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# SIMILARITY SOLUTIONS OF THE INCOMPRESSIBLE BOUNDARY LAYER SLIP CONDITIONS FOR A VERTICAL SURFACE WITH INTERNAL HEAT GENERATION & TEMPERATURE DEPENDENT VISCOSITY

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***ABSTRACT***

*The problem of viscous flow & heat transfer over a 2-D steady free convection boundary layer from a vertical surface is studied taking into account slip boundary conditions & exponential decay internal heat generation on variation of fluid viscosity with temperature. The governing equations of the problem are steady reduced to couple non-linear moment & the energy equation to be similar by introducing suitable similarity transformation. Dimensionless velocity and temperature profiles are presented graphically for various values of Prandtl number $(\Pr)$, velocity slip parameter $(a)$ & temperature slip parameter $(b)$ with & without internal heat generation. Velocity gradient & temperature gradient are also discussed & are given in tables.*

## INTRODUCTION

The study of two-dimensional boundary layer flow, heat and mass transfer over a steady free convection boundary layer from a vertical surface is very important as it finds many practical applications in different areas. Cheng and Minkowycz [1] presented similarity solutions for free convection from a vertical plate in a fluid-saturated medium. Gorla and co-workers [2, 3, 4] investigated the corresponding non-similar free convection boundary layer problem due to variations in surface temperature or heat flux, Bejan and Khair[5]. A comprehensive survey of relevant papers may be found in the recent monograph by Nield and Bejan [6]. A study on steady laminar free convection flow in an electrically conducting fluid along a porous vertical plate in the presence of heat source was carried out Sharma and Pankaj Mathur [7]. The effect of thermal diffusion on steady laminar free convective flow along a moving porous hot vertical plate in the presence of heat source with mass transfer was studied by Varshney and Shilendra kumar[8]. Heat transfer in MHD free convection flow over an infinite vertical plate with time-dependent suction was investigated in detail by Basant Kumar Mishra[9].Sharma and Chaudhary[10] analyzed the effect of variable suction on transient free convection viscous incompressible flow past a vertical plate with periodic temperature variations in slip-flow regime. A study of vorticity of fluctuating flow of a visco-elastic fluid past an infinite plate with variable suction in slip flow regime was made by Mittal and Mukesh Bijalwan[11].Free convection flow of magnetopolar fluid through porous medium in slip flow regime with mass transfer was studied by Rajput et al.[12]. The analysis of the results obtained shows that the flow field is influenced appreciably by the presence of internal heat generation and velocity slip parameter on variation of fluid viscosity with temperature.

## ANALYSIS

Governing equations of two dimensional steady flow and heat transfer in free convection of an incompressible viscous fluid past a vertical surface with variation of fluid viscosity can be written as

**Mass equation**

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=0$$

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**Momentum equation**

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)$$

**Energy equation**

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+q^{///}$$

Slip boundary conditions are as follows:

$$\left.\begin{array}{l} u=U_w+N_1 v\dfrac{\partial u}{\partial y},\quad v=0 \\ T=T_w+D_1\dfrac{\partial T}{\partial y}, \end{array}\right\} \text{ at } y=0$$

$$u=0,\ T=T_\infty,\ \text{as}\ \ y\to\infty$$

Let us introduce the following transformations

$$\eta=\frac{y}{x}\sqrt{\mathrm{Re}_x},\ u=U_w\ f(\eta),\ v=\frac{U_w}{2\sqrt{\mathrm{Re}_x}}(\eta f'-f),$$

$$\mathrm{Re}_x=\frac{U_w x}{\upsilon_x},\mu=\frac{\mu_\infty}{1+\gamma(T-T_\infty)},\theta_r=\frac{T_r-T_\infty}{T_w-T_\infty}=-\frac{1}{\gamma(T_w-T_\infty)},T_r=T_\infty-\frac{1}{r},\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}$$

Finally, we get the following transformed equations

$$(\theta-\theta_r)f^{///}-f^{//}\theta^{/}-\frac{(\theta-\theta_r)^2}{2\theta_r}ff^{//}=0$$

$$\theta^{//}+\frac{1}{2}\Pr\theta^{/}+c\Pr e^{-\eta}=0$$

where $\Pr=\dfrac{\nu}{\alpha}$ is the Prandtl number.

The transformed boundary conditions are given by

$$f=0,\ \theta=1+b\theta^{/},\ f^{/}=1+af^{//}\ \text{ at }\eta=0$$

$$f^{/}=0,\ \theta=0\ \text{ as }\eta\to\infty$$

**ANALYSIS**

In this paper Maple software was used to solve the similarity ordinary equations with boundary conditions.

Figure1 illustrates the influence of variable viscosity parameter $(\theta_r)$ on the velocity profile with & without internal heat generation. It can be seen that with increasing $\theta_r$, the velocity inside the boundary layer become smaller. It is also seen that internal heat generation reduced & reduced boundary layer thickness in the presence of internal heat generation $(c=1)$ than that of without internal heat generation $(c=0)$ when $\theta_r<0$ & $\theta_r>0$ respectively.

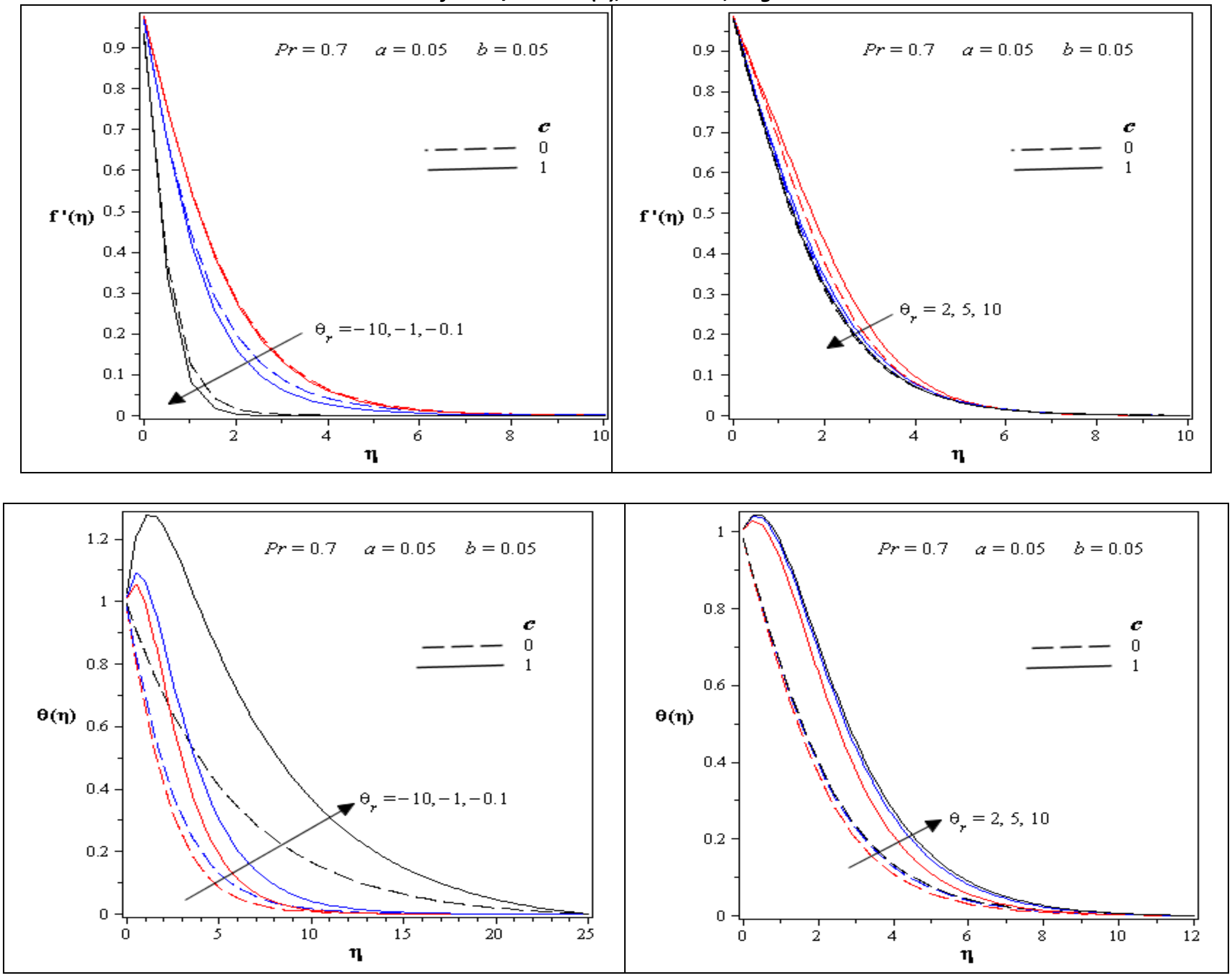


**Fig 1:** Velocity profiles for various values of $\theta_r$ .

Figure 2 illustrates the influence of prandtl number $(\mathrm{Pr})$ on the velocity & temperature profiles with & without internal heat generation. It can be seen that with increasing $\mathrm{Pr}$ , the velocity & temperature inside the boundary layer become smaller. Also internal heat generation induced more flow than that of without internal heat generation, i.e., the mechanical strength in the fluid motion is increased. Note that temperature has a significant impact due to prandtl number.

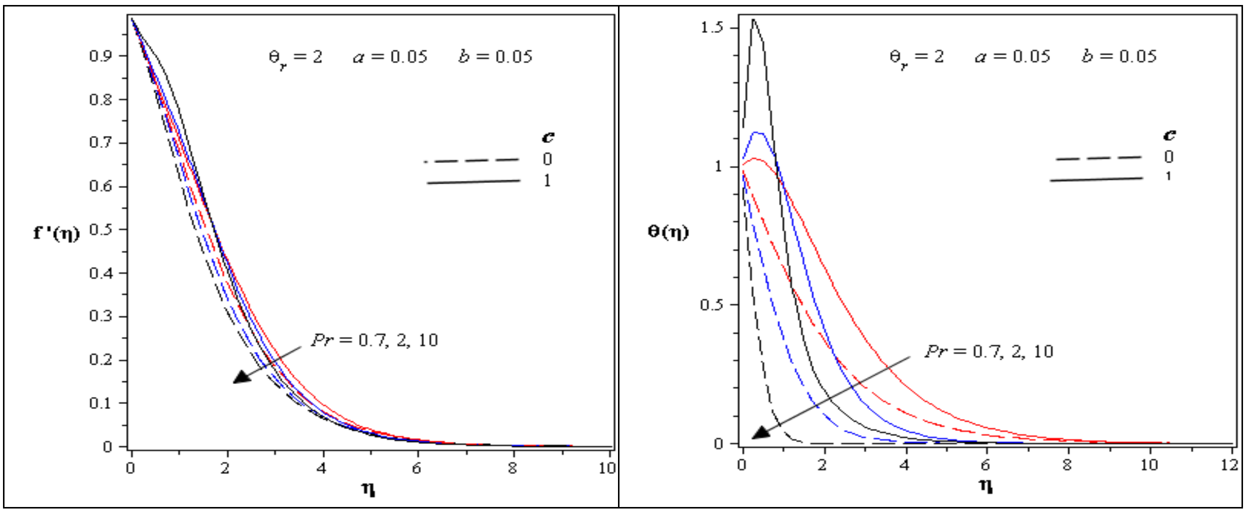


**Fig 2:** Velocity &Temperature profiles for various values of $\mathrm{Pr}$ .

Figure 3 illustrates the influence of velocity slip parameter $(a)$ on the velocity & temperature profiles with & without internal heat generation. It is seen that mere & far from the wall velocity profiles decreases & increases respectively with increasing $a$. Also more flow induced in the velocity & temperature boundary in the presence of internal heat generation.

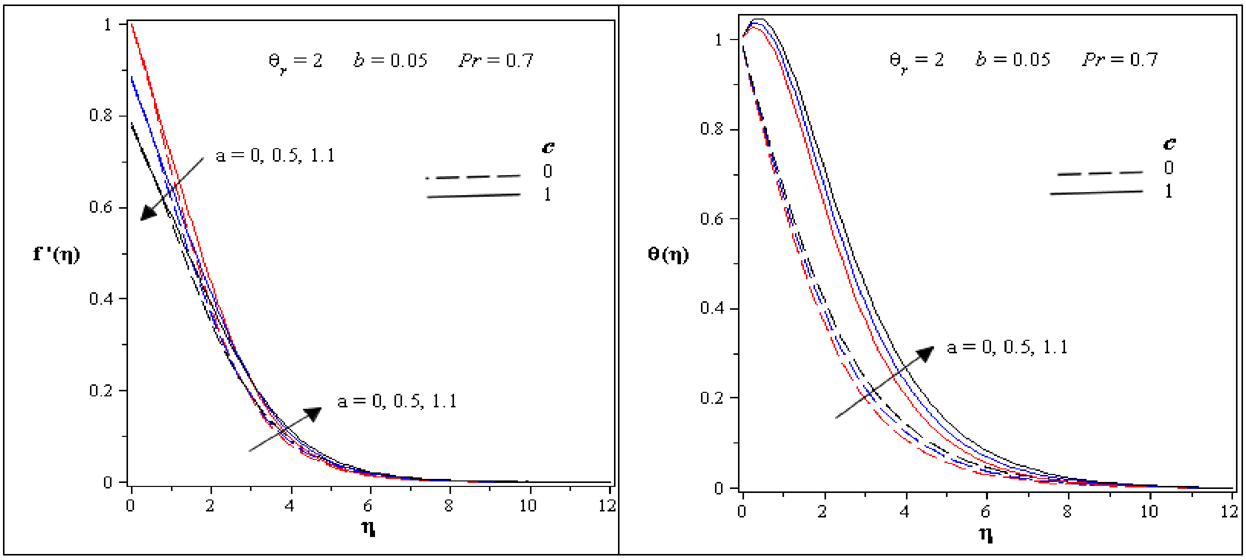


**Fig 3:** Velocity &Temperature profiles for various values of $a$.

Figure 43 illustrates the influence of temperature slip parameter $(b)$ on the velocity & temperature profiles with & without internal heat generation. It is seen that velocity and temperature both decrease with the increase in b.

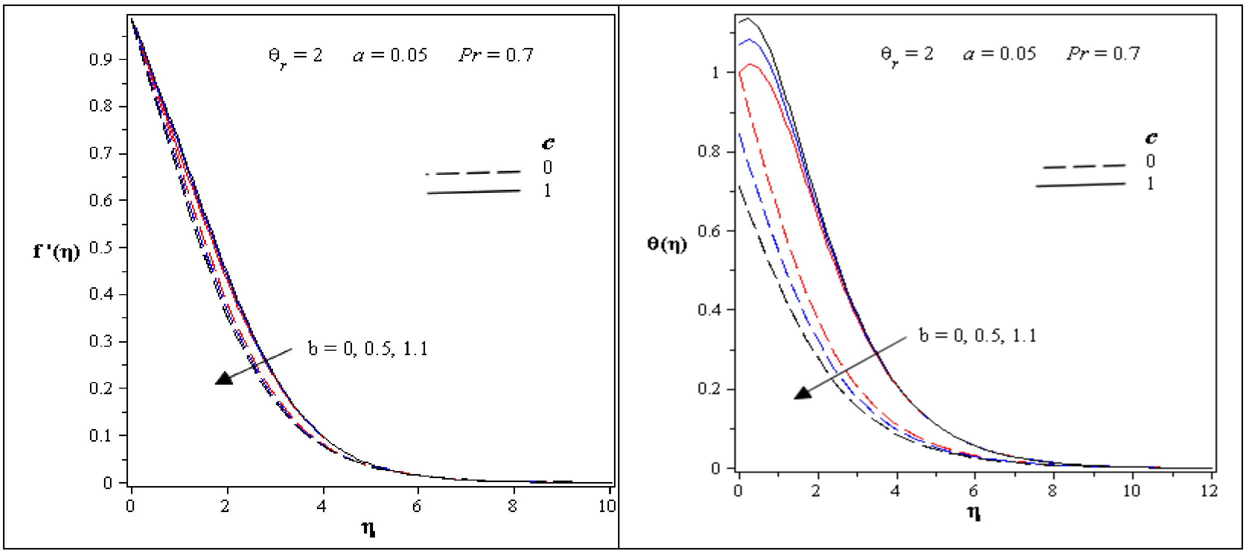


**Fig4:** Velocity &Temperature profiles for various values of b.

## REFERENCES


[1].Cheng P and Minkowycz WJ (1977). Free Convection about a Vertical Flat Plate Embedded in a Porous Medium with Application to Heat Transfer from a Dike, J. of Geophyiscal Res., 82, pp. 2040-2044.

[2].Bakier AY, Mansour MA, Gorla RSR, and Ebiana AB (1997). Non-similar solutions for free convection from a vertical plate in porous media, Heat and Mass Transfer, 33, pp. 145-148

[3]Gorla RSR and Zinolabedini A (1987)., Free convection from a vertical plate with nonuniform surface temperature and embedded in a porous medium, Transactions of ASME, 109, pp. 26-30.


 

[4].Gorla RSR and Tornabene R (1988). Free convection from a vertical plate with nonuniform heat flux and embedded in a porous medium, Transport in Porous Media, , pp.3 95-106.

[5].Bejan A and Khair KR (1985).Heat and mass transfer by natural convection in a porous mediumInt. J. Heat Mass Transfer, 28, pp. 909-918.

[6].Nield AD and Bejan A (1999).Convection in Porous Media, Springer, Berlin.

[7]. Sharma PR and Pankaj Mathur (1995). Steady laminar free convection flow of an electrically conducting fluid along a porous hot vertical plate in the presence of heat source/sink. *Indian J. Pure Appl. Math.*, **26(11)**, pp. 1125.

[8]. Varshney NK and Shilendra Kumar (2004).Effect of thermal diffusion on steady laminar free convective flow along a moving porous hot vertical plate in the presence of heat source with mass transfer. *Acta Ciencia Indica*, **XXX M** No.2, pp.489.

[9]. Basant Kumar Mishra (2005).Heat transfer in MHD free convection flow over an infinite vertical plate with time-dependent suction. *Acta Ciencia Indica*, **XXXI M** No.2, pp.371.

[10].Sharma PK and Chaudhary RC (2003).Effect of variable suction on transient free convection viscous incompressible flow past a vertical plate with periodic temperature variations in slip-flow regime. *Emirates Journal of Engineering Research*, **8**(2), pp. 33.

[11]. Mittal PK and Mukesh Bijalwan (2005). A Study of vorticity of fluctuating flow of an visco-elastic fluid past an infinite plate with variable suction in slip flow regime. *Acta Ciencia Indica*, **XXXI M** No.3, pp.903.

[12].Rajput US, Varshney NK and Debangana Rajput (2008). Free convection flow of magnetopolar fluid through porous medium in slip flow regime with mass transfer. *Acta Ciencia Indica*, **XXXIV M,** No.4, pp.2045.

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