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MAGNETO-HYDRODYNAMIC FLOW OF VISCO-ELASTIC [OLDROYD (1958) MODEL] LIQUID THROUGH POROUS MEDIUM BETWEEN TWO INFINITE CO-AXIAL RIGHT CIRCULAR CYLINDERS

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ABSTRACT

The aim of the present paper is to study the effect of magnetic field on oscillatory motion of visco-elastic [Oldroyd (1958) model] liquid through porous medium between two infinite co-axial right circular cylinders when both the cylinders execute simple harmonic motion along the common axis of the cylinders. The amplitudes and frequencies have been taken different for both the cylinders. Some particular cases have also been deduced.

INTRODUCTION

The fluids which exhibit the elasticity property of solids and viscous property of liquids are called visco-elastic fluids or non-Newtonian fluids. For this one may refer to the review literature given by Bhatnagar (1967), Joseph (1989), Kapur, Bhatt and Sacheti (1992), Huilgol and Phan Thien (1997) and others. Moreover some interesting problems in this area have been investigated by Das and Sengupta (1993), Ghosh and Sengupta (1993, 1996), Sengupta and Kundu (1999), Hassanien (2002), Sengupta and Basak (2002), Pundhir and Pundhir (2003), Rehman and Alam Sarkar (2004), Saroa (2006), Sharma and Pareek (2006), Agarwal and Agarwal (2006), Kumar, Sharma and Singh (2008) and Kumar, Singh and Sharma (2010) etc.

In the present paper we study the effect of magnetic field on unsteady oscillatary motion of visco-elastic Oldroyd (1958) type liquid through porous medium lying inside the annulus of right circular cylinders when both the cylinders execute SHM along the axis of annuals. The amplitudes and frequencies are taken different for both the co-axial cylinders of the annulus. Some particular cases have also been discussed.

GOVERNING EQUATIONS

The Rheological equations satisfied by visco-elastic Oldroyd (1958) liquid are:

$$P_{ik} = -p\,\delta_{ik} + P'_{ik}$$

$$P'_{ik} + \lambda_1 \frac{D}{Dt} P'_{ik} + \mu_0 P'_{ij}\, e_{ik} - \mu_1\left(P'_{ij}\, e_{ik} + P'_{jk}\, e_{ij}\right) - \upsilon_1 P'_{jl}\, e_{jl}\,\delta_{ik}$$
$$= 2\eta_0\left[e_{ik} + \lambda_2 \frac{D}{Dt} e_{ik} - 2\mu_2\, e_{ij}\, e_{jk} + \upsilon_2 e_{jl}\,\delta_{ik}\right]$$

with the equation of incompressibility

$$e_{ii} = 0$$

where

$$\frac{D}{Dt} b_{ik} = \frac{\partial}{\partial t} b_{ik} + v_{ij}\, b_{ik,j} + w_{ij}\, b_{jk} + w_{kj}\, b_{ij}$$

$$e_{ik} = \frac{1}{2}\left(v_{k,i} + v_{i,k}\right)$$

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$w_{ik} = \frac{1}{2}\left(v_{k,i} - v_{i,k}\right)$

e_{ik} = rate of stress tensor

P_{ik} = stress tensor

λ_1 = relaxation time

λ_2 = retardation time

η_0 = coefficient of viscosity

and $\mu_0, \mu_1, \mu_2, \upsilon_1$ and υ_2 are material constants, each being of the dimension of time. For $\eta_0 > 0,\ \lambda_1 = \mu_1 = \mu_2 = \lambda_2 = 0,\ \mu_0 = \upsilon_1 = \upsilon_2 = 0$

the liquid will behave as ordinary viscous liquid.

EQUATION OF MOTION

Let (r, θ, z) be the cylindrical polar coordinates, the z-axis coinciding with the axis of annulus of right circular cylinders and if u, v, w are the components of velocity of the visco-elastic liquid in the increasing directions of r, θ, z respectively.

Following Ghosh (1968) the equation of motion for Oldroyd (1958) type liquid through porous medium under the influence of magnetic field within annulus is given by

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial w}{\partial t} = -\frac{1}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial z} + \upsilon\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) - \left(\frac{\sigma B_0^2}{\rho} + \frac{\upsilon}{K}\right)\left(1+\lambda_1\frac{\partial}{\partial t}\right)w \tag{1}$$

where $\upsilon = \frac{\mu}{\rho} =$ kinematic viscosity, μ the coefficient of viscosity, w the velocity of liquid in the direction of oscillation i.e. along central axis of annulus, p the fluid pressure, t the time, K the permeability of porous medium, B_0 the magnetic inductivity and σ is the conductivity of fluid.

Here we consider the motion of the visco-elastic liquid through porous medium bounded between two co-axial right circular cylinders of radii a and b (a > b) executing longitudinal harmonic oscillations along their common axis with different amplitudes and frequencies.

Assuming the pressure gradient to be zero, the equation (1) becomes

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial w}{\partial t} = \upsilon\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) - \left(\frac{\sigma B_0^2}{\rho} + \frac{\upsilon}{K}\right)\left(1+\lambda_1\frac{\partial}{\partial t}\right)w \tag{2}$$

The boundary conditions are:

$$\left.\begin{aligned} w &= v_1 e^{-i\omega_1 t}, \qquad \text{when } r = a \\ w &= v_2 e^{-i\omega_2 t}, \qquad \text{when } r = b \end{aligned}\right\} \tag{3}$$

where $v_1,\ \omega_1$ *and* $v_2,\ \omega_2$ are the amplitudes and frequencies of outer and inner cylinders respectively.

Introducing the following non-dimensional quantities:

$$r^* = \frac{r}{a},\quad t^* = \frac{\upsilon}{a^2}t,\quad w^* = \frac{a}{\upsilon}w,\quad \lambda_1^* = \frac{\upsilon}{a^2}\lambda_1,\quad \lambda_2^* = \frac{\upsilon}{a^2}\lambda_2,$$

$$v_1^* = \frac{a}{\upsilon}v_1,\quad v_2^* = \frac{a}{\upsilon}v_2,\quad \omega_1^* = \frac{a^2}{\upsilon}\omega_1,\quad \omega_2^* = \frac{a^2}{\upsilon}\omega_2 \quad and \quad K^* = \frac{1}{a^2}K$$

In (2) and (3) and then dropping the stars, we get

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial w}{\partial t} = \left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)-\left(M^2+\frac{1}{K}\right)\left(1+\lambda_1\frac{\partial}{\partial t}\right)w \tag{4}$$

where $M = aB_0\sqrt{\dfrac{\sigma}{\mu}}$ (Hatrmann Number) and boundary conditions become

$$\left.\begin{aligned} w&=v_1 e^{-i\omega_1 t}, &&\text{when r} = 1\\ w&=v_2 e^{-i\omega_2 t}, &&\text{when r} = \frac{\text{b}}{\text{a}}\end{aligned}\right\} \tag{5}$$

SOLUTION OF THE PROBLEM

We look for a solution of equation (4) in the form

$$w=v_1\, f_1(r)e^{-i\omega_1 t}+v_2\, g(r)e^{-i\omega_2 t} \tag{6}$$

which is evidently periodic in t

Substituting (6) in (4), we get

$$\begin{aligned}&\left\{(1-i\,\omega_1\,\lambda_2)\left(\frac{d^2 f}{dr^2}+\frac{1}{r}\frac{df}{dr}\right)+(1-i\,\lambda_1\,\omega_1)\left(i\,\omega_1-M^2-\frac{1}{K}\right)f\right\}v_1\,e^{-i\omega_1 t}+\\ &\left\{(1-i\,\omega_2\,\lambda_2)\left(\frac{d^2 g}{dr^2}+\frac{1}{r}\frac{dg}{dr}\right)+(1-i\,\lambda_1\,\omega_2)\left(i\,\omega_2-M^2-\frac{1}{K}\right)g\right\}v_2\,e^{-i\omega_2 t}\end{aligned} \tag{7}$$

By assumption that v_1 and v_2 are not zero, we have

$$\frac{d^2 f}{dr^2}+\frac{1}{r}\frac{df}{dr}+\frac{(1-i\,\lambda_1\,\omega_1)\left(i\,\omega_1-M^2-\frac{1}{K}\right)}{(1-i\,\lambda_2\,\omega_1)}f = 0$$

or $$\frac{d^2 f}{dr^2}+\frac{1}{r}\frac{df}{dr}+m^2 f = 0 \tag{8}$$

and $$\frac{d^2 g}{dr^2}+\frac{1}{r}\frac{dg}{dr}+\frac{(1-i\,\lambda_1\,\omega_2)\left(i\,\omega_2-M^2-\frac{1}{K}\right)}{(1-i\,\lambda_2\,\omega_2)}g = 0$$

or $$\frac{d^2 g}{dr^2}+\frac{1}{r}\frac{dg}{dr}+n^2 g = 0 \tag{9}$$

where $$m^2 = \frac{(1-i\,\lambda_1\,\omega_1)\left(i\,\omega_1-M^2-\frac{1}{K}\right)}{(1-i\,\lambda_2\,\omega_1)}$$

and $$n^2 = \frac{(1-i\,\lambda_1\,\omega_2)\left(i\,\omega_2-M^2-\frac{1}{K}\right)}{(1-i\,\lambda_2\,\omega_2)}$$

The boundary conditions are transformed to

$$\left.\begin{array}{lll} f(r)=1, & g(r)=0 & \text{when } r = 1 \\ f(r)=0, & g(r)=1 & \text{when } r = \dfrac{\mathrm{b}}{\mathrm{a}} \end{array}\right\} \tag{10}$$

Now the solutions of (8) and (9) subject to the boundary conditions (10) are

$$f(r)=\frac{J_0(mr)\,Y_0\left(m\frac{b}{a}\right)-Y_0(mr)\,J_0\left(m\frac{b}{a}\right)}{J_0(m)Y_0\left(m\frac{b}{a}\right)-J_0\left(m\frac{b}{a}\right)Y_0(m)}$$

and

$$g(r)=\frac{J_0(nr)\,Y_0\left(n\frac{b}{a}\right)-Y_0(nr)\,J_0\left(n\frac{b}{a}\right)}{J_0(n)\,Y_0\left(n\frac{b}{a}\right)-Y_0(n)\,J_0\left(n\frac{b}{a}\right)}$$

Putting the values of f(r) and g(r) in (6), we get the velocity of visco-elastic Oldroyd (1958) type liquid through porous medium between two oscillating co-axial right circular cylinders under the influence of magnetic field

$$w=v_1\left\{\frac{J_0(mr)\,Y_0\left(m\frac{b}{a}\right)-Y_0(mr)\,J_0\left(m\frac{b}{a}\right)}{J_0(m)\,Y_0\left(m\frac{b}{a}\right)-Y_0(m)\,J_0\left(m\frac{b}{a}\right)}\right\}e^{-i\omega_1 t}-v_2\left\{\frac{J_0(nr)\,Y_0(n)-Y_0(nr)\,J_0(n)}{J_0(n)\,Y_0\left(n\frac{b}{a}\right)-Y_0(n)\,J_0\left(n\frac{b}{a}\right)}\right\}e^{-i\omega_2 t} \tag{11}$$

PARTICULAR CASES

Case–I: If both cylinders oscillate with same amplitudes and different frequencies, then $v_1=v_2$ = v(say) and from (11)

$$w=v\left[\left\{\frac{J_0(mr)\,Y_0\left(m\frac{b}{a}\right)-Y_0(mr)J_0\left(m\frac{b}{a}\right)}{J_0(m)Y_0\left(m\frac{b}{a}\right)-Y_0(m)J_0\left(m\frac{b}{a}\right)}\right\}e^{-i\omega_1 t}-\left\{\frac{J_0(nr)\,Y_0(n)-Y_0(nr)\,J_0(n)}{J_0(n)\,Y_0\left(n\frac{b}{a}\right)-Y_0(n)\,J_0\left(n\frac{b}{a}\right)}\right\}e^{-i\omega_2 t}\right] \tag{12}$$

Case - II: If both cylinders oscillate with same frequencies but different amplitudes, then $\omega_1=\omega_2=\omega(say)$ and from (11)

$$w=\left[v_1\left\{\frac{J_0(mr)\,Y_0\left(m\frac{b}{a}\right)-Y_0(mr)\,J_0\left(m\frac{b}{a}\right)}{J_0(m)\,Y_0\left(m\frac{b}{a}\right)-Y_0(m)\,J_0\left(m\frac{b}{a}\right)}\right\}-v_2\left\{\frac{J_0(nr)\,Y_0(n)-Y_0(nr)\,J_0(n)}{J_0(n)\,Y_0\left(n\frac{b}{a}\right)-Y_0(n)\,J_0\left(n\frac{b}{a}\right)}\right\}\right]e^{-i\omega t} \tag{13}$$

where $m^2=n^2=\dfrac{(1-i\lambda_1\omega)\left(i\omega-M^2-\frac{1}{K}\right)}{(1-i\lambda_2\omega)}$

Case – III: If both cylinders oscillate with same amplitudes and frequencies, then

$v_1=v_2=v(say)$ *and* $\omega_1=\omega_2=\omega(say)$ and from (11)

$$w = v\left[\frac{J_0(mr)\left\{Y_0\left(m\frac{b}{a}\right) - Y_0(m)\right\} - Y_0(mr)\left\{J_0\left(m\frac{b}{a}\right) - J_0(m)\right\}}{J_0(m)Y_0\left(m\frac{b}{a}\right) - Y_0(m)J_0\left(m\frac{b}{a}\right)}\right]e^{-i\omega t} \tag{14}$$

where $m^2 = \dfrac{(1 - i\lambda_1\omega)\left(i\omega - M^2 - \dfrac{1}{K}\right)}{(1 - i\lambda_2\omega)}$

Case – IV: If porous medium and magnetic field is withdrawn, then $K = \infty$, $B_0 = 0$ and from (11)

$$w = v_1\left\{\frac{J_0(mr)\,Y_0\left(m\frac{b}{a}\right) - Y_0(mr)\,J_0\left(m\frac{b}{a}\right)}{J_0(m)\,Y_0\left(m\frac{b}{a}\right) - Y_0(m)\,J_0\left(m\frac{b}{a}\right)}\right\}e^{-i\omega_1 t} - v_2\left\{\frac{J_0(nr)\,Y_0(n) - Y_0(nr)\,J_0(n)}{J_0(n)\,Y_0\left(n\frac{b}{a}\right) - Y_0(n)\,J_0\left(n\frac{b}{a}\right)}\right\}e^{-i\omega_2 t} \tag{15}$$

where $m^2 = \dfrac{\omega_1(i + \lambda_1\omega_1)}{(1 - i\lambda_2\omega_1)}$, $\quad n^2 = \dfrac{\omega_2(i + \lambda_1\omega_2)}{(1 - i\lambda_2\omega_2)}$

DEDUCTION

If magnetic field is withdrawn i.e. $B_0 = 0$ then all the expressions are agree with Kumar, Sharma and Singh (2008).

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