International Journal of Mathematical Archive-2(2), Feb. - 2011, Page: 294- 299
IJMA Available online through www.ijma.info ISSN 2229 – 5046

$\tilde{g}$ - PRECLOSED SETS IN TOPOLOGY

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(Received on:04-02-11; Accepted on:13-02-11)

ABSTRACT

In this paper, we introduce a new class of sets namely, $\tilde{g}$ -preclosed sets in topological spaces. This class lies between the classes of $\tilde{g}_\alpha$-closed sets and the classes of gp-closed sets. We study some of its basic properties. Applying this we introduce a new class of spaces called $T\tilde{g}_p$-spaces.

***2000 Mathematics Subject Classification:** 54C10, 54C08, 54C05*

***Key words and Phrases:** Topological space, $\tilde{g}$ -closed set, #gs-closed set, $\tilde{g}_\alpha$-closed set, gp-closed set, gsp-closed set.*

1. INTRODUCTION:

Sundaram et al [23] introduced $\tilde{g}$ -semi-closed sets in topological spaces. Recently Jafari et al [10] introduced $\tilde{g}_\alpha$-closed sets in topological spaces. Sarsak and Rajesh [20] introduced π-Generalized semi-preclosed sets. After the advent of these notions, many topologists introduced various types of generalized closed sets and studied their fundamental properties.

In this paper, we introduce a new class of sets, namely $\tilde{g}$ -preclosed sets in topological spaces and study their basic properties. We obtain many interesting results in topological spaces. To substantiate these results, suitable examples are given at the respective places.

2. PRELIMINARIES:

Throughout this paper (X, τ) and (Y, σ) (or X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively.

We recall the following definitions which are useful in the sequel.

Definition: 2.1
A subset A of a space (X, τ) is called
(i) semi-open set [12] if $A \subseteq cl(int(A))$;
(ii) preopen set [15] if $A \subseteq int(cl(A))$;
(iii) α -open set [16] if $A \subseteq int(cl(int(A)))$;
(iv) β-open set [1] (= semi-preopen [2]) if $A \subseteq cl(int(cl(A)))$;
(v) regular open set [21] if A = int(cl(A));
(vi) π-open set [28] if A is the union of regular open sets.

The complements of the above mentioned sets are called their respective closed sets.

The preclosure [17] (resp. semi-closure [6], α -closure [16], semi-pre-closure [2]) of a subset A of X, pcl(A) (resp. scl(A), αcl(A), spcl(A)) is defined to be the intersection of all preclosed (resp. semi-closed, α -closed, semi-preclosed) sets of (X, τ) containing A. It is known that pcl(A) (resp. scl(A), αcl(A), spcl(A)) is a preclosed (resp. semi-closed, α -closed, semi-preclosed) set. For any subset A of an arbitrarily chosen topological spaces, the preinterior [17] of A, denoted by pint(A), is defined to be the union of all preopen sets of (X, τ) contained in A.

Definition: 2.2

A subset A of a space (X, τ) is called:

(i) a generalized closed (briefly g-closed) set [11] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ). The complement of g-closed set is called g-open set;
(ii) a semi-generalized closed (briefly sg-closed) set [4] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ). The complement of sg-closed set is called sg-open set;
(iii) a generalized semi-closed (briefly gs-closed) set [3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ). The complement of gs-closed set is called gs-open set;
(iv) an α -generalized closed (briefly αg-closed) set [13] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ). The complement of αg-closed set is called αg-open set;

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(v) a generalized preclosed (briefly gp-closed) set [17] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ). The complement of gp-closed set is called gp-open set;

(vi) a g*-preclosed (briefly g*p-closed) set [27] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in (X, τ). The complement of g*p-closed set is called g*p-open set;

(vii) a generalized semi-preclosed (briefly gsp-closed) set [7] if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ). The complement of gsp-closed set is called gsp-open set;

(viii) a ĝ-closed set [24] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in (X, τ). The complement of ĝ-closed set is called ĝ-open set;

(ix) a *g-closed set [25] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is ĝ-open in (X, τ). The complement of *g-closed set is called *g-open set;

(x) a #g-semi-closed (briefly #gs-closed) set [26] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is *g-open in (X, τ). The complement of #gs-closed set is called #gs-open set;

(xi) a $\tilde{g}$ -closed set [9] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is #gs-open in (X, τ). The complement of $\tilde{g}$ -closed set is called $\tilde{g}$ -open set;

(xii) a $\tilde{g}$ -semi-closed (briefly $\tilde{g}$ s-closed) set [23] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is #gs-open in (X, τ). The complement of $\tilde{g}$ s-closed set is called $\tilde{g}$ s-open set;

(xiii) a $\tilde{g}_\alpha$-closed set [10] if α cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is #gs-open in (X, τ). The complement of $\tilde{g}_\alpha$-closed set is called $\tilde{g}_\alpha$-open set;

(xiv) a generalized semi-preregular closed (briefly gspr-closed) set [8] if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in (X, τ). The complement of gspr-closed set is called gspr-open set;

(xv) a π-generalized semi-preclosed (briefly πgsp-closed) set [20] if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is π-open in (X, τ). The complement of πgsp-closed set is called πgsp-open set;

(xvi) a πgp-closed set [18] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is π-open in (X, τ). The complement of πgp-closed set is called πgp-open set.

Definition: 2.3

A topological space X is called a

(i) T$\tilde{g}$ -space [23] if every $\tilde{g}$ -closed set in it is closed;

(ii) #T$\tilde{g}$ α -space [10] if every $\tilde{g}_\alpha$-closed set in it is closed;

(iii) *sT_p-space [27] if every gsp-closed set in it is g*p-closed;

(iv)*T_p-space [27] if every gp-closed set in it is g*p-closed;

(v) α T_p*-space [27] if every g*p-closed set in it is preclosed;

(vi) α -space [16] if every α -closed set in it is closed.

Result: 2.4 [10]

(1) Every closed set is $\tilde{g}_\alpha$-closed but not conversely.

(2) Every $\tilde{g}_\alpha$-closed set is $\tilde{g}$ s-closed but not conversely.

(3) Every α -closed set is $\tilde{g}_\alpha$-closed but not conversely.

(4) Every $\tilde{g}$ -closed set is $\tilde{g}_\alpha$-closed but not conversely.

Result: 2.5 [23]

(1) Every open set is #gs-open but not conversely.

(2) Every closed set is $\tilde{g}$ -closed but not conversely.

(3) Every semi-open set is #gs-open but not conversely.

(4) Every closed set is $\tilde{g}$ s-closed but not conversely.

(5) Every $\tilde{g}$ -closed set is $\tilde{g}$ s-closed but not conversely.

(6) Every $\tilde{g}$ s-closed set is gsp-closed but not conversely.

Result: 2.6 [20]

Every πgsp-closed set is gspr-closed but not conversely.

Result: 2.7 [18]

Every gp-closed set is πgp-closed but not conversely.

3. BASIC PROPERTIES OF $\tilde{g}$ -PRECLOSED SETS:

We introduce the following notion.

Definition: 3.1

A subset A of X is called a $\tilde{g}$ -preclosed (briefly $\tilde{g}$ p-closed) set if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is #gs-open in (X, τ).

Proposition: 3.2

Every closed set is $\tilde{g}$ p-closed but not conversely.

Proof: Let A be a closed set and U be any #gs-open set containing A. Since A is closed we have pcl(A) $\subseteq$ cl(A) = A $\subseteq$ U. Hence A is $\tilde{g}$ p-closed.

Example: 3.3

Let X = {a, b, c} with τ = {ϕ, {c}, X}. Then {a} is $\tilde{g}$ p-closed set but not closed.

Proposition: 3.4

Every $\tilde{g}_\alpha$-closed set is $\tilde{g}$ p-closed but not conversely.

Proof: Let A be a $\tilde{g}_\alpha$-closed set and U be any #gs-open set containing A. Since A is $\tilde{g}_\alpha$-closed we have pcl(A) $\subseteq$ αcl(A) $\subseteq$ U. Hence A is $\tilde{g}$ p-closed.

Example: 3.5

Let X = {a, b, c} with τ = {ϕ, {a}, {b, c}, X}. Then {a, b} is $\tilde{g}$ p-closed set but not $\tilde{g}_\alpha$-closed.

Proposition: 3.6

Every $\tilde{g}$ p-closed set is gsp-closed but not conversely.

Proof: Let A be a $\tilde{g}$ p-closed set and U be any open set containing A. Since A is $\tilde{g}$ p-closed we have spcl(A) $\subseteq$ pcl(A) $\subseteq$ U. Hence A is gsp-closed.

Example: 3.7

Let X = {a, b, c, d} with τ = {ϕ, {a}, {b}, {a, b}, {a, c}, {a, b, c}, X}. Then {a, b, d} is gsp-closed set but not $\tilde{g}$ p-closed.

Proposition: 3.8
Every $\tilde{g}$ p-closed set is gp-closed but not conversely.

Proof: Let A be a $\tilde{g}$ p-closed set and U be any open set containing A. Since A is $\tilde{g}$ p-closed we have pcl(A) $\subseteq$ U. Hence A is gp-closed.

Example: 3.9
Let X = {a, b, c, d} with τ = {ϕ, {a}, {a, c}, {a, b, c}, X}. Then {a, d} is gp-closed set but not $\tilde{g}$ p-closed.

Proposition: 3.10
Every gsp-closed set is πgsp-closed but not conversely.

Proof: It follows from the Definition 2.2 (vii) and (xv).

Example: 3.11
Let X = {a, b, c, d} with τ = {ϕ, {a}, {a, b}, {a, c}, {a, b, c}, X}. Then {a} is πgsp-closed set but not gsp-closed.

Proposition: 3.12
Every gp-closed set is πgsp-closed but not conversely.

Proof: It follows from the Definition 2.2 (v) and (xv).

Example: 3.13
Let X = {a, b, c, d} with τ = {ϕ, {a}, X}. Then {a} is πgsp-closed set but not gp-closed.

Proposition: 3.14
Every πgp-closed set is gspr-closed but not conversely.

Proof: It follows from the Definition 2.2 (xiv) and (xvi).

Example: 3.15
Let X = {a, b, c, d} with τ = {ϕ, {a}, {b, c}, {a, b, c}, X}. Then {a} is gspr-closed set but not πgp-closed.

Remark: 3.16
$\tilde{g}$ -preclosedness is independent from semi-closedness, gs-closedness, sg-closedness, g-closedness, α g-closedness, $\tilde{g}$ s-closedness and *g-closedness.

Example: 3.17
Let X = {a, b, c, d} with τ = {ϕ, {a}, {b}, {a, b}, X}. Then {a, c} is semi-closed, gs-closed, sg-closed, g-closed, α g-closed, $\tilde{g}$ s-closed, *g-closed set but not $\tilde{g}$ p-closed.

Example: 3.18
Let X = {a, b, c} with τ = {ϕ, {a, b}, X}. Then {a} is $\tilde{g}$ p-closed set but not semi-closed, gs-closed, sg-closed, g-closed, α g-closed, $\tilde{g}$ s-closed, *g-closed.

Remark: 3.19
The following diagrams show the relationships established between $\tilde{g}$ p-closed sets and other sets where A $\rightarrow$ B (resp. A $\leftrightarrow$ B) represents A implies B but not conversely (resp. A and B are independent of each other).

Diagram 1

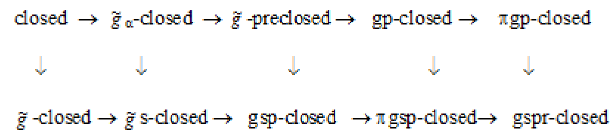
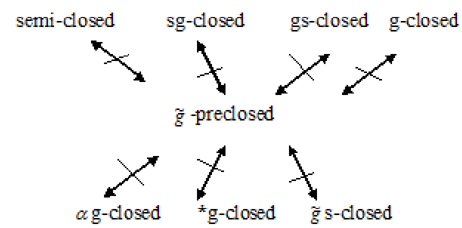


Diagram 2



4. CHARACTERIZATIONS OF $\tilde{g}$ -PRECLOSED SETS:

Theorem: 4.1
If A and B are $\tilde{g}$ p-closed sets such that cl(A) = pcl(A) and cl(B) = pcl(B), then A $\cup$ B is also $\tilde{g}$ p-closed in (X, τ).

Proof: Let A $\cup$ B $\subset$ U where U is #gs-open. Then A $\subset$ U and B $\subset$ U. Since A and B are $\tilde{g}$ p-closed, pcl(A) $\subset$ U and pcl(B) $\subset$ U. Now, cl(A $\cup$ B) = cl(A) $\cup$ cl(B) = pcl(A) $\cup$ pcl(B) $\subset$ U. But pcl(A $\cup$ B) $\subset$ cl(A $\cup$ B). So, pcl(A $\cup$ B) $\subset$ U and hence A $\cup$ B is $\tilde{g}$ p-closed.

Remark: 4.2
The following example shows that the union of two $\tilde{g}$ p-closed sets in (X, τ) is not, in general, $\tilde{g}$ p-closed in (X, τ).

Example:4.3
Let X= {a, b, c} with τ = { ϕ, {a, b}, X}. Then A= {a} and B = {b} are $\tilde{g}$ p-closed sets in (X, τ). But A $\cup$ B = {a, b} is not $\tilde{g}$ p-closed in (X, τ).

Proposition: 4.4
(1) If A is #gs-open and $\tilde{g}$ p-closed set, then A is preclosed.
(2) If A is open and $\tilde{g}$ p-closed set, then A is clopen.

Proof: (1) Since A is #gs-open and $\tilde{g}$ p-closed set, pcl(A) $\subset$ A. Therefore A = pcl(A). Hence A is preclosed.

(2) Since A is #gs-open and $\tilde{g}$ p-closed set, A is preclosed. Since A is preclosed and open, A is closed. Hence A is clopen.

Proposition: 4.5
Let A be a $\tilde{g}$ p-closed (X, τ). Then pcl(A) \ A does not contain any nonempty #gs-closed set.

Proof: Let F be a nonempty #gs-closed subset of pcl(A) \ A. Then A$\subset$X \ F, where A is $\tilde{g}$ p-closed and X \ F is #gs-open.

Thus pcl(A) ⊂ X \ F, or equivalently, F ⊂ X \ pcl(A). Since by assumption F ⊂ pcl(A), we get a contradiction.

Corollary: 4.6
Let A be $\tilde{g}$ p-closed in (X, τ). Then A is preclosed if and only if pcl(A) \ A is #gs-closed.

Proof: Necessity. Let A be $\tilde{g}$ p-closed. By hypothesis pcl(A) = A and so pcl(A) \ A = ϕ which is #gs-closed.

Sufficiency. Suppose pcl(A) \ A is #gs-closed. Then by Proposition 4.5, pcl(A) \ A =ϕ, that is, pcl(A) = A. Hence A is preclosed.

Proposition: 4.7
If A is a $\tilde{g}$ p-closed subset of (X, τ) such that A ⊂ B ⊂ pcl(A), then B is $\tilde{g}$ p-closed subset of (X, τ).

Proof: Let U be a #gs-open set in (X, τ) such that B ⊂ U. Then A ⊂ U. Since A is $\tilde{g}$ p-closed, then pcl(A) ⊂ U. Now, pcl(B) ⊂ pcl(A) ⊂ U. Therefore, B is a $\tilde{g}$ p-closed.

Proposition: 4.8
For every point x of a space X, X \ {x} is $\tilde{g}$ p-closed or #gs-open.

Proof: Suppose X \ {x} is not #gs-open. Then X is the only #gs-open set containing X \ {x}. Hence, X \ {x} is $\tilde{g}$ p-closed.

5. PROPERTIES OF $\tilde{g}$ -PREOPEN SETS:

The following corollary is an immediate consequence of the fact that pcl(X \A) = X \ pint(A).

Proposition: 5.1
A set A in a topological space (X, τ) is $\tilde{g}$ p-open if and only if F ⊆ pint(A) whenever F is #gs-closed in (X, τ) and F ⊆ A.

Proof: Necessity. Let A be $\tilde{g}$ p-open in (X, τ) and suppose F ⊆ A where F is #gs-closed. By definition X|A is $\tilde{g}$ p-closed. Also X |A is contained in the #gs-open set X|F. This implies pcl(X|A) ⊆ X|F. It means X| pint(A) ⊆ X| F. Hence F ⊆ pint(A). Sufficiency. If F is #gs-closed set with F ⊆ pint(A) whenever F ⊆ A, it follows that X|A ⊆ X|F and X| pint(A) ⊆ X|F, i.e., pcl(X|A) ⊆ X|F. Hence X|A is $\tilde{g}$ p-closed and A becomes $\tilde{g}$ p-open in (X, τ).

Proposition: 5.2
If pint (A) ⊂ B ⊂ A and A is $\tilde{g}$ p-open, then B is $\tilde{g}$ p-open.

Proof: Follows from Proposition 4.7.

Proposition: 5.3
If a set A is $\tilde{g}$ p-open in a topological space (X, τ), then G = X whenever G is #gs-open in (X, τ) and pint(A) ∪ A^c ⊂ G.

Proof: Suppose that G is #gs-open and pint(A) ∪ A^c ⊂ G. Now G^c ⊂ pcl(A^c) ∩ A = pcl(A^c) \ A^c. Since G^c is #gs-closed and A^c is $\tilde{g}$ p-closed, by Proposition 4.5, G^c = ϕ and hence G = X.

Remark: 5.4
The following example shows that the intersection of two $\tilde{g}$ p-open sets in X is not, in general, $\tilde{g}$ p-open in (X, τ).

Example: 5.5
Let X = {a, b, c} with τ = {ϕ, {a, c}, X}. Then A = {a, b} and B = {b, c} are $\tilde{g}$ p-open sets in (X, τ). But A ∩ B = {b} is not $\tilde{g}$ p-open in (X, τ).

Proposition: 5.6
Let (X, τ) be a space such that the family PO(X, τ) of all preopen subsets of (X, τ) be closed under finite intersections. If A and B are $\tilde{g}$ p-open in (X, τ), then A ∩ B is $\tilde{g}$ p-open.

Proof: Let X \ (A ∩ B) = (X \A) ∪ (X \ B) ⊂ U, where U is #gs-open. Then X \ A ⊂ U and X \ B ⊂ U. Since A and B are $\tilde{g}$ p-open, pcl(X \ A) ⊂ U and pcl(X \ B) ⊂ U. By hypothesis, pcl((X \A) ∪ (X \ B)) = pcl(X \A) ∪ pcl(X \ B) ⊂ U. Hence A ∩ B is $\tilde{g}$ p-open.

Proposition: 5.7
Let A be $\tilde{g}$ p-open in (X, τ) and B be open. Then A ∩ B is $\tilde{g}$ p-open in (X, τ).

Proof: Let F be any #gs-closed subset of X such that F⊂ A ∩ B. Hence F ⊂ A and by Proposition 5.1, F ⊂ pint(A) = ∪ { U: U is preopen and U ⊂ A}. Obviously, F ⊂ ∪ (U ∩ B), where U is a preopen set in X contained in A. Since (U ∩ B) is a preopen set contained in A ∩ B for each preopen set U contained in A, F ⊂ pint (A ∩ B), and by Proposition 5.1, A ∩ B is $\tilde{g}$ p-open in X.

Proposition: 5.8
Let (X, τ) be a topological space and A, B ⊂ X. If B is $\tilde{g}$ p-open and pint(B) ⊂ A , then A ∩ B is $\tilde{g}$ p-open.

Proof: Since B is $\tilde{g}$ p-open and pint(B) ⊂ A, pint(B) ⊂ A ∩ B ⊂ B. By Proposition 5.2, A ∩ B is $\tilde{g}$ p-open.

Proposition: 5.9
If A ⊂ X is $\tilde{g}$ p-closed, then pcl(A) \ A is $\tilde{g}$ p-open.

Proof: Let A be $\tilde{g}$ p-closed and F be a #gs-closed set such that F ⊂ pcl(A) \A. Then by Proposition 4.5, F = ϕ. So, F ⊂ pint (pcl(A) \ A). By Proposition 5.1, pcl(A) \ A is $\tilde{g}$ p-open.

The following Lemma can be easily verified.

Lemma: 5.10
For every subset A of a space (X, τ), pint(pcl(A) \A) =ϕ.

Proposition: 5.11
Let $A \subset B \subset X$ and pcl(A) \A be $\widetilde{g}$ p-open. Then pcl(A) \B is also $\widetilde{g}$ p-open.

Proof: Suppose pcl(A) \ A is $\widetilde{g}$ p-open and let F be a $^{\#}$gs-closed subset of (X, τ) with $F \subset$ pcl(A) \ B. Then $F \subset$ pcl(A) \ A. By Proposition 5.1 and Lemma 5.10, $F \subset$ pint(pcl(A) \ A) $=\phi$. Thus $F = \phi$ and hence, $F \subset$ pint(pcl(A) \ B). Hence pcl(A) \ B is $\widetilde{g}$ p-open.

6. APPLICATIONS:

Definition: 6.1
A space X is called a T$\widetilde{g}_p$-space if every $\widetilde{g}$ p-closed set in it is preclosed.

Theorem: 6.2
For a space X the following conditions are equivalent.

(i) X is a T$\widetilde{g}_p$-space.
(ii) Every singleton of X is either $^{\#}$gs-closed or preopen.

Proof: (i)→(ii). Let $x \in X$. Suppose that {x} is not a $^{\#}$gs-closed set of X. Then X−{x} is not a $^{\#}$gs-open set. So X is the only $^{\#}$gs-open set containing X −{x}. Then X − {x} is an $\widetilde{g}$ p-closed set of X. Since X is a T$\widetilde{g}_p$-space, X −{x} is a preclosed set of X and hence {x} is a preopen set of X.
(ii)→(i). Let A be a $\widetilde{g}$ p-closed set of X. We have $A \subseteq$ pcl(A). Let $x \in$ pcl(A) by (ii) {x} is either $^{\#}$gs-closed or preopen. Case (i) Suppose that {x} is $^{\#}$gs-closed. If $x \notin A$, pcl(A) −A contains a nonempty $^{\#}$gs-closed set {x}. By Theorem 4.5, we arrive at a contradiction. Thus $x \in A$. Case (ii) Suppose that {x} is preopen. Since $x \in$ pcl(A), $\{x\} \cap A \neq \phi$. This implies $x \in A$. Thus in any case $x \in A$. So pcl(A) $\subseteq$ A. Therefore pcl(A) = A or equivalently A is preclosed. Hence X is a T$\widetilde{g}_p$-space.

Definition: 6.3
A topological space X is called a $^{\#}$T$\widetilde{g}_p$-space if every $\widetilde{g}$ p-closed set in it is closed.

Theorem: 6.4
Every $^{\#}$T$\widetilde{g}_p$-space is a T$\widetilde{g}_p$-space but not conversely.

Proof: Since every closed set is preclosed, the result follows.

Example: 6.5
Let X = {a, b, c, d} with τ = {ϕ, {b}, {a, b}, X}. Then (X, τ) is a T$\widetilde{g}_p$-space but not a $^{\#}$T$\widetilde{g}_p$-space.

Theorem: 6.6
Every $^{\#}$T$\widetilde{g}_p$-space is a $^{\#}$T$\widetilde{g}\,\alpha$-space but not conversely.

Proof: Let A be a $\widetilde{g}_\alpha$-closed subset of X. Then A is $\widetilde{g}$ p-closed. Since X is a $^{\#}$T$\widetilde{g}_p$-space, A is closed in X. Hence X is a $^{\#}$T$\widetilde{g}\,\alpha$-space.

Example: 6.7
Let X = {a, b, c} with τ = {ϕ, {a}, {b, c}, X}. Then (X, τ) is a $^{\#}$T$\widetilde{g}\,\alpha$-space but not a $^{\#}$T$\widetilde{g}_p$-space.

Theorem: 6.8
If X is $*_sT_p$-space and αT_p*-space then it is T$\widetilde{g}_p$-space.

Proof: Let A be $\widetilde{g}$ p-closed set. Then A is gsp-closed set. Since X is $*_sT_p$-space, A is g*p-closed set. Since X is αT_p*-space, A is preclosed. Hence X is T$\widetilde{g}_p$-space.

Example: 6.9
Let X = {a, b, c, d} with τ = {ϕ, {a}, {a, b}, X}. Then (X, τ) is a T$\widetilde{g}_p$-space but it is neither $*sT_p$-space nor αT_p*-space.

Theorem: 6.10
If X is $*T_p$-space and αT_p*-space then it is T$\widetilde{g}_p$-space.

Proof: Let A be $\widetilde{g}$ p-closed set. Then A is gp-closed set. Since X is $*T_p$-space, A is g*p-closed set. Since X is αT_p*-space, A is preclosed. Hence X is T$\widetilde{g}_p$-space.

Example: 6.11
Let X = {a, b, c, d} with τ = {ϕ, {b}, {a, b}, X}. Then (X, τ) is a T$\widetilde{g}_p$-space but it is neither $*T_p$-space nor αT_p*-space.

Theorem: 6.12
Every $^{\#}$T$\widetilde{g}_p$-space is α-space but not conversely.

Proof: Let A be a α-closed subset of X. Then A is $\widetilde{g}$ p-closed. Since X is a $^{\#}$T$\widetilde{g}_p$-space, A is closed in X. Hence X is a α-space.

Example: 6.13
Let X = {a, b, c} and τ = {ϕ, {b}, {a, c}, X}. Then (X, τ) is a α-space but not a $^{\#}$T$\widetilde{g}_p$-space.

Theorem: 6.14
Every $^{\#}$T$\widetilde{g}_p$-space is a T$\widetilde{g}$-space but not conversely.

Proof: Let A be a $\widetilde{g}$-closed subset of X. Then A is $\widetilde{g}$ p-closed. Since X is a $^{\#}$T$\widetilde{g}_p$-space, A is closed in X. Hence X is a T$\widetilde{g}$-space.

Example: 6.15
Let X = {a, b, c, d} and τ = {ϕ, {b}, {a, b}, X}. Then (X, τ) is a T$\widetilde{g}$ - space but not a $^{\#}$T$\widetilde{g}_p$-space.

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