $\mu(b) \le t$.

Let x ∈ A (a,b). Then $a * (b * x) = 1$.

Now,

$\mu(x) = \mu(1 * x)$
$= \mu((a * (b * x)) * x)$
$\leq \text{Max}\{\mu(a), \mu(b)\}$
$\leq \text{Max}\{t, t\}$
$= t$
$\Rightarrow \mu(x) \leq t$.
$\Rightarrow x \in \mu^t$.

Therefore $A(a,b) \subseteq \mu^t$.

Conversely suppose that $A(a,b) \subseteq \mu^t$.

Obviously $1 \in A(a,b) \subseteq \mu^t$ for all $a,b \in X$.

Let $x,y,z \in X$ be such that $x * (y * z) \in \mu^t$ and $y \in \mu^t$.

Since $(x * (y * z)) * (y * (x * z)) = (x * (y * z)) * (x * (y * z)) = 1$. [By (2.1) and (2.3)] , We have $x * z \in A(x * (y * z), y) \subseteq \mu^t$ .It follows from theorem 2.10 that μ^t is an ideal of X.

Hence,by theorem 2.9, μ is an antifuzzy ideal of X.

Theorem 2.12: Let μ be a fuzzy set in CI-algebra X.If μ is an antifuzzy ideal of X then

$$(\forall t \in [0,1])\ \mu^t \neq \emptyset \Rightarrow \mu^t = \bigcup_{a,b \in \mu^t} A(a,b). \quad (2.23)$$

Proof : Let $t \in [0,1]$ be such that $\mu^t \neq \emptyset$. Since $1 \in \mu^t$, we have $\mu^t \subseteq \bigcup_{a \in \mu^t} A(a,1) \subseteq \bigcup_{a,b \in \mu^t} A(a,b)$.

Now, let $x \in \bigcup_{a,b \in \mu^t} A(a,b)$.

Then there exists $u,v \in \mu^t$ such that $x \in A(u,v) \subseteq \mu^t$ by theorem 2.11. Thus $\bigcup_{a,b \in \mu^t} A(a,b) \subseteq \mu^t$.

This completes the proof.

CONCLUSION

In this article we have discussed anti fuzzy ideal of CI-algebras and its lower level cuts in detail. It has been observed that the CI-algebra as a generation of BE-algebras. These concepts can further be generalized.

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