

DECOMPOSITION OF $*$ - CONTINUITY IN IDEAL TOPOLOGICAL SPACES

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(Received on: 02-06-12; Accepted on: 20-06-12)

ABSTRACT

In this paper, we introduce and investigate the notions of $I\omega$ -continuous maps and $I\omega$ -irresolute maps in ideal topological spaces. Also we introduce the notions of slc^* - I -sets, $\wedge s$ -sets, λs - I -closed sets, slc^* - I -continuous maps and λs - I -continuous maps. Finally, we obtain the decompositions of $*$ -continuity.

2000 Mathematics subject classification: Primary 54A05, Secondary 54D15, 54D30.

Keywords: $I\omega$ -closed set, slc^* - I - set, $\wedge s$ -set, λs - I -closed set, slc^* - I - continuous and λs - I -continuous.

1 INTRODUCTION AND PRELIMINARIES

The concept of ideals in topological spaces is treated in the classic text by Kuratowski [10] and Vaidyanathaswamy [16]. The notion of I -open sets in topological spaces was introduced by Jankovic and Hamlett [8]. Dontchev et al. [4] introduced and studied the notion of Ig -closed sets. Recently, Navaneethakrishnan and Paulraj Joseph [13] have studied further the properties of Ig -closed sets and Ig -open sets. An ideal I on a topological space (X, τ) is a non-empty collection of subsets of X satisfying the following properties: (1) $A \in I$ and $B \subset A$ imply $B \in I$ (heredity), (2) $A \in I$ and $B \in I$ imply $A \cup B \in I$ (finite additivity). A topological space (X, τ) with an ideal I on X is called an ideal topological space and is denoted by (X, τ, I) . For a subset $A \subset X$, $A^*(I) = \{x \in X : U \cap A \notin I \text{ for every } U \in \tau(x)\}$ is called the local function [10] of A with respect to I and τ . We simply write A^* in case there is no chance for confusion. A Kuratowski closure operator $cl^*(\cdot)$ for a topology $\tau^*(I)$ called the $*$ -topology finer than τ is defined by $cl^*(A) = A \cup A^*$ [16]. Let (X, τ) denote a topological space on which no separation axioms are assumed unless explicitly stated. In a topological space (X, τ) , the closure and the interior of any subset A of X will be denoted by $cl(A)$ and $int(A)$, respectively. A subset A of a space is said to be semi-open [12] if $A \subset cl(int(A))$. A subset A of a topological space (X, τ) is said to be g -closed [11] (resp. ω -closed [15]) if $cl(A) \subset U$ whenever $A \subset U$ and U is open (resp. semi-open) in X . A subset A of a topological space (X, τ) is locally closed [5] (briefly LC) if $A = U \cap V$, where U is open and V is closed.

A subset A of a topological space (X, τ) is semi-locally closed [6] (briefly slc) if $A = U \cap V$ where U is semi-open and V is semi-closed. A subset A of a topological space (X, τ) is called slc^* -set if $A = U \cap V$ where U is semi-open and V is closed. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is g -continuous [11] (resp. ω -continuous [15]) if $f^{-1}(V)$ is g -closed (resp. ω -closed) in (X, τ) for every closed set V of (Y, σ) .

A subset A of an ideal space (X, τ, I) is $*$ -closed [8] if $A^* \subset A$. A subset A of an ideal space (X, τ, I) is Ig -closed [4] (resp. $I\omega$ -closed [14]) if $A^* \subset U$ whenever $A \subset U$ and U is open (resp. semi-open). An ideal space (X, τ, I) is called a TI -space [4] (resp. $TI\omega$ -space [14]) if every Ig -closed (resp. $I\omega$ -closed) subset of X is $*$ -closed.

Lemma 1.1: [8] Let (X, τ, I) be a space with an ideal I on X , and A is a subset of X . Then,

1. Every $*$ -closed set is $I\omega$ -closed,
2. Every $I\omega$ -closed set is Ig -closed.

We denote the family of all $I\omega$ -closed (resp. $I\omega$ - open) subsets of an ideal space (X, τ, I) by $I\omega(X)$ (resp. $I\omega O(X)$).

Definition 1.2: A subset A of an ideal space (X, τ, I) is called

1. strongly- I -locally closed [7] (briefly, strongly- I -LC) if $A = U \cap V$ where U is regular open and V is $*$ - closed.

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2. weakly- I -locally closed [9] (briefly, weakly- I -LC) if $A = U \cap V$, where U is open and V is *-closed.
3. mildly- I -locally closed [2] (briefly, mildly- I -LC) if $A = U \cap V$, where U is π -open and V is *-closed.

Definition 1.3: A function $f: (X, \tau, I) \rightarrow (Y, \sigma)$ is said to be

1. *-continuous [7] if $f^{-1}(A)$ is *-closed in X for every closed set A in Y.
2. Ig -continuous [7] if $f^{-1}(A)$ is Ig -closed in X for every closed set A in Y.
3. strongly- I -LC-continuous [7] if $f^{-1}(A)$ is a strongly- I -LC-set in X for every closed set A in Y.
4. weakly- I -LC-continuous [9] if $f^{-1}(A)$ is a weakly- I -LC-set in X for every closed set A in Y.
5. mildly- I -LC-continuous [2] if $f^{-1}(A)$ is a mildly- I -LC-set in X for every closed set A in Y.

Lemma 1.4: [7] Let (X, τ, I) be an ideal space and A be a subset of X. If A is *-closed then A is strongly- I -LC-set but not conversely.

2. $I\omega$ -continuity and $I\omega$ -irresoluteness

Definition 2.1: A function $f: (X, \tau, I) \rightarrow (Y, \sigma)$ is said to be $I\omega$ -continuous if $f^{-1}(A)$ is $I\omega$ -closed in X for every closed set A in Y.

Remark 2.2: If $I = \{\phi\}$ in the above definition, then the notion of $I\omega$ -continuity coincides with the notion of ω -continuity.

Definition 2.3: A function $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ is said to be $I\omega$ -irresolute if $f^{-1}(A)$ is $I\omega$ -closed in (X, τ, I) for every $I\omega$ -closed set A in (Y, σ, J) .

Theorem 2.4: For a function $f: (X, \tau, I) \rightarrow (Y, \sigma)$, the following hold:

1. Every continuous function is $I\omega$ -continuous,
2. Every *-continuous function is $I\omega$ -continuous,
3. Every ω -continuous function is $I\omega$ -continuous,
4. Every $I\omega$ -continuous function is Ig -continuous.

Proof:

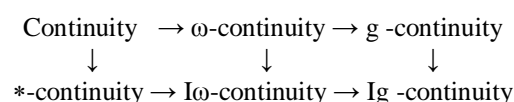
(1) Let f be a continuous function and V be a closed set in (Y, σ) . Then $f^{-1}(V)$ is closed in (X, τ, I) . Since every closed set is *-closed and hence $I\omega$ -closed, $f^{-1}(V)$ is $I\omega$ -closed in (X, τ, I) . Therefore, f is $I\omega$ -continuous.

(2) Let V be a closed set in (Y, σ) . Then $f^{-1}(V)$ is *-closed in (X, τ, I) because f is *-continuous in X. Since every *-closed set is $I\omega$ -closed, $f^{-1}(V)$ is $I\omega$ -closed in (X, τ, I) . Therefore, f is $I\omega$ -continuous.

(3) Let f be an ω -continuous function. Then $f^{-1}(V)$ is ω -closed in (X, τ, I) for every closed set V in (Y, σ) . Since every ω -closed set is $I\omega$ -closed [14], $f^{-1}(V)$ is $I\omega$ -closed in (X, τ, I) . Therefore, f is $I\omega$ -continuous.

(4) Let V be a closed set in (Y, σ) and f be an $I\omega$ -continuous function. Then $f^{-1}(V)$ is $I\omega$ -closed in (X, τ, I) . Since every $I\omega$ -closed set is Ig -closed, $f^{-1}(V)$ is Ig -closed in (X, τ, I) . Therefore, f is Ig -continuous.

Remark 2.5: The relationships defined above, are shown in the following diagram:



None of these implications is reversible as shown by the following examples.

Example 2.6: Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$, $I = \{\phi, \{c\}\}$ and $\sigma = \{\phi, \{a\}, X\}$. Define $f: (X, \tau, I) \rightarrow (Y, \sigma)$ as $f(a) = b$, $f(b) = a$, $f(c) = c$. Then f is $I\omega$ -continuous but not continuous.

Example 2.7: Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$, $I = \{\phi, \{c\}\}$ and $\sigma = \{\phi, \{a, c\}, X\}$. The identity map $f: (X, \tau, I) \rightarrow (Y, \sigma)$ is $I\omega$ -continuous but not *-continuous.

Example 2.8: Let $X = \{a, b, c, d\}$, $\tau = \{\phi, \{b\}, \{b, c, d\}, X\}$, $I = \{\phi, \{a\}\}$ and $\sigma = \{\phi, \{c, d\}, X\}$. Then the identity map $f: (X, \tau, I) \rightarrow (Y, \sigma)$ is Ig - continuous but not $I\omega$ -continuous.

Theorem 2.9: A function $f: (X, \tau, I) \rightarrow (Y, \sigma)$ is $I\omega$ -continuous if and only if $f^{-1}(V)$ is $I\omega$ -open in (X, τ, I) for every open set V in (Y, σ) .

Proof: Let V be an open set in (Y, σ) and $f: (X, \tau, I) \rightarrow (Y, \sigma)$ be $I\omega$ -continuous. Then V^c is closed in (Y, σ) and $f^{-1}(V^c)$ is $I\omega$ -closed in (X, τ, I) . But $f^{-1}(V^c) = (f^{-1}(V))^c$ and so $f^{-1}(V)$ is $I\omega$ -open in (X, τ, I) .

Conversely, Suppose that $f^{-1}(V)$ is $I\omega$ -open in (X, τ, I) for each open set V in (Y, σ) . Let F be a closed set in (Y, σ) . Then F^c is open in (Y, σ) and by hypothesis $f^{-1}(F^c)$ is $I\omega$ -open in (X, τ, I) . Since $f^{-1}(F^c) = (f^{-1}(F))^c$, we have $f^{-1}(F)$ is $I\omega$ -closed in (X, τ, I) and so f is $I\omega$ -continuous.

Definition 2.10: For a function $f: (X, \tau, I) \rightarrow (Y, \sigma)$, the subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of f and is denoted by $G(f)$.

Lemma 2.11: The following properties are equivalent for a graph $G(f)$ of a function f :

1. $G(f)$ is $I\omega$ -continuous,
2. For each $(x, f(x)) \in X \times Y \setminus G(f)$, there exists an $I\omega$ -open set U_ω of X containing x and an open set V containing $f(x)$ such that $g(U_\omega) \cap V \neq \emptyset$.

Theorem 2.12: Let $f: (X, \tau, I) \rightarrow (Y, \sigma)$ be a function and $g: X \rightarrow X \times Y$ be the graph function of f . If g is $I\omega$ -continuous, then f is $I\omega$ -continuous.

Proof: Suppose that g is $I\omega$ -continuous. Let $x \in X$ and V be any open set of Y containing $f(x)$. Then $X \times V$ is open in $X \times Y$ and by $I\omega$ -continuity of g , there exists $U \in I\omega O(X, \tau)$ containing x such that $g(U) \subset X \times V$. Therefore we obtain $f(U) \subset V$. This shows that f is $I\omega$ -continuous.

Theorem 2.13: Let (X, τ, I) be an ideal topological space. If $A \in I\omega(X)$ and $A \subset X_0 \subset I\omega(X)$ then $A \in I\omega(X_0)$.

Proof: Let $A \in I\omega(X)$ and $A \subset X_0 \subset I\omega(X)$ then $cl^*(A) \cap X_0 \subset U$ whenever $A \subset U$ and U is semi-open.

Also $cl^*_{X_0}(A) = cl^*(A) \cap X_0$ (Lemma 3.2 [1]). Hence $A \in I\omega(X_0)$.

Theorem 2.14: If $f: (X, \tau, I) \rightarrow (Y, \sigma)$ is an $I\omega$ -continuous function and $X_0 \in I\omega O(X)$. Then the restriction $f / X_0: (X_0, \tau / X_0, I / X_0) \rightarrow (Y, \sigma)$ is $I\omega$ -continuous.

Proof: Let V be any open set of (Y, σ) . Since f is $I\omega$ -continuous, $f^{-1}(V)$ is $I\omega$ -open in (X, τ, I) and $f^{-1}(V) \cap X_0 = (f / X_0)^{-1}(V) \in I\omega O(X)$. Moreover by Theorem 2.13 we have $(f / X_0)^{-1}(V) \in I\omega O(X_0)$. This shows that f / X_0 is $I\omega$ -continuous.

Remark 2.15: The composition of two $I\omega$ -continuous maps need not be $I\omega$ -continuous as seen from the following example:

Example 2.16: Let $X = Y = Z = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{a, b, c\}, X\}$ and $\sigma = \{\emptyset, \{c\}, Y\}$, $\eta = \{\emptyset, \{b, d\}, Z\}$, $I = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$, $J = \{\emptyset, \{c\}\}$. Let $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ and $g: (Y, \sigma, J) \rightarrow (Z, \eta)$ be identity maps. Then the maps f and g are $I\omega$ -continuous but $g \circ f$ is not $I\omega$ -continuous.

Theorem 2.17: Let $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ be $I\omega$ -continuous and $g: (Y, \sigma, J) \rightarrow (Z, \eta)$ be continuous. Then $g \circ f: (X, \tau, I) \rightarrow (Z, \eta)$ is $I\omega$ -continuous.

Theorem 2.18: A function $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ is $I\omega$ -irresolute if and only if the inverse image of every $I\omega$ -open set in (Y, σ, J) is $I\omega$ -open in (X, τ, I) .

Theorem 2.19: Let $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ and $g: (Y, \sigma, J) \rightarrow (Z, \eta, K)$ be $I\omega$ -irresolute. Then $(g \circ f): (X, \tau, I) \rightarrow (Z, \eta, K)$ is $I\omega$ -irresolute.

Proof: Let g be an $I\omega$ -irresolute function and V be any $K\omega$ -open set in (Z, η, K) . Then $g^{-1}(V)$ is $J\omega$ -open in (Y, σ, J) . Since f is $I\omega$ -irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is $I\omega$ -open in (X, τ, I) . Hence $g \circ f$ is $I\omega$ -irresolute.

Theorem 2.20: If $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ is $I\omega$ -irresolute and $g: (Y, \sigma, J) \rightarrow (Z, \eta)$ is $*$ -continuous then $g \circ f: (X, \tau, I) \rightarrow (Z, \eta)$ is $I\omega$ -continuous.

Proof: Let V be any closed set of (Z, η) . Then $g^{-1}(V)$ is $*$ -closed in (Y, σ, J) . Therefore, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is $I\omega$ -closed in (X, τ, I) , since every $*$ -closed set is $I\omega$ -closed. Hence $g \circ f$ is $I\omega$ -continuous.

Theorem 2.21: Let (X, τ, I) be an ideal topological space, (Y, σ, J) be a TI_ω -space and (Z, η) be a topological space. If $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ is I_ω -irresolute and $g: (Y, \sigma, J) \rightarrow (Z, \eta)$ is I_ω -continuous then $g \circ f$ is I_ω -continuous.

Theorem 2.22: Let (X, τ, I) be an ideal topological space, (Y, σ, J) be a TI -space and (Z, η) be a topological space. If $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ is I_ω -irresolute and $g: (Y, \sigma, J) \rightarrow (Z, \eta)$ is I_g -continuous then $g \circ f$ is I_ω -continuous.

3 slc^* - I -sets

In this section, we introduce the notions of slc^* - I -sets in ideal topological spaces and study some of its properties.

Definition 3.1: A subset A of an ideal topological space (X, τ, I) is called slc^* - I -set if $A = U \cap F$ where U is semi-open and F is $*$ -closed.

Proposition 3.2: Let (X, τ, I) be an ideal space and A be a subset of X . Then the following holds.

1. If A is semi-open then A is slc^* - I -set.
2. If A is $*$ -closed then A is slc^* - I -set.
3. If A is weakly- I -LC-set then A is slc^* - I -set.

Example 3.3: Let $X = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{a, b, c\}, X\}$ and $I = \{\phi, \{a\}, \{b\}, \{a, b\}\}$. Then the set $A = \{c\}$ is an slc^* - I -set but not a $*$ -closed set.

Example 3.4: Let $X = \{a, b, c, d\}$, $\tau = \{\phi, \{a, b\}, X\}$ and $I = \{\phi, \{a\}\}$. Then the set $A = \{a, c\}$ is an slc^* - I -set but not a weakly- I -LC-set.

Remark 3.5: The notions of I_ω -closed sets and slc^* - I -sets are independent.

Example 3.6: Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $I = \{\phi, \{c\}\}$. Then the set $A = \{b\}$ is I_ω -closed but not an slc^* - I -set.

Example 3.7: Let $X = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{a, b, c\}, X\}$ and $I = \{\phi, \{a\}, \{b\}, \{a, b\}\}$. Then the set $A = \{c\}$ is slc^* - I -set but not an I_ω -closed set.

Theorem 3.8: A subset of an ideal topological space (X, τ, I) is $*$ -closed if and only if it is both I_ω -closed and an slc^* - I -set.

Proof: Necessity is trivial. To prove the sufficiency, assume that A is both I_ω -closed and an slc^* - I -set. Then $A = U \cap F$ where U is semi-open and F is $*$ -closed. Therefore $A \subset U$ and $A \subset F$ and so by hypothesis, $A^* \subset U$ and $A^* \subset F$. Thus $A^* \subset U \cap F = A$. Hence A is $*$ -closed.

Remark 3.9: From [2] and Proposition 3.2, We have the following implications:

strongly- I -LC-set \rightarrow mildly- I -LC-set \rightarrow weakly- I -LC-set \rightarrow slc^* - I -set.

Theorem 3.10: For a subset A of an ideal topological space (X, τ, I) , the following are equivalent.

1. A is a $*$ -closed set.
2. A is a strongly- I -LC-set and an I_ω -closed set.
3. A is a mildly- I -LC-set and an I_ω -closed set.
4. A is a weakly- I -LC-set and an I_ω -closed set.
5. A is an slc^* - I -set and an I_ω -closed set.

Corollary 3.11: For a subset A of an ideal topological space (X, τ, I) , the following are equivalent.

1. A is a $*$ -closed set.
2. A is a weakly- I -LC-set and an I_ω -closed set.
3. A is a weakly- I -LC-set and an I_g -closed set. [7]

4 A new subset of an ideal topological space

Definition 4.1: [3] Let A be a subset of a topological space (X, τ) . Then the s -kernel of the set A , denoted by $s\text{-ker}(A)$ is the intersection of all semi-open supersets of A .

Definition 4.2: [3] A subset A of a topological space (X, τ) is called $\wedge s$ -set if $A = s - \ker(A)$.

Definition 4.3: A subset A of an ideal space (X, τ, I) is called $\lambda s - I$ -closed if $A = U \cap V$ where U is a $\wedge s$ -set and V is $*$ -closed.

Proposition 4.4: In an ideal space (X, τ, I) , every $*$ -closed set is $\lambda s - I$ -closed but not conversely.

Example 4.5: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$ and $I = \{\emptyset, \{a\}\}$. Then the set $A = \{b\}$ is $\lambda s - I$ -closed but not $*$ -closed.

Lemma 4.6: For a subset A of an ideal space (X, τ, I) , the following are equivalent.

1. A is $\lambda s - I$ -closed.
2. $A = U \cap \text{cl}^*(A)$ where U is a $\wedge s$ -set.
3. $A = s - \ker(A) \cap \text{cl}^*(A)$.

Lemma 4.7: A subset A of an ideal space (X, τ, I) is $I\omega$ -closed if and only if $\text{cl}^*(A) \subset s - \ker(A)$.

Remark 4.8: The notions of $I\omega$ -closed sets and $\lambda s - I$ -closed sets are independent.

Example 4.9: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$ and $I = \{\emptyset, \{a\}\}$. Then the set $A = \{c\}$ is $\lambda s - I$ -closed but not $I\omega$ -closed.

Example 4.10: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ and $I = \{\emptyset, \{c\}\}$. Then the set $A = \{b\}$ is $I\omega$ -closed but not $\lambda s - I$ -closed.

Theorem 4.11: A subset of an ideal topological space (X, τ, I) is $*$ -closed if and only if it is both $I\omega$ -closed and $\lambda s - I$ -closed.

5. Decompositions of *-continuity

In this section, we obtain decompositions of $*$ -continuity in ideal topological spaces. In order to obtain the decompositions of $*$ -continuity we introduce the notion of $\text{slc}^* - I$ -continuity and $\lambda s - I$ -continuity in ideal topological spaces.

Definition 5.1: A function $f: (X, \tau, I) \rightarrow (Y, \sigma)$ is said to be $\text{slc}^* - I$ -continuous (resp. $\lambda s - I$ -continuous) if $f^{-1}(V)$ is an $\text{slc}^* - I$ -set (resp. $\lambda s - I$ -closed set) in (X, τ, I) for every closed set V in (Y, σ) .

Example 5.2: Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{a, b, c\}, X\}$, $I = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{\emptyset, \{c\}, X\}$. Then the identity map $f: (X, \tau, I) \rightarrow (Y, \sigma)$ is an $\text{slc}^* - I$ -continuous map.

Remark 5.3: Every $*$ -continuous map is $\text{slc}^* - I$ -continuous, but the converse is not true.

Example 5.4: Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{a, b, c\}, X\}$, $I = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{\emptyset, \{a, b, d\}, X\}$. Let $f: (X, \tau, I) \rightarrow (Y, \sigma)$ be the identity map. Then f is $\text{slc}^* - I$ -continuous but not $*$ -continuous.

Remark 5.5: The concepts of $I\omega$ -continuity and $\text{slc}^* - I$ -continuity are independent as seen from the following examples.

Example 5.6: Let X, τ, σ and I be defined as in Example 5.4. Then the map $f: (X, \tau, I) \rightarrow (Y, \sigma)$ is $\text{slc}^* - I$ -continuous but not $I\omega$ -continuous.

Example 5.7: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$, $I = \{\emptyset, \{c\}\}$ and $\sigma = \{\emptyset, \{a, c\}, X\}$. Let the map $f: (X, \tau, I) \rightarrow (Y, \sigma)$ be the identity map. Then f is $I\omega$ -continuous but not $\text{slc}^* - I$ -continuous.

Theorem 5.8: For a function $f: (X, \tau, I) \rightarrow (Y, \sigma)$, the following are equivalent.

1. f is $*$ -continuous.
2. f is strongly- I -LC-continuous and $I\omega$ -continuous.
3. f is mildly- I -LC-continuous and $I\omega$ -continuous.
4. f is weakly- I -LC-continuous and $I\omega$ -continuous.
5. f is $\text{slc}^* - I$ -continuous and $I\omega$ -continuous.

Corollary 5.9: For a function $f: (X, \tau, I) \rightarrow (Y, \sigma)$, the following are equivalent.

1. f is $*$ -continuous.
2. f is weakly- I -LC-continuous and $I\omega$ -continuous.
3. f is weakly- I -LC-continuous and I_g -continuous. [7]

Theorem 5.10: A function $f: (X, \tau, I) \rightarrow (Y, \sigma)$ is $*$ -continuous if and only if it is both $I\omega$ -continuous and λ_s - I -continuous.

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Source of support: Nil, Conflict of interest: None Declared