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VISCOUS DISSIPATION AND HEAT SOURCE/SINK EFFECTS ON FLOW OVER STRETCHING SHEET

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ABSTRACT

Flow of an incompressible viscous fluid over a stretching surface played an important rule due to its important from industry such as the extrusion of polymers, the cooling of metallic plates, etc. The steady state analysis of an incompressible viscous fluid over a flat sheet is studied and the effects of viscous dissipation and heat source are taken into account in the energy equation. The governing equations for the fluid over stretching surface are solved numerically with the boundary conditions using Nachtsheim-Swigert iteration technique. Numerical results are presented in the form of velocity and temperature profiles with the boundary layer for different parameters.

MATHEMATICAL FORMULATION AND DISCUSSIONS

Consider the steady two dimensional flow of a viscous and incompressible fluid near the stagnation point on a stretching surface placed in a plane y = 0. The stretching surface has a uniform temperature T_w and a linear velocity u_w, while the velocity of the flow external to the boundary layer is $u_e(x)$. Also, the effects of viscous dissipation and heat source are included in the energy equation. Governing equation of the present problem can be written as [1]

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+u_e\frac{du_e}{dx} \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+q\left(T-T_\infty\right) \tag{3}$$

The boundary conditions are

$$u=bx,\ v=0,\ T=T_w \quad \text{at} \quad y=0 \tag{4}$$

$$u=u_e(x)=ax,\ T=T_\infty \quad \text{as} \quad y\to\infty \tag{5}$$

To facilitate the analysis, we introduce the following dimensionless variables:

$$\eta=y\left(\frac{b}{\upsilon}\right)^{1/2},\ u=bxf'(\eta),\ v=-(b\upsilon)^{1/2}f(\eta),\ \theta=\frac{T-T_\infty}{T_w-T_\infty}=\frac{T-T_\infty}{cx^2} \tag{6}$$

Substituting (6) into (2)-(3), we have

$$f'''+ff''-(f')^2+\lambda^2=0 \tag{7}$$

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$$\frac{1}{Pr}\theta'' + f\theta' - 2f'\theta + E_c f''^2 + Q\theta = 0 \qquad (8)$$

$$\begin{aligned} &f = 0,\ \ f' = 1,\ \ \theta = 1\, at\ \eta = 0 \\ &f' = \lambda,\ \ \theta = 0, at\ \eta = \infty \end{aligned} \qquad (9)$$

The parameters appear in the above equations are given by

$Pr = \dfrac{k}{\mu c_p}$, Prandtl number

$E_c = \dfrac{1}{c_p}\dfrac{b^2}{c}$ Eckert number

$Q = \dfrac{q}{\rho b c_{\mathrm{p}}}$, heat source parameter

And $\lambda = \dfrac{a}{b}$, velocity parameter

Equations (7) and (8) subject to the boundary conditions (9) have been solved numerically using Runge Kutta method [2] coupled with a shooting technique for some values of the parameters $\lambda,\ P_r,\ E_C,\ Q$.

RESULTS AND DISCUSSIONS

Figures 1 to 2 show the variation of the non-dimensional velocity and temperature profiles with some values of the parameters of interest namely $P_r = 0.71$. It is seen from figure-1 that the velocity of the fluid flow become fuller and increase with the increase of λ. Further (figure-2) the increase of λ leads to decrease of the temperature profiles when $E_c = \pm 0.01$ and $Q = \pm 0.5$. It may be mention that $E_c \rangle 0$ corresponding to cooling of the plate while the case $E_c \langle 0$ corresponds to the heating of the plate [3].

Figure 3, shows the temperature profiles for different values of dimensionless Prandtl number when $\lambda = 0.2, E_C = \pm 0.01, Q = \pm 0.5$. It is observed that at higher Prandtl number the thermal boundary layer reduces the thickness. It is evident that θ decreases with an decrease in the E_c and Q. Also the thermal boundary layer thickness increases for the cooling of the plate ($E_c \rangle 0$) than that of heating of the plate ($E_c \langle 0$) i.e. the effects of $E_c \rangle 0,\ E_c \langle 0$ on heat transfer are found to be more pronounced at $E_c \rangle 0$.

In figure 4 the graphs of temperature versus η for heat source/sink are drawn. For $Q \rangle 0$ and $Q \langle 0$ we have from equation (8) that there is heat source in the boundary layer when $T_w \langle T_\infty$ and a heat sink when $T_w \rangle T_\infty$.

- Physically these might correspond respectively to recombination and dissociation within boundary layer [4]. It is seen that wall temperature increases as heat source/sink parameter increases.
- Furthermore, the more flow induced when $Q > 0$.

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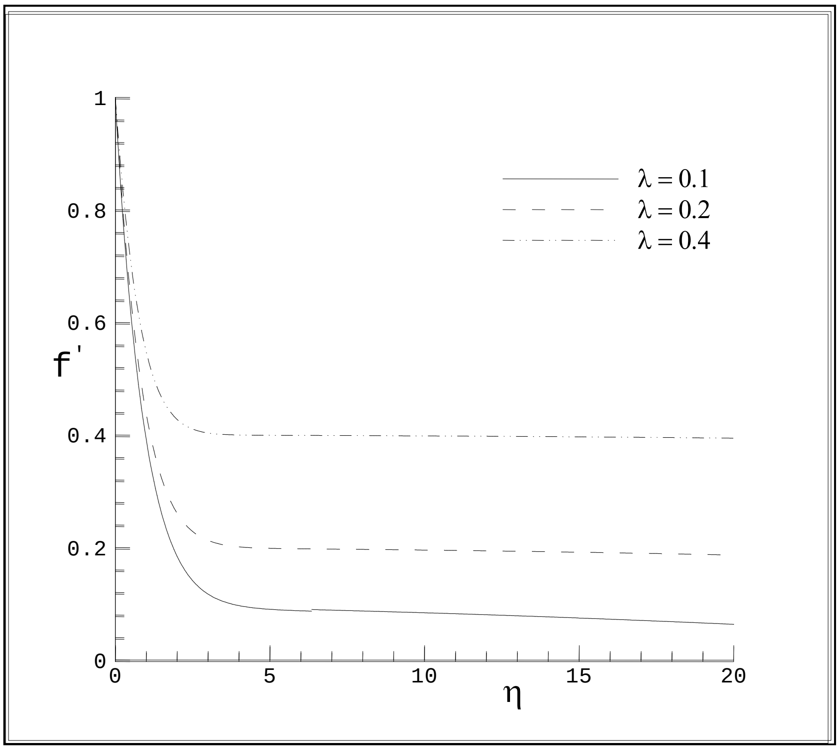

Figure 1: Velocity profiles for different values of λ

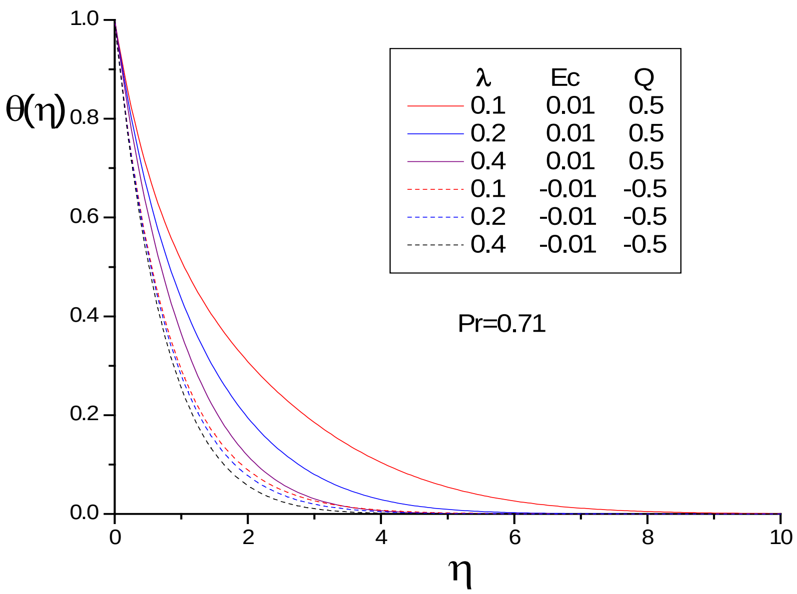

Figure 2: Temperature profiles for different values of λ

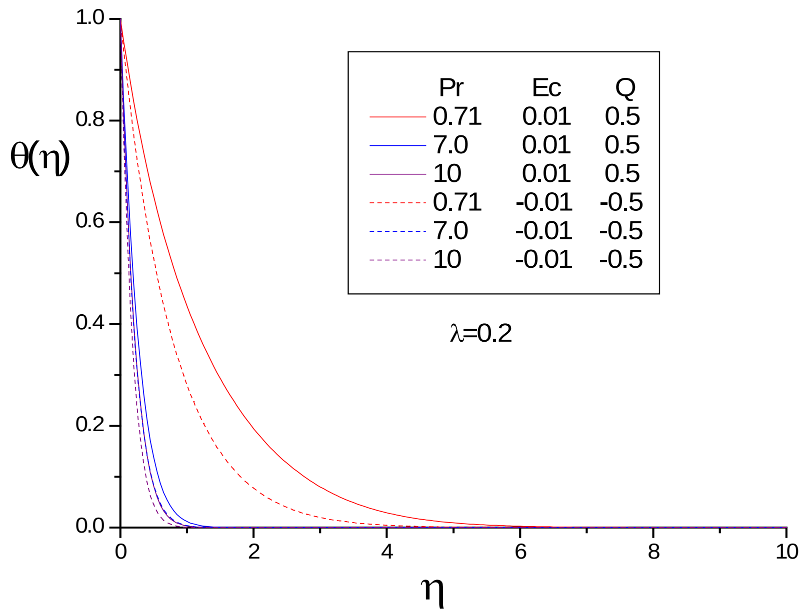

Figure 3: Temperature Profiles for different values of Pr

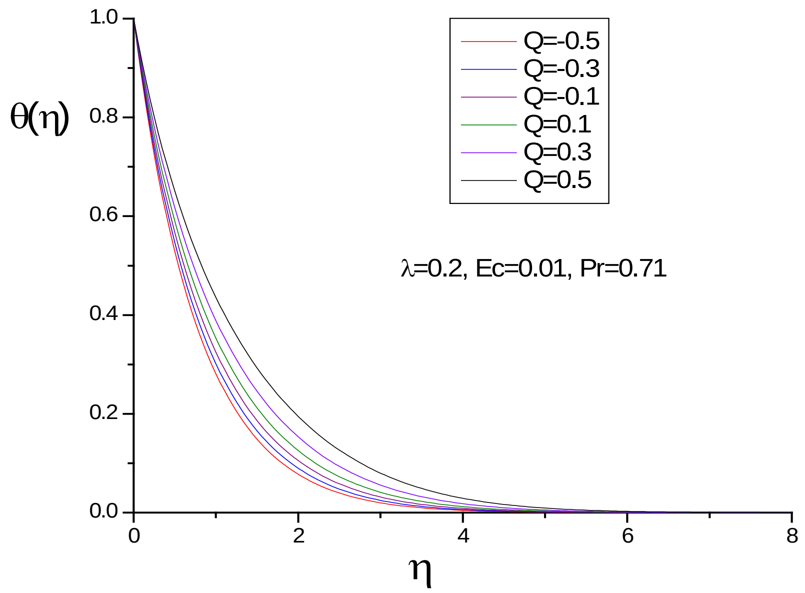

Figure 4: Temperature Profiles for different values of Q

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