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COMMON FIXED POINTS OF WEAKLY COMPATIBLE MAPPINGS ON NONARCHIMEDEAN MENGER SPACES

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ABSTRACT

In this paper we prove a common fixed point theorem for weakly compatible mappings on a nonarchimedean Menger space. This improves a number of fixed point theorems in metric and Menger spaces.

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Key words: *Nonarchimedean Menger space, fixed point, compatible mapping.*

INTRODUCTION

The existence of fixed point of mappings on nonarchimedean Menger space has been establishsd by Istratescu [4]. Achari [1] has proved some fixed point theorems for quasi-contraction type mappings on a nonarchimedean Menger space. Also Cho et al. [3] have proved a common fixed point theorem for compatible mappings in nonarchimedean Menger PM- space.

On the other hand, Singh and Pant [6] proved a common fixed point theorem for weakly commuting mappings on a nonarchimedean Menger space. In this paper our attempt has been to combine the ideas of [3] and [6] and to prove a common fixed point theorem for weakly compatible mappings on a nonarchimedean Menger space.

PRELIMINARIES

Definition 1: A t- norm is a function t: $[0, 1] \times [0, 1] \to [0, 1]$ which is associative, commutative, non decreasing in each coordinate and $t(a, 1) = a \ \forall\ a \in [0, 1]$

Definition 2: let $F_{u,v}$ denotes the value at $(u, v) \in X \times X$ of the function $F: X \times X \to L$ the collection of all distribution functions. A nonarchimedean Menger space is a triplet (X, F, t), where (X, F) is a nonarchimedean probabilistic metric space (PM-space) and t is t-norm such that the nonarchimedean triangle inequality

$F_{u,w}(\max\{x, y\}) \geq t\{F_{u,v}(x), F_{v,w}(y)\}$ for all $u, v, w \in X$ and $x, y \geq 0$.

Definition 3: Two self mappings P and S on a PM-space X will be called weakly compatible iff Pu = Su implies that SPu = PSu, for all $u \in X$.

RESULTS

In order to state the theorem we need the following lemmas:

Lemma (1) Let X be a nonarchimedean Menger space and $\Phi: [0, \infty) \to [0, \infty)$ be an upper semicontinuous function with $\Phi(0) = 0$ and $\Phi(t) < t$ for all $t > 0$. If $\{u_n\}$ is a sequence in X such that for any $\varepsilon > 0$ and any $n \in N$

$$F_{u_n u_{n+1}}(\Phi(\varepsilon)) \geq F_{u_{n-1} u_n}(\varepsilon)$$

Then $\{u_n\}$ is a Cauchy sequence in X.

Lemma (2): Let (X, F) be a PM-space and $\beta(0) = 0$ and $\beta(t) < t$ for all $t > 0$.

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If for u, v ϵ X and for all $\varepsilon > 0$

$F_{u,v}(\beta(\varepsilon)) > F_{u,v}(\varepsilon)$, then u = v.

Now we state our main theorem:

Theorem 1: Let (X, F, t) be a nonarchimedean Menger space and P, Q, S, T: X → X satisfy the following:

(1). $P(X) \subseteq T(X)$ and $Q(X) \subseteq S(X)$;
(2). T(X) or S(X) is a complete subspace of X;
(3). (P, S) and (Q, T) are weakly compatible pairs;
(4). There exists an upper semi continuous function with $\Phi(0) = 0$ and $\Phi(t) < t$ for all $t > 0$ such that

$$F_{Pu,Qv}(\Phi(\varepsilon)) \geq \max\{F_{Su,Tv}(\varepsilon), F_{Su,Pu}(\varepsilon), F_{Qv,Tv}(\varepsilon), F_{Su,Qv}(\varepsilon), F_{Tv,Pu}(\varepsilon)$$

For all u, v in X and $\varepsilon > 0$. Then P, Q, S and T have a unique common fixed point.

Proof: Let $u_0 \in X$. We define the sequence $\{u_n\}$ in X given by the rule

$Tu_{2n+1} = Pu_{2n} = v_{2n}$ (say), and $Su_{2n+2} = Qu_{2n+1} = v_{2n+1}$ (say)
where $n \in N \cup \{0\}$.

This can be done by the virtue of (1).

Without loss of generality, we can assume that $v_n \neq v_{n+1}$ for all n ϵ N. By (4) and for n ϵ N and all $\varepsilon > 0$,

$$F_{v2n,v2n+2}(\varepsilon) = F_{v2n,v2n+2}(\max\{\varepsilon, \Phi(\varepsilon)\})$$

$$\geq \max\{F\, v_{2n}, v_{2n+1}(\varepsilon), F\, v_{2n+2}, v_{2n+1}(\Phi(\varepsilon))\}$$

Now

$$F\, v_{2n+2}, v_{2n+1}(\Phi(\varepsilon)) = F_{Pu2n+2,Qu2n+1}(\Phi(\varepsilon))$$
$$\geq \max\{F_{Su2n+2,Tu2n+1}(\varepsilon), \quad F_{Su2n+2,Pu2n+2}(\varepsilon),$$

$$F_{Qu2n+1,Tu2n+1}(\varepsilon), F_{Su2n+2,Qu2n+1}(\varepsilon), F_{Tu2n+1,Pu2n+2}(\varepsilon)\}$$
$$= \max\{F\, v_{2n+1}, v_{2n}(\varepsilon), F\, v_{2n+1}, v_{2n+2}(\varepsilon), F\, v_{2n+1}\, v_{2n}(\varepsilon)\}$$

$$F\, v_{2n+1}\, v_{2n+1}(\varepsilon), F\, v_{2n}, v_{2n+2}(\varepsilon)\} = \max\{Fv_{2n+1}, v_{2n}(\varepsilon), F\, v_{2n+1}, v_{2n+2}(\varepsilon)\}$$

Thus in view of Lemma (2), we have

$$F\, v_{2n+1}, v_{2n+2}(\Phi(\varepsilon)) \geq Fv_{2n+1}, v_{2n}(\varepsilon),$$

Similarly, we can prove that for n ϵ N and for all $\varepsilon > 0$.

$$F\, v_{2n+3}, v_{2n+2}(\Phi(\varepsilon)) \geq F\, v_{2n+2}, v_{2n+1}(\varepsilon).$$

Thus in general

$$F\, v_{n+1}, v_n(\Phi(\varepsilon)) \geq F\, v_n, v_{n-1}(\varepsilon), \text{ for all n and every } \varepsilon > 0.$$

So, by Lemma (1), $\{v_n\}$ is a Cauchy sequence. Suppose T(X) is complete, then $\{v_n\}$ has a limit in T(X). Call it w. Hence there exists a point p in X such that Tp = w. Consequently, the subsequences $\{Pu_{2n}\}$, $\{Qu_{2n+1}\}$ and $\{Su_{2n}\}$ also converge to w.

For $\varepsilon > 0$, by (4)

$$F_{Pu2n,Qp}(\Phi(\varepsilon)) \geq \max\{F_{Su2n,Tp}(\varepsilon), F_{Su2n,Pu2n}(\varepsilon), F_{Qp,Tp}(\varepsilon), F_{Su2n,Qp}(\varepsilon), F_{Tp,Pu2n}(\varepsilon)\}.$$

Letting $n \to \infty$,

$F_{w, Qp}\ (\Phi\ (\varepsilon)) \geq \max\ \{ F_{w, w}\ (\varepsilon), F_{w, w}\ (\varepsilon), F_{Qp, w}\ (\varepsilon)\}$

Which implies that Qp = w.

Hence w = Tp = Qp. But $Q(X) \subseteq S(X)$, so there exists a point $q \in X$ such that

Sq = Qp = Tp.

Using (4), we get

$$F_{Pq, Qp}\ (\Phi\ (\varepsilon)) \geq \max\ \{F_{Sq, Tp}\ (\varepsilon), F_{Sq, Pq}\ (\varepsilon), F_{Qp, T}(\varepsilon), F_{Sq, Qp}\ (\varepsilon), F_{Tp, Pq}\ (\varepsilon)\}$$

$$= F_{Pq, Qp}\ (\varepsilon).$$

Therefore by Lemma (2), we have Pq = Qp.

Thus w = Pq = Sq = Qp = Tp. This shows that p and q are the coincidence points of (Q, T) and (P, S) respectively.

Now, let Pq = Sq. By the hypothesis of weak compatibility of P and S, we have SPq = PSq. This implies that Sw = Pw.

To prove Pw = w, suppose $Pw \neq w$, then by (4),

$$F_{Pw, w}\ (\Phi\ (\varepsilon)) = F_{Pw, Qp}\ (\Phi\ (\varepsilon))$$

$$\geq \max\ \{F_{Sw, Tp\ (\varepsilon),} F_{Sw, Pw}\ (\varepsilon), F_{Qp, Tp}\ (\varepsilon), F_{Sw, Qp}\ (\varepsilon), F_{Tp, Pw}\ (\varepsilon)\}$$

$$= F_{Pw, w}\ (\varepsilon)$$

Therefore Pw = w = Sw, by Lemma (2)

In similar way, taking Qp = Tp and weak compatibility of Q and T, we can prove Qw = Tw and then Qw = w = Tw.

Thus, we have proved that w is common fixed point of P, Q, S and T. Same result holds good if S(X) is taken to be complete.

Finally, we have to prove the uniqueness of w as a common fixed point of P, Q, S and T and this can be done easily using (4).

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