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COMMON FIXED POINTS OF WEAKLY COMPATIBLE MAPPINGS ON NONARCHIMEDEAN MENGER SPACES

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ABSTRACT

In this paper we prove a common fixed point theorem for weakly compatible mappings on a nonarchimedean Menger space. This improves a number of fixed point theorems in metric and Menger spaces.

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Key words: *Nonarchimedean Menger space, fixed point, compatible mapping.*

INTRODUCTION

The existence of fixed point of mappings on nonarchimedean Menger space has been establishsd by Istratescu [4]. Achari [1] has proved some fixed point theorems for quasi-contraction type mappings on a nonarchimedean Menger space. Also Cho et al. [3] have proved a common fixed point theorem for compatible mappings in nonarchimedean Menger PM- space.

On the other hand, Singh and Pant [6] proved a common fixed point theorem for weakly commuting mappings on a nonarchimedean Menger space. In this paper our attempt has been to combine the ideas of [3] and [6] and to prove a common fixed point theorem for weakly compatible mappings on a nonarchimedean Menger space.

PRELIMINARIES

Definition 1: A t- norm is a function t: [0, 1] x [0, 1] → [0, 1] which is associative, commutative, non decreasing in each coordinate and t (a, 1) = a ∀ a ϵ [0, 1]

Definition 2: let $F_{u,v}$ denotes the value at (u, v) ϵ X x X of the function F: X x X → L the collection of all distribution functions. A nonarchimedean Menger space is a triplet (X, F, t), where (X, F) is a nonarchimedean probabilistic metric space (PM-space) and t is t-norm such that the nonarchimedean triangle inequality

$F_{u,w}(\max\{x, y\}) \geq t\{F_{u,v}(x), F_{v,w}(y)\}$ for all u, v, w ϵ X and x, y ≥ 0.

Definition 3: Two self mappings P and S on a PM-space X will be called weakly compatible iff Pu = Su implies that SPu = PSu, for all u ϵ X.

RESULTS

In order to state the theorem we need the following lemmas:

Lemma (1) Let X be a nonarchimedean Menger space and Φ: [0, ∞)→[0, ∞) be an upper semicontinuous function with Φ (0) = 0 and Φ (t) < t for all t > 0. If $\{u_n\}$ is a sequence in X such that for any ε > 0 and any n ϵ N

$$F_{u_n u_{n+1}}(\Phi(\varepsilon)) \geq F_{u_{n-1} u_n}(\varepsilon)$$

Then $\{u_n\}$ is a Cauchy sequence in X.

Lemma (2): Let (X, F) be a PM-space and β (0) = 0 and β (t) < t for all t > 0.

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If for u, v ϵ X and for all $\varepsilon > 0$

$F_{u,v}(\beta(\varepsilon)) > F_{u,v}(\varepsilon)$, then u = v.

Now we state our main theorem:

Theorem 1: Let (X, F, t) be a nonarchimedean Menger space and P, Q, S, T: X → X satisfy the following:

(1). $P(X) \subseteq T(X)$ and $Q(X) \subseteq S(X)$;
(2). T(X) or S(X) is a complete subspace of X;
(3). (P, S) and (Q, T) are weakly compatible pairs;
(4). There exists an upper semi continuous function with $\Phi(0) = 0$ and $\Phi(t) < t$ for all $t > 0$ such that

$F_{Pu,Qv}(\Phi(\varepsilon)) \geq \max\{F_{Su,Tv}(\varepsilon), F_{Su,Pu}(\varepsilon), F_{Qv,Tv}(\varepsilon), F_{Su,Qv}(\varepsilon), F_{Tv,Pu}(\varepsilon)\}$

For all u, v in X and $\varepsilon > 0$. Then P, Q, S and T have a unique common fixed point.

Proof: Let u_0 ϵ X. We define the sequence $\{u_n\}$ in X given by the rule

$Tu_{2n+1} = Pu_{2n} = v_{2n}$ (say), and $Su_{2n+2} = Qu_{2n+1} = v_{2n+1}$ (say)
where n ϵ N ∪ {0}.

This can be done by the virtue of (1).

Without loss of generality, we can assume that $v_n \neq v_{n+1}$ for all n ϵ N. By (4) and for n ϵ N and all $\varepsilon > 0$,

$$F_{v_{2n}, v_{2n+2}}(\varepsilon) = F_{v_{2n}, v_{2n+2}}(\max\{\varepsilon, \Phi(\varepsilon)\})$$

$$\geq \max\{F_{v_{2n}, v_{2n+1}}(\varepsilon), F_{v_{2n+2}, v_{2n+1}}(\Phi(\varepsilon))\}$$

Now

$F_{v_{2n+2}, v_{2n+1}}(\Phi(\varepsilon)) = F_{Pu_{2n+2}, Qu_{2n+1}}(\Phi(\varepsilon))$
$\geq \max\{F_{Su_{2n+2}, Tu_{2n+1}}(\varepsilon), \; F_{Su_{2n+2}, Pu_{2n+2}}(\varepsilon),$

$F_{Qu_{2n+1}, Tu_{2n+1}}(\varepsilon), F_{Su_{2n+2}, Qu_{2n+1}}(\varepsilon), F_{Tu_{2n+1}, Pu_{2n+2}}(\varepsilon)\}$
$= \max\{F_{v_{2n+1}, v_{2n}}(\varepsilon), F_{v_{2n+1}, v_{2n+2}}(\varepsilon), F_{v_{2n+1} v_{2n}}(\varepsilon)\}$

$F_{v_{2n+1} v_{2n+1}}(\varepsilon), F_{v_{2n}, v_{2n+2}}(\varepsilon)\} = \max\{F_{v_{2n+1}, v_{2n}}(\varepsilon), F_{v_{2n+1}, v_{2n+2}}(\varepsilon)\}$

Thus in view of Lemma (2), we have

$F_{v_{2n+1}, v_{2n+2}}(\Phi(\varepsilon)) \geq F_{v_{2n+1}, v_{2n}}(\varepsilon)$,

Similarly, we can prove that for n ϵ N and for all $\varepsilon > 0$.

$F_{v_{2n+3}, v_{2n+2}}(\Phi(\varepsilon)) \geq F_{v_{2n+2}, v_{2n+1}}(\varepsilon)$.

Thus in general

$F_{v_{n+1}, v_n}(\Phi(\varepsilon)) \geq F_{v_n, v_{n-1}}(\varepsilon)$, for all n and every $\varepsilon > 0$.

So, by Lemma (1), $\{v_n\}$ is a Cauchy sequence. Suppose T(X) is complete, then $\{v_n\}$ has a limit in T(X). Call it w. Hence there exists a point p in X such that Tp = w. Consequently, the subsequences $\{Pu_{2n}\}$, $\{Qu_{2n+1}\}$ and $\{Su_{2n}\}$ also converge to w.

For $\varepsilon > 0$, by (4)

$F_{Pu_{2n}, Qp}(\Phi(\varepsilon)) \geq \max\{F_{Su_{2n}, Tp}(\varepsilon), F_{Su_{2n}, Pu_{2n}}(\varepsilon), F_{Qp, Tp}(\varepsilon), F_{Su_{2n}, Qp}(\varepsilon), F_{Tp, Pu_{2n}}(\varepsilon)\}$.

Letting $n \to \infty$,

$F_{w, Qp}\ (\Phi(\varepsilon)) \geq \max\{F_{w, w}(\varepsilon), F_{w, w}(\varepsilon), F_{Qp, w}(\varepsilon)\}$

Which implies that Qp = w.

Hence w = Tp = Qp. But $Q(X) \subseteq S(X)$, so there exists a point $q \in X$ such that

Sq = Qp = Tp.

Using (4), we get

$$F_{Pq, Qp}(\Phi(\varepsilon)) \geq \max\{F_{Sq, Tp}(\varepsilon), F_{Sq, Pq}(\varepsilon), F_{Qp, T}(\varepsilon), F_{Sq, Qp}(\varepsilon), F_{Tp, Pq}(\varepsilon)\}$$

$$= F_{Pq, Qp}(\varepsilon).$$

Therefore by Lemma (2), we have Pq = Qp.

Thus w = Pq = Sq = Qp = Tp. This shows that p and q are the coincidence points of (Q, T) and (P, S) respectively.

Now, let Pq = Sq. By the hypothesis of weak compatibility of P and S, we have SPq = PSq. This implies that Sw = Pw.

To prove Pw = w, suppose $Pw \neq w$, then by (4),

$$F_{Pw, w}(\Phi(\varepsilon)) = F_{Pw, Qp}(\Phi(\varepsilon))$$

$$\geq \max\{F_{Sw, Tp}(\varepsilon), F_{Sw, Pw}(\varepsilon), F_{Qp, Tp}(\varepsilon), F_{Sw, Qp}(\varepsilon), F_{Tp, Pw}(\varepsilon)\}$$

$$= F_{Pw, w}(\varepsilon)$$

Therefore Pw = w = Sw, by Lemma (2)

In similar way, taking Qp = Tp and weak compatibility of Q and T, we can prove Qw = Tw and then Qw = w = Tw.

Thus, we have proved that w is common fixed point of P, Q, S and T. Same result holds good if S(X) is taken to be complete.

Finally, we have to prove the uniqueness of w as a common fixed point of P, Q, S and T and this can be done easily using (4).

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