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FIXED POINT THEOREMS IN POLISH SPACE TAKING CONCEPT OF 2-METRIC

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ABSTRACT

In this paper, We established Some Common Fixed Point Theorems for Random Operator in polish spaces, by using some new type of contractive condition taking concept of 2-metric space motivated by Sharma, Sharma and Iskey[16]. Our result is generalization of our result for polish space and also various known results.

Key Words: *Polish Space, Random Operator, Random Multivalued Operator, Random Fixed Point, Measurable Mapping, 2-metruc spaces.*

AMS Subject Classification: *47H10, 54H25.*

1. INTRODUCTION

Probabilistic functional analysis has emerged as one of the important mathematical disciplines in view of its role in analyzing Probabilistic models in the applied sciences. The study of fixed point of random operator forms a central topic in this area. Random fixed point theorem for contraction mappings in Polish spaces and random Fixed point theorems are of fundamental importance in probabilistic functional analysis. There study was initiated by the Prague school of Probabilistics, in 1950, with their work of Spacek [15] and Hans [5,6]. For example survey are refer to Bharucha-Reid[4]. Itoh [8] proved several random fixed point theorems and gave their applications to Random differential equations in Banach spaces. Random coincidence point theorems and random fixed point theorems are stochastic generalization of classical coincidence point theorems and classical fixed point theorems.

Random fixed point theorems are stochastic generalization of classical fixed point theorems. Itoh [8] extended several well known fixed point theorems, thereafter; various stochastic aspects of Schauder's fixed point theorem have been studied by Sehgal and Singh [14], Papageorgiou [12], Lin [13] and many authors. In a separable metric space, random fixed point theorems for contractive mappings were proved by Spacek [15], Hans [5, 6]. Afterwards, Beg and Shahzad [2], Badshah and Sayyad studied the structure of common random fixed points and random coincidence points of a pair of compatible random operators and proved the random fixed point theorems for contraction random operators in Polish spaces.

2. PRELIMINARIES

In this section, we give some definitions which are useful to prove our results.

Definition: 2.1: A metric space (X, d) is said to be a Polish Space, if it satisfying following conditions:
(i) X, is complete,
(ii) X is separable,

Definition 2.2: 2-Metric space
(2.2.1) A 2- metric space is a space X in which for each triple of points x, y, z, there exists a real function d (x, y, z) such that

$[M_1]$ to each pair of distinct points x, y, z, $d(x, y, z) \neq 0$
$[M_2]$ $d(x, y, z) = 0$ when at lest two of x, y, z are equal
$[M_3]$ $d(x, y, z) = d(y, z, x) = d(x, z, y)$
$[M_4]$ $d(x, y, z) \leq d(x, y, v) + d(x, v, z) + d(v, y, z)$ for all x, y, z, v in X.

Definition (2.2.2): A sequence $\{x_n\}$ in a 2-metic space (X, d) is said to be convergent at x if limit $d(x_n, x, z) = 0$ for all z in X as $n \to \infty$

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Definition (2.2.3) A sequence $\{x_n\}$ in a 2-metric space, (x, d) is said to be Cauchy sequence if limit d $(x_n, x, z) = 0$ for all z in X and $m, n \to \infty$

Definition (2.2.4) A 2-metic space (X, d) is said to be complete if every Cauchy sequence in X is convergent.

Before we describe our next hierarchy of set of reals of ever increasing complexity, we would like to consider a class of metric spaces under which we can unify 2^ω, ω^ω, $\mathcal{R}$ and there products. This will be helpful in formulating this hierarchy. (as well as future ones) Recall that a metric space (X, d) is complete if whenever $(x_n: n \in \omega)$ is a sequence of member of X, such that for every $\epsilon > 0$ there is an N, such that $m, n \geq N$ implies $d(x_n, x_m) < \epsilon$, there is a single x in X such that $\lim_{n<\omega} x_n = x$. It is easy to see that 2^ω, ω^ω are polish space, So in fact is ω under the discrete topology, whose metric is given by letting $d(x, y) = 1$ when $x \neq y$ and $d(x, y) = 0$ when $x = y$. Let (X, d) be a Polish space that is a separable complete metric space and (Ω, q) be Measurable space. Let 2^x be a family of all subsets of X and $CB(X)$ denote the family of all nonempty bounded closed subsets of X. A mapping $T: \Omega \to 2^X$ is called measurable if for any open subset C of X, $T^{-1}(C) = \{\omega \in \Omega: f(\omega) \cap C \neq \phi \in q$. A mapping $\xi: \Omega \to X$ is said to be measurable selector of a measurable mapping $T: \Omega \to 2X$, if ξ is measurable and for any $\omega \in \Omega$, $\xi(\omega) \in T(\omega)$. A mapping $f: \Omega \times X \to X$ is called random operator, if for any $x \in X$, $f(\cdot, x)$ is measurable. A Mapping $T: \Omega \times X \to CB(X)$ is a random multivalued operator, if for every $x \in X$, $T(\cdot, x)$ is measurable. A measurable mapping $\xi: \Omega \to X$ is called random fixed point of a random multivalued operator $T: \Omega \times X \to CB(X)$ $(f: \Omega \times X \to X)$ if for every $\omega \in \Omega$, $\xi(\omega) \in T(\omega, \xi(\omega))$, $f(\omega), \xi(\omega) = \xi(\omega))$. Let $T: \Omega \times X \to CB(X)$ be a random operator And $\{\xi_n\}$ a sequence of measurable mappings , $\xi_n: \Omega \to X$. The sequence $\{\xi_n\}$ is said to be asymptotically T-regular if $d(\xi_n(\omega), T(\omega, \xi_n(w)) \to 0$.

3. MAIN RESULTS

Theorem 3.1 Let X be a Polish space associated with 2-metric. Let $T, S: \Omega \times X \to CB(X)$ be two continuous random multivalued operators. If there exists measurable mappings $\alpha, \beta, \gamma, \delta : \Omega \to (0,1)$ and $a(w)$ is also a measurable function greater than zero such that,

$$H(S(\omega, x), T(\omega, y), a(w)) \leq \alpha(\omega) \left[\frac{\begin{array}{c} d(x, S(\omega, x), a(w)) d(y, T(\omega, y), a(w)) d(x, T(\omega, y)) \\ + d(x(\omega), y(\omega), a(w)) d(y, S(\omega, x), a(w)), d(x, T(\omega, y), a(w)) \end{array}}{1 + d(y(\omega), S(\omega, x), a(w)) d(y(\omega), T(\omega, y), a(w))} \right]$$

$$+ \beta(\omega)[d(x, S(\omega, x), a(w)) + d(y, T(\omega, y), a(w))]$$

$$+ \gamma(\omega)[d(y, S(\omega, x), a(w)) + d(x, T(\omega, y), a(w))] + \delta(\omega) d(x(\omega), y(\omega), a(w)) \quad 3.1(a)$$

For each $x, y \in X$, $\omega \in \Omega$ and $\alpha, \beta, \gamma, \delta \in R^+$ with $0 \leq \alpha(\omega) + 2\beta(\omega) + 2\gamma(\omega) + \delta(\omega) < 1$, and $1 - \alpha(\omega) - \beta(\omega) - \gamma(\omega) \neq 0, a(w) > 0$ there exists a common random fixed point of S and T.

(hence H represents the Hausdroff $2 -$ metric on $CB(X)$ induced by the $2 -$ metric d)

Proof: Let $\xi_0 : \Omega \to X$ be an arbitrary measurable mapping and choose a measurable mapping $\xi_1 : \Omega \to X$ such that $\xi_1(\omega) \in S(\omega, \xi_0(\omega), a(w))$ for each $\omega \in \Omega$. then for each $\omega \in \Omega$.

$$H\left(S\left(\omega, \xi_0(\omega)\right), T\left(\omega, \xi_1(\omega)\right), a(w)\right) \leq \alpha(\omega) \left[\frac{\begin{array}{c} d\left(\xi_0(\omega), S\left(\omega, \xi_0(\omega)\right), a(w)\right) d\left(\xi_1(\omega), T\left(\omega, \xi_1(\omega)\right), a(w)\right) d\left(\xi_0(\omega), T\left(\omega, \xi_1(\omega)\right), a(w)\right) \\ + d(\xi_0(\omega), \xi_1(\omega), a(w)) d\left(\xi_1(\omega), S\left(\omega, \xi_0(\omega)\right), a(w)\right), d\left(\xi_0(\omega), T\left(\omega, \xi_1(\omega)\right), a(w)\right) \end{array}}{1 + d\left(\xi_0(\omega), S\left(\omega, \xi_0(\omega)\right), a(w)\right) d\left(\xi_1(\omega), T\left(\omega, \xi_1(\omega)\right), a(w)\right)} \right]$$

$$+ \beta(\omega) \left[d\left(\xi_0(\omega), S\left(\omega, \xi_0(\omega)\right), a(w)\right) + d\left(\xi_1(\omega), T\left(\omega, \xi_1(\omega)\right), a(w)\right) \right]$$

$$+ \gamma(\omega) \left[d\left(\xi_1(\omega), S\left(\omega, \xi_0(\omega)\right), a(w)\right) + d\left(\xi_0(\omega), T\left(\omega, \xi_1(\omega)\right), a(w)\right) \right]$$

$$+ \delta(\omega) d\left(\xi_0(\omega), \xi_1(\omega), a(w)\right)$$

Further there exists a measurable mapping $\xi_2 : \Omega \to X$ such that for all $\omega \in \Omega, \xi_2(\omega) \in T\left(\omega, \xi_1(\omega)\right), a(w)$ and

$$d\big(\xi_1(\omega),\xi_2(\omega),a(w)\big) \le \alpha(\omega)\left[\frac{\begin{array}{c} d\big(\xi_0(\omega),\xi_1(\omega),a(w)\big)d\big(\xi_1(\omega),\xi_2(\omega),a(w)\big)d\big(\xi_0(\omega),\xi_2(\omega),a(w)\big) \\ +d\big(\xi_0(\omega),\xi_1(\omega),a(w)\big)d\big(\xi_1(\omega),\xi_1(\omega),a(w)\big),d\big(\xi_0(\omega),\xi_2(\omega),a(w)\big)\end{array}}{1+d\big(\xi_0(\omega),\xi_1(\omega),a(w)\big)d\big(\xi_1(\omega),\xi_2(\omega),a(w)\big)}\right]$$

$$+\beta(\omega)\big[d\big(\xi_0(\omega),\xi_1(\omega),a(w)\big)+d\big(\xi_1(\omega),\xi_2(\omega),a(w)\big)\big]$$

$$+\gamma(\omega)\big[d\big(\xi_1(\omega),\xi_1(\omega),a(w)\big)+d\big(\xi_0(\omega),\xi_2(\omega),a(w)\big)\big]+\delta(\omega)d\big(\xi_0(\omega),\xi_1(\omega),a(w)\big)$$

$$d\big(\xi_1(\omega),\xi_2(\omega),a(w)\big) \le \frac{\alpha(\omega)+\beta(\omega)+\gamma(\omega)+\delta(\omega)}{1-\alpha(\omega)-\beta(\omega)-\gamma(\omega)}d\big(\xi_0(\omega),\xi_1(\omega),a(w)\big)$$

Let $k = \frac{\alpha(\omega)+\beta(\omega)+\gamma(\omega)+\delta(\omega)}{1-\alpha(\omega)-\beta(\omega)-\delta(\omega)}$

This gives

$$d\big(\xi_1(\omega),\xi_2(\omega),a(w)\big) \le k\, d\big(\xi_0(\omega),\xi_1(\omega),a(w)\big)$$

By Beg and Shahzad [2, lemma 2.3],, we obtain a measurable mapping $\xi_3 : \Omega \to X$ such that for all $\omega \in \Omega$, $\xi_3(\omega) \in S\big(\omega,\xi_2(\omega),a(w)\big)$ and

$$d\Big(\xi_2(\omega),\xi_3(\omega),a(w)\Big) \le d\Big(\xi_1(\omega),\xi_2(\omega),a(w)\Big)$$

$$\le \alpha(\omega)\left[\frac{\begin{array}{c} d\big(\xi_1(\omega),\xi_2(\omega),a(w)\big)d\big(\xi_2(\omega),\xi_3(\omega),a(w)\big)d\big(\xi_1(\omega),\xi_3(\omega),a(w)\big) \\ +d\big(\xi_1(\omega),\xi_2(\omega),a(w)\big)d\big(\xi_2(\omega),\xi_2(\omega),a(w)\big),d\big(\xi_1(\omega),\xi_3(\omega),a(w)\big)\end{array}}{1+d\big(\xi_1(\omega),\xi_2(\omega)\big)d\big(\xi_2(\omega),\xi_3(\omega),a(w)\big)}\right]$$

$$+\beta(\omega)\big[d\big(\xi_1(\omega),\xi_2(\omega),a(w)\big)+d\big(\xi_2(\omega),\xi_3(\omega),a(w)\big)\big]$$

$$+\gamma(\omega)\big[d\big(\xi_2(\omega),\xi_2(\omega),a(w)\big)+d\big(\xi_1(\omega),\xi_3(\omega),a(w)\big)\big]$$

$$+\delta(\omega)d\big(\xi_1(\omega),\xi_2(\omega),a(w)\big)D\Big(\xi_2(\omega),\xi_3(\omega),a(w)\Big)$$

$$\le k\, d\Big(\xi_1(\omega),\xi_2(\omega),a(w)\Big)\ k^2 d\big(\xi_0(\omega),\xi_1(\omega),a(w)\big)$$

Similarly, proceeding the same way, by induction, we get a sequence of measurable mapping $\xi_n : \Omega \to X$ suct that for $n > 0$ and for any $\omega \in \Omega$,

$$\xi_{2n+1}(\omega) \in S\big(\omega,\xi_{2n}(\omega),a(w)\big)\ \text{, and }\ \xi_{2n+2}(\omega) \in T\big(\omega,\xi_{2n+1}(\omega),a(w)\big)$$

This gives,

$$d\big(\xi_n(\omega),\xi_{n+1}(\omega),a(w)\big) \le kd\big(\xi_{n-1}(\omega),\xi_n(\omega),a(w)\big) \le \cdots \ldots \ldots \ldots \le k^n d\big(\xi_0(\omega),\xi_1(\omega),a(w),a(w)\big)$$

For any $m, n \in N$ such that $m > n$, also by using triangular inequality we have

$$d\big(\xi_n(\omega),\xi_m(\omega),a(w)\big) \le \frac{k^n}{1-k}d\big(\xi_0(\omega),\xi_1(\omega),a(w)\big)$$

Which tends to zero as $n \to \infty$. It follows that $\{\xi_n(\omega)\}$ is a Cauchy sequence and there exists a measurable mapping $\xi : \Omega \to X$ such that $\xi_n(\omega) \to \xi(\omega)$ for each $\omega \in \Omega$. It implies that $\xi_{2n+1}(\omega) \to \xi(\omega)$. Thus we have for any $\omega \in \Omega$,

$$d\big(\xi(\omega),S\big(\omega,\xi(\omega)\big),a(w)\big) \le d\big(\xi(\omega),\xi_{2n+2}(\omega),a(w)\big)+ d\Big(\xi(\omega),S\Big(\omega,\xi_{2n+2}(\omega)\Big),a(w)\Big)$$

$$d\big(\xi(\omega),S\big(\omega,\xi(\omega)\big),a(w)\big) \le \big(\xi(\omega),\xi_{2n+2}(\omega),a(w)\big)+ H\Big(T\big(\omega,\xi_{2n+1}(\omega),a(w)\big),S\big(\omega,\xi_{2n+2}(\omega),a(w)\big)\Big)$$

Therefore,

$$d\left(\xi(\omega), S(\omega, \xi(\omega)), a(w)\right) \leq d\left(\xi(\omega), \xi_{2n+2}(\omega), a(w), a(w)\right)$$

$$+\alpha(\omega)\left[\frac{d\left(\xi_{2n+2}(\omega), S\left(\omega, \xi_{2n+2}(\omega)\right), a(w)\right) d\left(\xi_{2n+1}(\omega), T\left(\omega, \xi_{2n+1}(\omega)\right), a(w)\right) d\left(\xi_{2n+2}(\omega), T\left(\omega, \xi_{2n+1}(\omega)\right), a(w)\right) + d\left(\xi_{2n+2}(\omega), \xi_{2n+1}(\omega)\right) d\left(\xi_{2n+1}(\omega), S\left(\omega, \xi_{2n+2}(\omega)\right)\right), d\left(\xi_{2n+2}(\omega), T(\omega, y)\right)}{1 + d\left(\xi_{2n+1}(\omega), S\left(\omega, \xi_{2n+2}(\omega)\right), a(w)\right) d\left(\xi_{2n+1}(\omega), T\left(\omega, \xi_{2n+1}(\omega)\right), a(w)\right)}\right]$$

$$+ \beta(\omega)\left[d\left(\xi_{2n+2}(\omega), S\left(\omega, \xi_{2n+2}(\omega)\right), a(w)\right) + d\left(\xi_{2n+1}(\omega), T\left(\omega, \xi_{2n+1}(\omega)\right), a(w)\right)\right]$$

$$+ \gamma(\omega)\left[d\left(\xi_{2n+1}(\omega), S\left(\omega, \xi_{2n+2}(\omega)\right), a(w)\right) + d\left(\xi_{2n+2}(\omega), T\left(\omega, \xi_{2n+1}(\omega)\right)\right)\right]$$

$$+\delta(\omega)\, d\left(\xi_{2n+2}(\omega), \xi_{2n+1}(\omega)), a(w)\right)$$

Taking as $n \to \infty$, we have

$$d\big(\xi(\omega), S\big(\omega, \xi(\omega)\big), a(w)\big) \leq \big(\alpha(\omega) + \beta(\omega) + \gamma(\omega) + \delta(\omega)\big)\, d\big(\xi(\omega), S\big(\omega, \xi(\omega)\big), a(w)\big)$$

Which contradiction, hence $\xi(\omega) = S(\omega, \xi(\omega), a(w))$ for al l$\omega \in \Omega$.

Similarly, for any $\omega \in \Omega$,

$$d\big(\xi(\omega), S\big(\omega, \xi(\omega)\big), a(w)\big) \leq d(\xi(\omega), \xi_{2n+1}(\omega), a(w)) + H\big(S\big(\omega, \xi_{2n}(\omega)\big), T\big(\omega, \xi_{2n+1}(\omega)\big), a(w)\big)$$

Hence $\xi(\omega) = T(\omega, \xi(\omega), a(w))$ $for\ all\ \omega \in \Omega$.

It is easy to see that, $\xi(\omega)$ is common fixed point for $S\ and\ T$ in X.

Uniqueness: Let us assume that, $\xi^*(\omega)$ is another fixed point of S and T in X, different from $\xi(\omega)$, then we have

$$d\big(\xi(\omega), \xi^*(\omega), a(w)\big) \leq d\big(\xi(\omega), S\big(\omega, \xi_{2n}(\omega)\big), a(w)\big) + H\big(S(\omega, \xi_{2n}(\omega), a(w)), T\big(\omega, \xi_{2n+1}(\omega)\big), a(w)\big) + d\big(T\big(\omega, \xi_{2n+1}(\omega)\big), \xi^*(\omega), a(w)\big)$$

By using $3.1(a)$ and $n \to \infty$ we have,

$$d(\xi(\omega), \xi^*(\omega), a(w)) \leq 0$$

Which contradiction,

So we have, $\xi(\omega)$ is unique common fixed point of S and T in X.

Corollary 3.2 Let X be a Polish space associated with 2-metric. Let $S^p, T^q : \Omega \times X \to CB(X)$ be two continuous random multivalued operators. If there exists measurable mappings $\alpha, \beta, \gamma, \delta : \Omega \to (0,1)$ such that,

$$H(S^p(\omega, x), T^q(\omega, y), a(w)) \leq \alpha(\omega)\left[\frac{d\big(x, S^p(\omega, x), a(w)\big) d\big(y, T^q(\omega, y), a(w)\big) d\big(x, T^q(\omega, y), a(w)\big) + d\big(x(\omega), y(\omega), a(w)\big) d\big(y, S^p(\omega, x), a(w)\big), d\big(x, T^q(\omega, y), a(w)\big)}{1 + d\big(y(\omega), S^p(\omega, x), a(w)\big) d\big(y(\omega), T^q(\omega, y), a(w)\big)}\right]$$

$$+ \beta(\omega)[\, d(x, S^p(\omega, x), a(w)) + d(y, T^q(\omega, y), a(w))]$$

$$+ \gamma(\omega)[d(y, S^p(\omega, x), a(w)) + d(x, T^q(\omega, y), a(w))] + \delta(\omega)\, d(x(\omega), y(\omega), a(w)) \qquad 3.2(a)$$

For each $x, y \in X$, $\omega \in \Omega$ and $\alpha, \beta, \gamma, \delta \in R^+$ with $0 \leq \alpha(\omega) + 2\beta(\omega) + 2\gamma(\omega) + \delta(\omega) < 1$,and $1 - \alpha(\omega) - \beta(\omega) - \gamma(\omega) \neq 0$, $p, q > 1$ there exists a common random fixed point of S and T.

$(hence\ H\ represents\ the\ Hausdroff 2-\ metric\ on\ CB(X)\ induced\ by\ the\ 2-metric\ d)$

Proof: From the theorem 3.1 and on taking $p = q = 1$ it is immediate to see that, the corollary is true. If not then we choose a $\xi_0 : \Omega \to X$ be an arbitrary measurable mapping and choose a measurable mapping $\xi_1 : \Omega \to X$ such that $\xi_1(\omega) \in S(\omega, \xi_0(\omega))$ for each $\omega \in \Omega$. then for each $\omega \in \Omega$, and by using $3.2(a)$ the result is follows.

Theorem 3.3 Let X be a Polish space associated with2-metrc. Let $T, S: \Omega \times X \to CB(X)$ be two continuous random multivalued operators. If there exists measurable mappings $\alpha, \beta, \gamma, \delta : \Omega \to (0,1)$ such that,

$$H\big(S(\omega,x),T(\omega,y),a(w)\big) \le \alpha(\omega)\left[\frac{d\big(x,S(\omega,x),a(w)\big)d\big(y,T(\omega,y),a(w)\big)d\big(x,T(\omega,y),a(w)\big) + d\big(x(\omega),y(\omega),a(w)\big)d\big(y,S(\omega,x),a(w)\big),d\big(x,T(\omega,y),a(w)\big)}{1+d\big(y(\omega),S(\omega,x),a(w)\big)d\big(y(\omega),T(\omega,y),a(w)\big)}\right]$$

$$+\ \beta(\omega)\,max[\,d(x,S(\omega,x),a(w)), d(y,T(\omega,y),a(w)), d(y,S(\omega,x),a(w)), d(x,T(\omega,y),a(w)), d(x(\omega),y(\omega),a(w))]$$

For each $x, y \in X$, $\omega \in \Omega$ and $\alpha, \beta, \gamma, \delta \in R^+$ with $0 \le \alpha(\omega) + 2\beta(\omega) + 2\gamma(\omega) + \delta(\omega) < 1$,
and $1 - \alpha(\omega) - \beta(\omega) \ne 0, a(w) > 0$ there exists a common random fixed point of S and T.

$(hence\ H\ represents\ the\ Hausdroff2 -\ metric\ on\ CB(X)\ induced\ by\ the\ 2 - metric\ d)$

Proof: Let $\xi_0 : \Omega \to X$ be an arbitrary measurable mapping and choose a measurable mapping $\xi_1 : \Omega \to X$ such that $\xi_1(\omega) \in S(\omega, \xi_0(\omega))$ for each $\omega \in \Omega$. then for each $\omega \in \Omega$.

$$H\big(S(\omega,\xi_0(\omega)),T(\omega,\xi_1(\omega)),a(w)\big) \le \alpha(\omega)\left[\frac{d\big(\xi_0(\omega),S(\omega,\xi_0(\omega),a(w)),a(w)\big)d\big(\xi_1(\omega),T(\omega,\xi_1(\omega)),a(w)\big)d\big(\xi_0(\omega),T(\omega,\xi_1(\omega)),a(w)\big) + d(\xi_0(\omega),\xi_1(\omega),a(w))d\big(\xi_1(\omega),S(\omega,\xi_0(\omega)),a(w)\big),d\big(\xi_0(\omega),T(\omega,\xi_1(\omega)),a(w)\big)}{1+d\big(\xi_0(\omega),S(\omega,\xi_0(\omega),a(w))\big)d\big(\xi_1(\omega),T(\omega,\xi_1(\omega)),a(w)\big)}\right]$$

$$+\beta(\omega)\,max\left[\begin{matrix} d\big(\xi_0(\omega),S(\omega,\xi_0(\omega)),a(w)\big),d\big(\xi_1(\omega),T(\omega,\xi_1(\omega)),a(w)\big) \\ ,d\big(\xi_1(\omega),S(\omega,\xi_0(\omega)),a(w)\big),d\big(\xi_0(\omega),T(\omega,\xi_1(\omega)),a(w)\big),d\big(\xi_0(\omega),\xi_1(\omega),a(w)\big)\end{matrix}\right]$$

Further there exists a measurable mapping $\xi_2 : \Omega \to X$ such that for all $\omega \in \Omega, \xi_2(\omega) \in T(\omega, \xi_1(\omega), a(w))$ and

$$d(\xi_1(\omega),\xi_2(\omega),a(w)) \le \alpha(\omega)\left[\frac{d\big(\xi_0(\omega),\xi_1(\omega),a(w)\big)d\big(\xi_1(\omega),\xi_2(\omega),a(w)\big)d\big(\xi_0(\omega),\xi_2(\omega),a(w)\big) + d(\xi_0(\omega),\xi_1(\omega),a(w))d(\xi_1(\omega),\xi_1(\omega),a(w)),d(\xi_0(\omega),\xi_2(\omega),a(w))}{1+d(\xi_0(\omega),\xi_1(\omega),a(w))d(\xi_1(\omega),\xi_2(\omega),a(w))}\right]$$

$$+\beta(\omega)\,max\left[\begin{matrix} d(\xi_0(\omega),\xi_1(\omega),a(w)),d(\xi_1(\omega),\xi_2(\omega),a(w)), \\ d(\xi_1(\omega),\xi_1(\omega),a(w)),d(\xi_0(\omega),\xi_2(\omega),a(w))\ ,d(\xi_0(\omega),\xi_1(\omega),a(w))\end{matrix}\right]$$

$$d(\xi_1(\omega),\xi_2(\omega),a(w)) \le \frac{\alpha(\omega)+\beta(\omega)}{1-\alpha(\omega)-\beta(\omega)} d(\xi_0(\omega),\xi_1(\omega),a(w))$$

Let $k = \frac{\alpha(\omega)+\beta(\omega)}{1-\alpha(\omega)-\beta(\omega)}$

This gives

$$d(\xi_1(\omega),\xi_2(\omega),a(w)) \le k\ d(\xi_0(\omega),\xi_1(\omega),a(w))$$

By Beg and Shahzad [2, $lemma$ 2.3],we obtain a measurable mapping $\xi_3 : \Omega \to X$ such that for all $\omega \in \Omega$,
$\xi_3(\omega) \in S(\omega, \xi_2(\omega), a(w))$ and

$$d(\xi_2(\omega),\xi_3(\omega),a(w)) \le d(\xi_1(\omega),\xi_2(\omega),a(w)) \le \alpha(\omega)\left[\frac{d\big(\xi_1(\omega),\xi_2(\omega),a(w)\big)d\big(\xi_2(\omega),\xi_3(\omega),a(w)\big)d\big(\xi_1(\omega),\xi_3(\omega),a(w)\big) + d(\xi_1(\omega),\xi_2(\omega),a(w))d(\xi_2(\omega),\xi_2(\omega),a(w)),d(\xi_1(\omega),\xi_3(\omega),a(w))}{1+d\big(\xi_1(\omega),\xi_2(\omega)\big)d\big(\xi_2(\omega),\xi_3(\omega),a(w)\big)}\right]$$

$$+\beta(\omega)\,max\left[\begin{matrix} d(\xi_1(\omega),\xi_2(\omega),a(w)),d(\xi_2(\omega),\xi_3(\omega),a(w))\ , \\ d(\xi_2(\omega),\xi_2(\omega),a(w)),d(\xi_1(\omega),\xi_3(\omega),a(w)),d(\xi_1(\omega),\xi_2(\omega),a(w))\end{matrix}\right]$$

$$d(\xi_2(\omega),\xi_3(\omega),a(w)) \le k\ d(\xi_1(\omega),\xi_2(\omega),a(w)) \le k^2 d(\xi_0(\omega),\xi_1(\omega),a(w))$$

Similarly, proceeding the same way, by induction, we get a sequence of measurable mapping $\xi_n : \Omega \to X$ suct that for $n > 0$ and for any $\omega \in \Omega$,

$$\xi_{2n+1}(\omega) \in S(\omega, \xi_{2n}(\omega), a(w)) \text{ , and } \xi_{2n+2}(\omega) \in T(\omega, \xi_{2n+1}(\omega), a(w))$$

This gives,

$$d(\xi_n(\omega), \xi_{n+1}(\omega), a(w)) \le kd(\xi_{n-1}(\omega), \xi_n(\omega), a(w)) \le \cdots \ldots \le k^n d(\xi_0(\omega), \xi_1(\omega), a(w))$$

For any $m, n \in N \text{ such that } m > n$, also by using triangular inequality we have

$$d(\xi_n(\omega), \xi_m(\omega), a(w)) \le \frac{k^n}{1-k} d(\xi_0(\omega), \xi_1(\omega), a(w))$$

Which tends to zero as $n \to \infty$. It follows that $\{\xi_n(\omega)\}$ is a Cauchy sequence and there exists a measurable mapping $\xi : \Omega \to X$ such that $\xi_n(\omega) \to \xi(\omega)$ for each $\omega \in \Omega$. It implies that $\xi_{2n+1}(\omega) \to \xi(\omega)$.

Thus we have for any $\omega \in \Omega$,

$$d\big(\xi(\omega), S\big(\omega, \xi(\omega)\big), a(w)\big) \le d(\xi(\omega), \xi_{2n+2}(\omega), a(w)) + d\big(\xi(\omega), S\big(\omega, \xi_{2n+2}(\omega)\big), a(w)\big)$$

$$d\big(\xi(\omega), S\big(\omega, \xi(\omega)\big), a(w)\big) \le d(\xi(\omega), \xi_{2n+2}(\omega), a(w)) + H\big(T\big(\omega, \xi_{2n+1}(\omega)\big), S\big(\omega, \xi_{2n+2}(\omega)\big), a(w)\big)$$

Therefore,

$$d\Big(\xi(\omega), S\big(\omega, \xi(\omega)\big), a(w)\Big) \le d\big(\xi(\omega), \xi_{2n+2}(\omega), a(w)\big)$$

$$+ \alpha(\omega) \left[\frac{\begin{array}{c} d\big(\xi_{2n+2}(\omega), S(\omega, \xi_{2n+2}(\omega)), a(w)\big) d\big(\xi_{2n+1}(\omega), T(\omega, \xi_{2n+1}(\omega)), a(w)\big) d\big(\xi_{2n+2}(\omega), T(\omega, \xi_{2n+1}(\omega)), a(w)\big) \\ + d\big(\xi_{2n+2}(\omega), \xi_{2n+1}(\omega), a(w)\big) d\big(\xi_{2n+1}(\omega), S(\omega, \xi_{2n+2}(\omega)), a(w)\big), d\big(\xi_{2n+2}(\omega), T(\omega, y), a(w)\big) \end{array}}{1 + d\big(\xi_{2n+1}(\omega), S(\omega, \xi_{2n+2}(\omega)), a(w)\big) d\big(\xi_{2n+1}(\omega), T(\omega, \xi_{2n+1}(\omega)), a(w)\big)} \right]$$

$$+ \beta(\omega) \max \begin{bmatrix} d\big(\xi_{2n+2}(\omega), S\big(\omega, \xi_{2n+2}(\omega)\big), a(w)\big), d\big(\xi_{2n+1}(\omega), T\big(\omega, \xi_{2n+1}(\omega)\big), a(w)\big), \\ d\big(\xi_{2n+1}(\omega), S\big(\omega, \xi_{2n+2}(\omega)\big), a(w)\big), d\big(\xi_{2n+2}(\omega), T\big(\omega, \xi_{2n+1}(\omega)\big), a(w)\big), \\ d(\xi_{2n+2}(\omega), \xi_{2n+1}(\omega)), a(w)) \end{bmatrix}$$

Taking as $n \to \infty$, we have

$$d\big(\xi(\omega), S\big(\omega, \xi(\omega)\big), a(w)\big) \le (\alpha(\omega) + \beta(\omega)) \, d\big(\xi(\omega), S\big(\omega, \xi(\omega)\big), a(w)\big)$$

Which contradiction, hence

$$\xi(\omega) = S(\omega, \xi(\omega), a(w)) \text{ for al l } \omega \in \Omega.$$

Similarly, for any $\omega \in \Omega$,

$$d\big(\xi(\omega), S\big(\omega, \xi(\omega)\big), a(w)\big) \le d(\xi(\omega), \xi_{2n+1}(\omega), a(w)) + H\big(S\big(\omega, \xi_{2n}(\omega)\big), T\big(\omega, \xi_{2n+1}(\omega)\big), a(w)\big)$$

Hence $\xi(\omega) = T(\omega, \xi(\omega), a(w))$ $for\ all\ \omega \in \Omega$.

It is easy to see that, $\xi(\omega)$ is common fixed point for $S\ and\ T$ in X.

Uniqueness: Let us assume that, $\xi^*(\omega)$ is another fixed point of S and T in X, different from $\xi(\omega)$, then we have

$$d(\xi(\omega), \xi^*(\omega), a(w)) \le d\big(\xi(\omega), S\big(\omega, \xi_{2n}(\omega)\big), a(w)\big) + H\big(S(\omega, \xi_{2n}(\omega), a(w)), T\big(\omega, \xi_{2n+1}(\omega)\big), a(w)\big) + d\big(T\big(\omega, \xi_{2n+1}(\omega)\big), \xi^*(\omega), a(w)\big)$$

By using $3.1(a)$ and $n \to \infty$ we have,

$$d(\xi(\omega), \xi^*(\omega), a(w)) \le 0$$

Which contradiction,

So we have, $\xi(\omega)$ is unique common fixed point of S and T in X.

Corollary 3.2 Let X be a Polish space associated with 2-metric. Let $S^p, T^q : \Omega \times X \to CB(X)$ be two continuous random multivalued operators. If there exists measurable mappings $\alpha, \beta, \gamma, \delta : \Omega \to (0,1)$, $a(w) > 0$ such that,

$$H(S^p(\omega,x),T^q(\omega,y),a(w)) \leq \alpha(\omega)\left[\frac{d(x,S^p(\omega,x),a(w))d(y,T^q(\omega,y),a(w))d(x,T^q(\omega,y),a(w)) + d(x(\omega),y(\omega),a(w))d(y,S^p(\omega,x),a(w)),d(x,T^q(\omega,y),a(w))}{1+d(y(\omega),S^p(\omega,x))d(y(\omega),T^q(\omega,y))}\right]$$

$$+\ \beta(\omega)\,max\begin{bmatrix} d(x,S^p(\omega,x),a(w)),d(y,T^q(\omega,y),a(w)), \\ d(y,S^p(\omega,x),a(w)),d(x,T^q(\omega,y),a(w)), \\ d(x(\omega),y(\omega),a(w)) \end{bmatrix} \qquad 3.2(a)$$

For each $x, y \in X$, $\omega \in \Omega$ and $\alpha, \beta, \gamma, \delta \in R^+$ with $0 \leq \alpha(\omega)+\beta(\omega) < 1$, and $1-\alpha(\omega)-\beta(\omega) \neq 0$, $p, q > 1$ there exists a common random fixed point of S and T.

(hence H represents the Hausdroff metric on CB(X) induced by the metric d)

Proof: From the theorem 3.1 and on taking $p = q = 1$ it is immediate to see that, the corollary is true. If not then we choose a $\xi_0 : \Omega \to X$ be an arbitrary measurable mapping and choose a measurable mapping $\xi_1 : \Omega \to X$ such that $\xi_1(\omega) \in S(\omega, \xi_0(\omega))$ for each $\omega \in \Omega$. then for each $\omega \in \Omega$, and by using 3.2(a) the result is follows.

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