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HYDROMAGNETIC OSCILLATORY FLOW OF DUSTY FLUID IN A ROTATING POROUS CHANNEL

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ABSTRACT

An analysis of the unsteady flow of a dusty viscous, incompressible, electrically conducting fluid in a horizontal porous channel rotating with an angular velocity under the influence of periodic pressure gradient is studied. The lower porous plate is subjected to a uniform injection and the upper porous plate to a uniform suction. A magnetic field of uniform strength is applied perpendicular to the plates. The magnetic Reynolds number is assumed to be small enough so that the induced magnetic field is negligible. The whole system in unison rotates about the axis normal to the plates. Analytical solutions for the velocities of the fluid and the dust particles are obtained. The effect of the various parameters on the governing equations of the fluid velocity and the dust particles has been numerically evaluated, shown graphically and discussed.

***Key Words**: Hydromagnetic, Oscillatory, Dusty, Rotating porous channel.*

1. INTRODUCTION

The influence of the dust particles on viscous fluid flow are of importance in many application such as wastewater treatment, power plant piping, purification of the crude oils, combustion and petroleum transport. Other important applications involving dust particles in boundary layer, including soil salvation by natural winds, lunar surface erosion by exhaust of landing vehicle and dust entrainment in a cloud formed during a nuclear explosion. Particularly, the flow and heat transfer of electrically conducting fluids in channels under the effect of a transverse magnetic field occur in magnetohydrodynamics accelerators, pumps and generators. This type of flow has uses in nuclear reactors, geothermal systems and filtration etc. Saffman [6] formulated the equation of the dust laden gas in a simplified form by making some assumptions on the dust particles. The study of dusty viscous fluid under the influence of different physical conditions has been carried out by several authors; Nag and Jana [5] have studied unsteady couette flow of a dusty gas between two infinite parallel plates, when one plate of the channel is kept stationary and other plate moves uniformly in its own plane. Dalal [4] analyzed the generalized couette flow of dusty gas due to an impulsive pressure gradient as well as due to impulsive start of the lower plate. Singh and Singh [7] studied the laminar convective flow of an incompressible, conducting viscous fluid embedded with non-conducting dust particles through a vertical parallel plate channel in the presence of uniform magnetic field and constant pressure gradient taking volume fraction of the particles into consideration when one plate of the channel is fixed and the other is oscillating in time and in magnitude about a constant non-zero mean. Attia [1] studied the effects of variable viscosity on the unsteady flow of an electrically conducting, viscous, incompressible dusty fluid and heat transfer between parallel non-conducting porous plates when a uniform magnetic field is applied perpendicular to the plates. Attia [2], investigated the time varying couette flow with heat transfer of a dusty viscous incompressible, electrically conducting fluid under the influence of constant pressure gradient without neglecting the hall current The governing equations are solved numerically using the finite differences to yield the velocity and the temperature distribution for both the fluid and the dust particles.

The aim of the present paper is to study the influence of rotation and the periodic pressure gradient on the flow of an unsteady, viscous, incompressible and electrically conducting fluid embedded with non-conducting dust particles in a horizontal porous channel rotating with constant angular velocity when a uniform magnetic field is applied perpendicular to the plates.

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2. DESCRIPTION OF THE PROBLEM

The dusty fluid is assumed to be flowing between two infinite horizontal porous plates located at $z = \pm\frac{d}{2}$ planes, as shown in the Fig. 1. The fluid is injected with constant velocity w_0 through the lower porous plate and simultaneously removed with same suction velocity through the upper porous plate. Thus the z^*-component of the velocity of the fluid is constant and denoted by w_0. The dust particles are assumed to be electrically non-conducting, spherical and uniformly distributed in the fluid and are big enough so that they are not pumped out through the porous plates and have no z^*-component of velocity. The two-plates are assumed to be electrically non-conducting and kept at two different temperatures; T_1^* for the lower plate and T_2^* for the upper plate with $T_2^* > T_1^*$. A periodic pressure gradient varying with time is applied in x^* −direction. A uniform magnetic field B_0 is applied in the positive z^* −direction. This is only magnetic field in the problem as the induced magnetic field is neglected by assuming very small magnetic Reynolds number (Crammer and Pai, 1973) [3]. The whole system is rotating with constant angular velocity about the z^* −axis. It is required to obtain the time varying velocity distribution for both the fluid and the dust particles. Since the plates are infinite in x^* and y^* directions, all physical quantities for this fully developed flow depends only on z^* coordinate except the pressure.

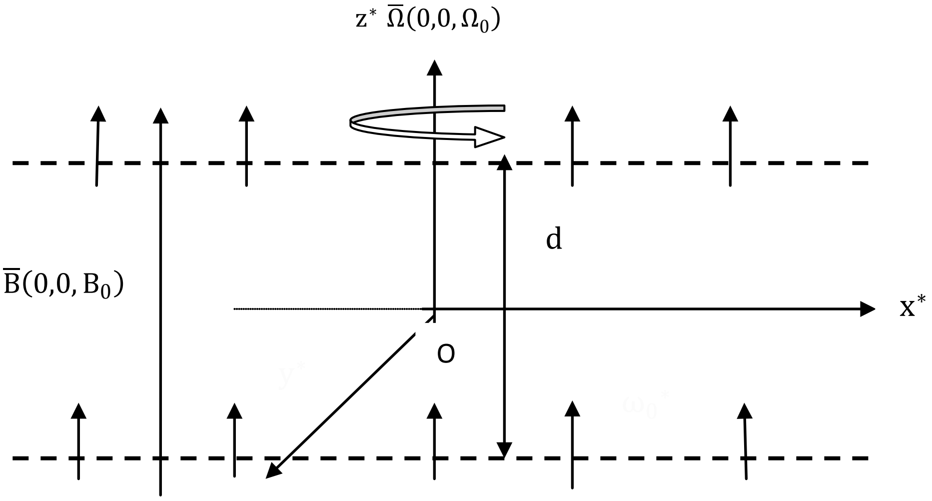

Fig. (1): The physical configuration of the problem

3. GOVERNING EQUATIONS

The governing equations for this study are based on the conservation laws of mass, linear momentum for both fluid and dust particles phases.

Let $\vec{u}(u, v\, w)$ and $\overrightarrow{u_p}(u_{p,}\, v_p, w_p)$ be the fluid and particle velocity respectively. The magnetic field and angular velocity for the present problem are $\vec{B}(0,0,B_0)$ and $\vec{\Omega}(0,0,\Omega_0)$ respectively.
The equations of conservation of mass for the fluid and particles are given by

$$\nabla.\vec{u} = 0, \qquad \nabla.\overrightarrow{u_p} = 0 \tag{1}$$

Momentum equation

The flow of the fluid is governed by the following momentum equation

$$\frac{\partial\vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} + 2\vec{\Omega}\times\vec{u} = \frac{-1}{\rho}\nabla p + \mu\nabla^2\vec{u} + \frac{KN}{\rho}\left(\overrightarrow{u_p} - \vec{u}\right) + \frac{1}{\rho}\vec{J}\times\vec{B} \tag{2}$$

where ρ is the density of the clean fluid, μ is the viscosity of the clean fluid , $\vec{u}$ is the fluid velocity $\overrightarrow{u_p}$ is the velocity of the dust particles, $\vec{J}$ is the current density, $\vec{B}$ is the magnetic flux density vector, p is the pressure distribution, $\vec{\Omega}$ is the constant angular velocity of the channel, N is the number of dust particle per unit volume, $K = 6\pi\mu a$ is the Stokes constant here a is the average radius of the dust particles.

The motion of the dust particles is governed by second law of Newton's and is given by

$$m_p\left[\frac{\partial\overrightarrow{u_p}}{\partial t} + (\overrightarrow{u_p}.\nabla)\overrightarrow{u_p} + 2\Omega\times\overrightarrow{u_p}\right] = K'(\vec{u} - \overrightarrow{u_p}) \tag{3}$$

Here, m_p is the average mass of the dust particles.

The momentum equation (2) for the fluid velocity in its component form is given by

$$\frac{\partial u^*}{\partial t^*}+w_0\frac{\partial u^*}{\partial z^*}-2\Omega_0 v^*=-\frac{1}{\rho}\frac{\partial p^*}{\partial x^*}+\mu\frac{\partial^2 u}{\partial z^{*2}}+\frac{KN}{\rho}\left(u_p^*-u^*\right)-\frac{\sigma B_0^2}{\rho}u^*\ , \tag{4}$$

$$\frac{\partial v^*}{\partial t^*}+w_0\frac{\partial v^*}{\partial z^*}+2\Omega_0 u^*=-\frac{1}{\rho}\frac{\partial p^*}{\partial y^*}+\mu\frac{\partial^2 v^*}{\partial z^{*2}}+\frac{KN}{\rho}\left(v_p^*-v^*\right)-\frac{\sigma B_0^2}{\rho}v^*\ , \tag{5}$$

$$0=-\frac{1}{\rho}\frac{\partial p^*}{\partial z^*}\ . \tag{6}$$

Similarly, the components of the momentum equation (3) for the dust particle phase is given by

$$m_p\left[\frac{\partial u_p^*}{\partial t^*}-2\Omega_0 v_p^*\right]=KN(u^*-u_p^*)\ , \tag{7}$$

$$m_p\left[\frac{\partial v_p^*}{\partial t^*}-2\Omega_0 u_p^*\right]=KN\left(v^*-v_p^*\right). \tag{8}$$

Boundary conditions relevant to the problem are given by

$$\left.\begin{array}{l} z^*=-\frac{d}{2},\quad u^*=v^*=u_p^*=v_p^*,\ =0\qquad T^*=0 \\ z^*=\frac{d}{2}, u^*=v^*=u_p^*=v_p^*,=0, T^*=T_0\cos\omega^* t^* \end{array}\right\} \tag{9}$$

Introducing the following non-dimensional quantities

$$\begin{gathered}(x,y,z)=\frac{(x^*,y^*,z^*)}{d},(u,v)=\frac{(u^*,v^*)}{w_0},(u_p,v_p)=\frac{(u_p^*,v_p^*)}{w_0},G=\frac{m_p\nu}{KNd^2}\\ \omega=\frac{\omega^* d}{w_0},t=\frac{t^* w_0}{d},\Omega=\frac{\Omega_0 d^2}{\nu},\lambda=\frac{w_0 d}{\nu},p=\frac{p^*}{\rho {w_0}^2}\ R=\frac{KN d^2}{\mu}\ ,M=B_0 d\sqrt{\frac{\sigma}{\mu}}\end{gathered} \tag{10}$$

where, G =Particle mass parameter, $R=$ Particle concentration parameter, ω =Frequency of oscillation, M =Magnetic field parameter, λ =suction parameter

In equations (4), (5), (7) and (8) the equations governing the flow of fluid and dust particle phases reduce to:

$$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial z}-\frac{2\Omega}{\lambda}v=\frac{-\partial p}{\partial x}+\frac{1}{\lambda}\frac{\partial^2 u}{\partial z^2}+\frac{R}{\lambda}\left(u_p-u\right)-\frac{M^2}{\lambda}u\ , \tag{11}$$

$$\frac{\partial v}{\partial t}+\frac{\partial v}{\partial z}+\frac{2\Omega u}{\lambda}=-\frac{\partial p}{\partial y}+\frac{1}{\lambda}\frac{\partial^2 v}{\partial z^2}+\frac{R}{\lambda}\left(v_p-v\right)-\frac{M^2}{\lambda}v\ , \tag{12}$$

$$\frac{\partial u_p}{\partial t}-2\frac{\Omega}{\lambda}v_p=\frac{1}{G\lambda}(u-u_p)\ \ , \tag{13}$$

$$\frac{\partial v_p}{\partial t}+2\frac{\Omega}{\lambda}u_p=\frac{1}{G\lambda}(v-v_p)\ . \tag{14}$$

The corresponding boundary conditions transformed to:

$$\left.\begin{array}{l} z=-\frac{1}{2}, u=v=u_p=v_p=0 \\ z=\frac{1}{2},\quad u=v=u_p=v_p=0 \end{array}\right\} \tag{15}$$

Continuity equations are identically satisfied and $\frac{-1}{\rho}\frac{\partial p^*}{\partial z^*}=0$, shows the constant fluid pressure along the z^*axis, the axis of rotation. We shall assume now that the fluid flows only under the pressure gradient along the x^*-axis only varying with time which is of the form

$$-\frac{\partial p^*}{\partial x^*}=A\cos\omega t\quad\text{and}\quad\frac{\partial p^*}{\partial y^*}=0\ . \tag{16}$$

In order to combine the velocity components for the fluid and the dust particle we assume the complex velocity in the form

$$F = u + iv \quad \text{and} \quad F_p = u_p + iv_p \tag{17}$$

Equations (11) and (12) with the help of (16) can be combined into a single equation of the form:

$$\frac{\partial F}{\partial t} + \frac{\partial F}{\partial z} + \frac{2i\Omega}{\lambda} F = -\frac{\partial p}{\partial x} + \frac{1}{\lambda}\frac{\partial^2 F}{\partial z^2} + \frac{R}{\lambda}\left(F_p - F\right) - \frac{M^2}{\lambda} F \tag{18}$$

Similarly, equations (13) and (14) can also be combined into a single equation of the form:

$$\frac{\partial F_p}{\partial t} + \frac{2i\Omega}{\lambda} F_p = \frac{1}{G\lambda}(F - F_p) \tag{19}$$

The boundary conditions reduce to:

$$\left.\begin{aligned} z = -\tfrac{1}{2}; &\quad F = F_p = 0; \\ z = \tfrac{1}{2}; &\quad F = F_p = 0. \end{aligned}\right\} \tag{20}$$

4. SOLUTION OF THE PROBLEM

To solve the equations (18) and (19), we assume in complex form the solution of problem as:

$$F(z,t) = \phi(z)e^{i\omega t}, -\frac{\partial p}{\partial x} = Ae^{i\omega t}, F_p(z,t) = \psi(z)e^{i\omega t}. \tag{21}$$

Substituting (21) in the equations (18) and (19), we obtain the following equations for the fluid and dust particle velocities

$$\phi'' - \lambda\phi' - m^2\phi = -A\lambda\,, \tag{22}$$

$$\psi = \frac{\phi}{1 + iG\,(\lambda\omega + 2\Omega)}\ . \tag{23}$$

These equations are solved under the following boundary conditions

$$\left.\begin{aligned} z = -\tfrac{1}{2}; &\quad \phi = \psi = 0, \\ z = \tfrac{1}{2}; &\quad \phi = \psi = 0. \end{aligned}\right\} \tag{24}$$

The expressions for the fluid and the dust particle velocities are obtained in the following form:

$$F(z,t) = \frac{A\lambda}{m^2} e^{i\omega t}\left[1 + \frac{e^{m_1 z}\sinh\frac{m_2}{2} - e^{m_2 z}\sinh\frac{m_1}{2}}{\sinh\frac{(m_1 - m_2)}{2}}\right] \tag{25}$$

$$F_p(z,t) = \frac{A\lambda}{m^2(1 + iG\,(\lambda\omega + 2\Omega))} e^{i\omega t}\left[1 + \frac{e^{m_1 z}\sinh\frac{m_2}{2} - e^{m_2 z}\sinh\frac{m_1}{2}}{\sinh\frac{(m_1 - m_2)}{2}}\right] \tag{26}$$

All the constants used above have been listed in the Appendix.

5. RESULTS AND DISCUSSION

The following discussion brings out the effects of some pertinent parameters such as the particle concentration parameter(R), the magnetic field parameter(M), the suction parameter(λ), the frequency of oscillation(ω), the amplitude of the pressure gradient(A), and the particle mass parameter(G) on the fluid and dust particles velocity. The numerical calculations have been carried out for small ($\Omega = 5$) and large($\Omega = 10$) values of the rotation parameter(Ω). The results have been graphically expressed for both the fluid and dust particles velocity. The variations in the fluid velocity are represented in Figures (2-7). The study of these figures show that the fluid velocity decrease with the increasing value of the particle concentration parameter(R), the magnetic field parameter(M), the suction parameter(λ), the frequency of the oscillation(ω), and the particle mass parameter(G) whereas it increases with the increasing value of the amplitude of the pressure gradient(A), for both small($\Omega = 5$) and large ($\Omega = 10$) value of rotation parameter(Ω). It is also clear from these figures that the fluid velocity is maximum at the centre of the channel.

The dust particles velocity variations with these parameters have been shown in Figures (8-13). It is observed from the study of these figures that for both small($\Omega = 5$) and large ($\Omega = 10$) value of rotation parameter (Ω), the dust particles velocity follows the same pattern as observed in case of the fluid velocity i.e the dust particles velocity decrease with the increasing value of the particle concentration parameter(R), the magnetic field parameter (M), the suction parameter (λ), the frequency of the oscillation(ω), and the particle mass parameter (G) whereas it increases with the increasing value of the amplitude of the pressure gradient(A). It is also clear from these figures that the dust particles velocity is maximum along the centre of the channel.

6. CONCLUSION

- The fluid and dust particle velocity is significantly enhanced by the amplitude of the pressure gradient.
- All other parameters diminish the fluid and the dust particle velocity.
- The fluid and the dust particle velocity decreases with the increasing value of rotation parameter.
- Fluid and the dust particle velocity are maximum along the centre of the channel.

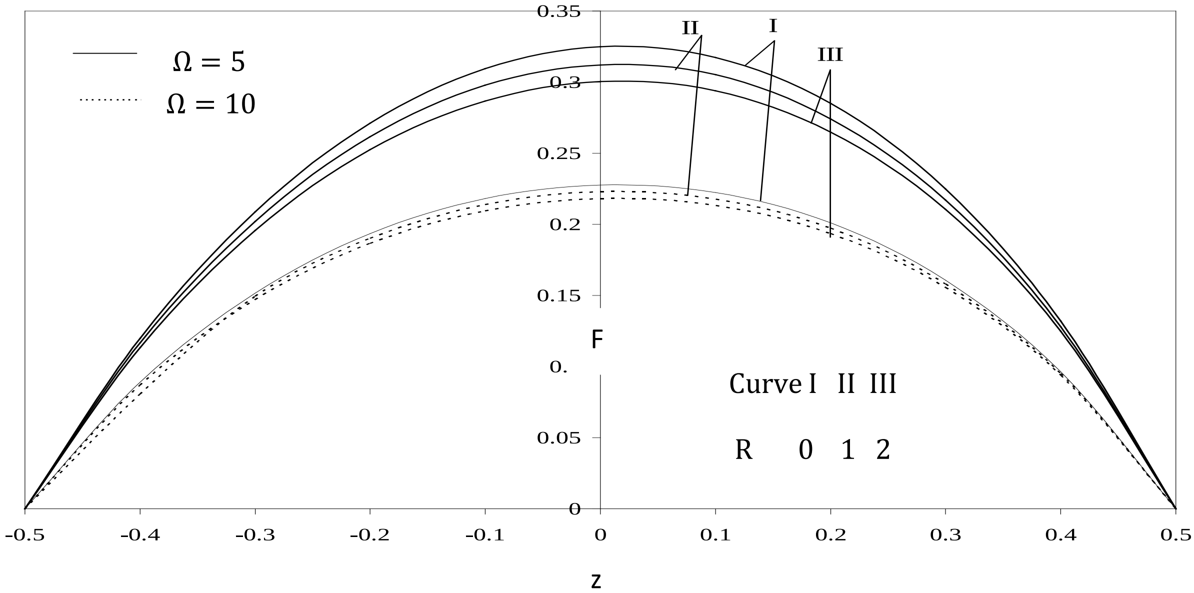

Fig. (2): Fluid velocity variations for $M = \angle$, $\lambda = 0.5$, $\omega = 5$, $A = 5$, $G = 1$ and $t = 0$.

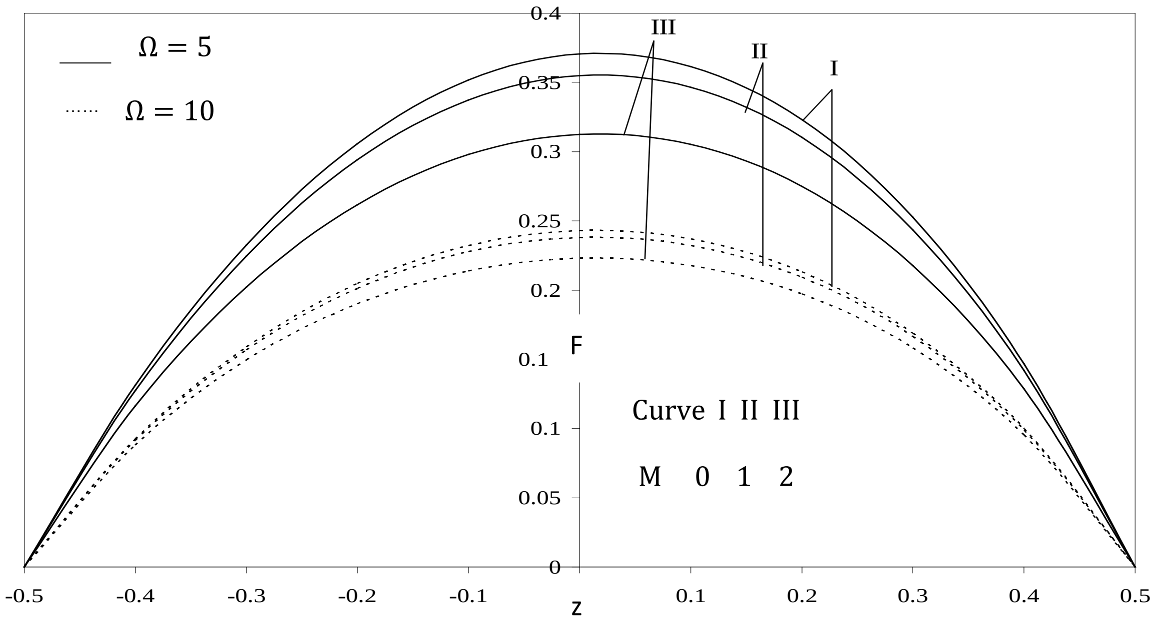

Fig. (3): Fluid velocity variations for$R = 1$, $\lambda = 0.5$, $\omega = 5$, $A = 5$, $G = 1$ and $t = 0$.

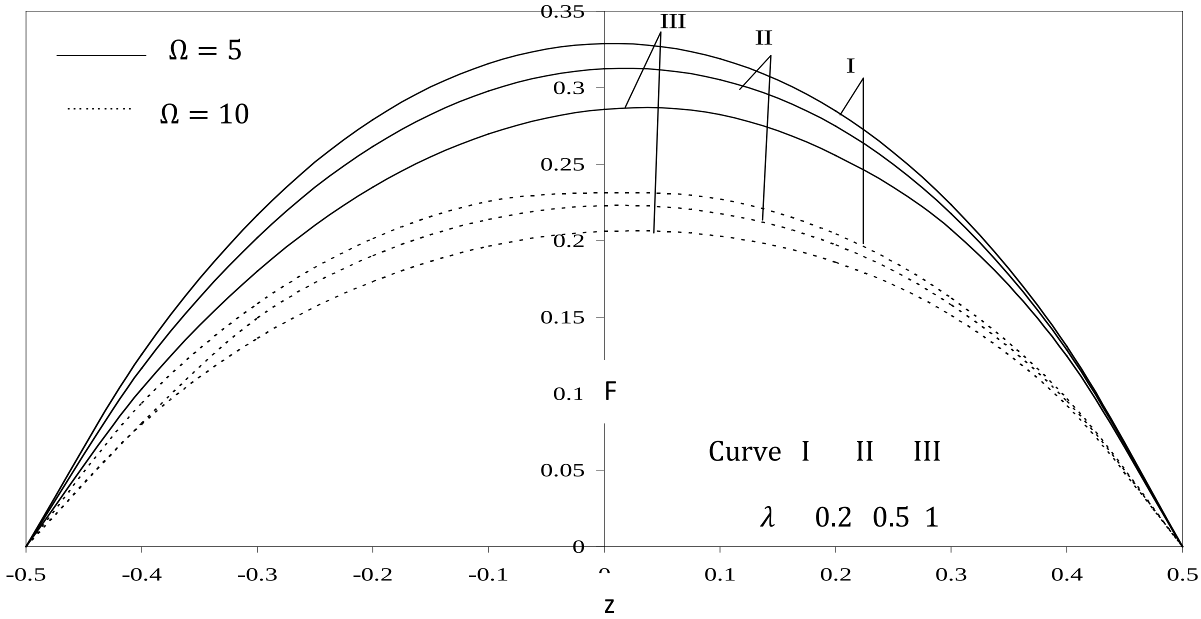

Fig. (4): Fluid velocity variations for R = 1, M = 2, ω = 5, A = 5, G = 1 and t = 0.

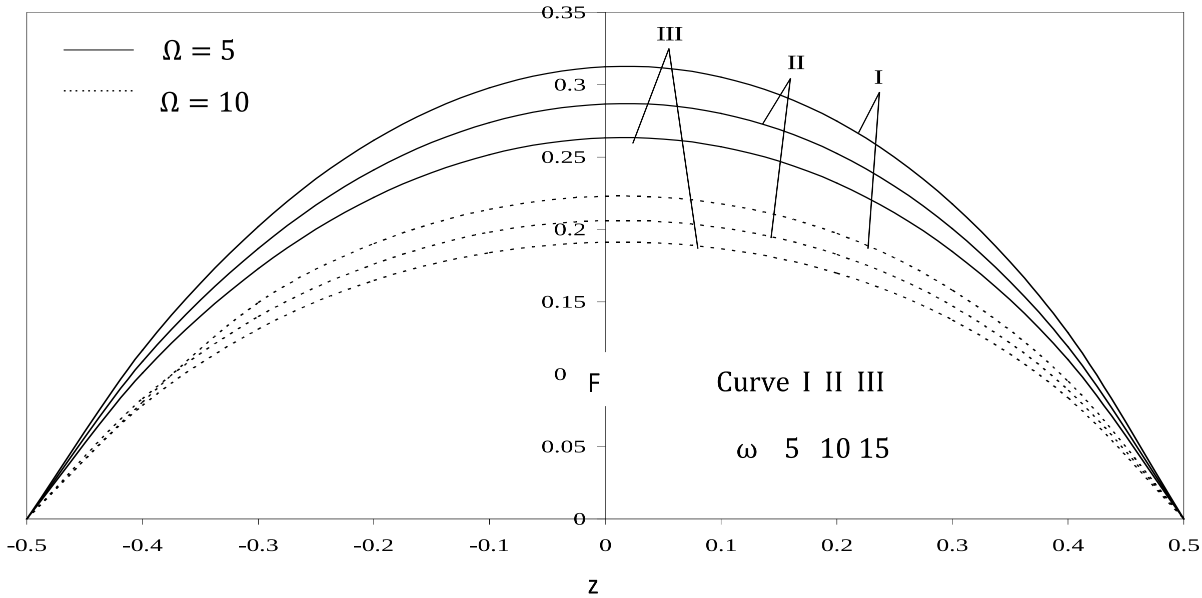

Fig. (5): Fluid velocity variations for R = 1, M = 2, λ = 0.5, A = 5, G = 1 and t = 0.

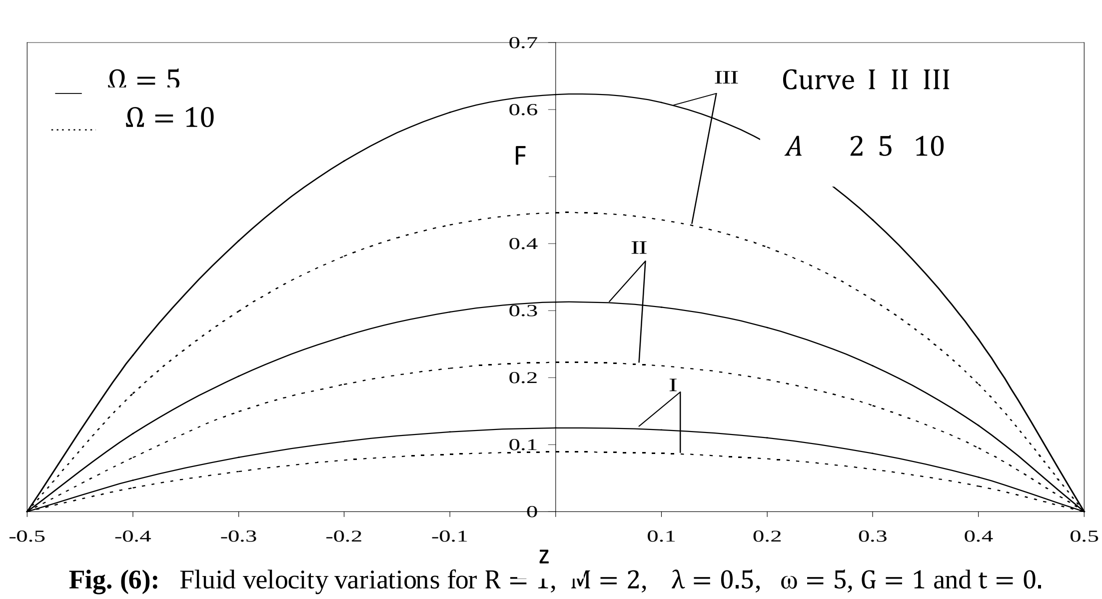

Fig. (6): Fluid velocity variations for R = 1, M = 2, λ = 0.5, ω = 5, G = 1 and t = 0.

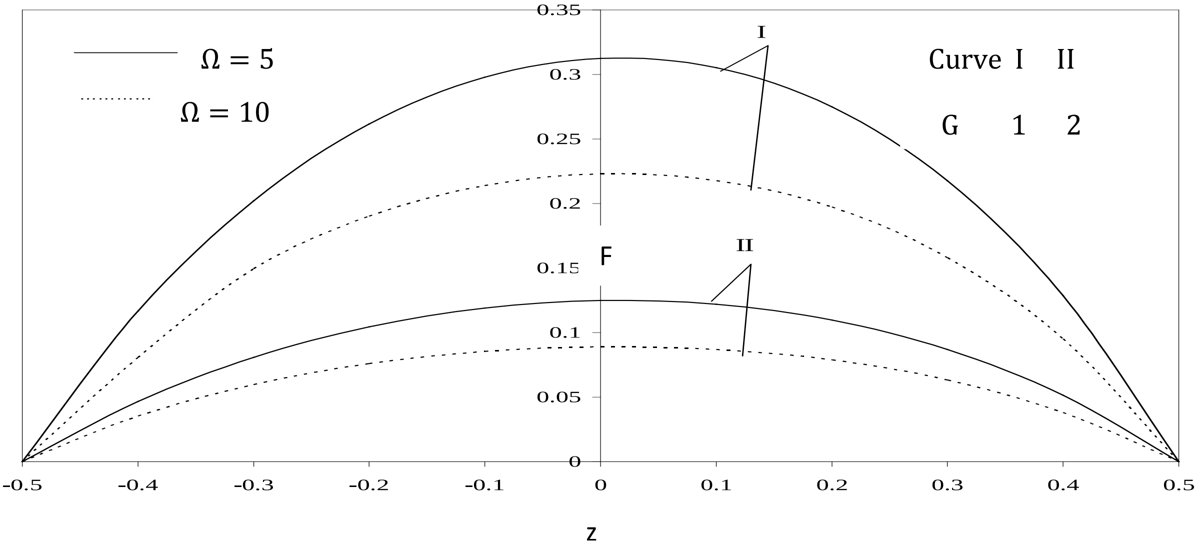

Fig. (7): Fluid velocity variations for R = 1, M = 2, λ = 0.5, ω = 5, A = 5 and t = 0.

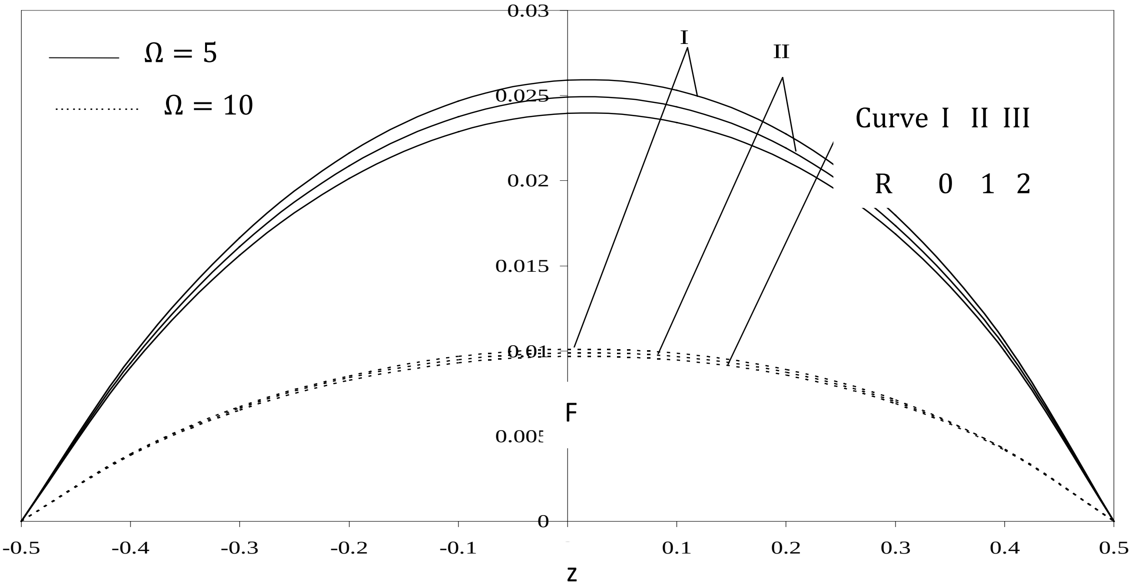

Fig. (8): Dust particles velocity variations for M = 2, λ = 0.5, ω = 5, A = 5, G = 1 and t = 0.

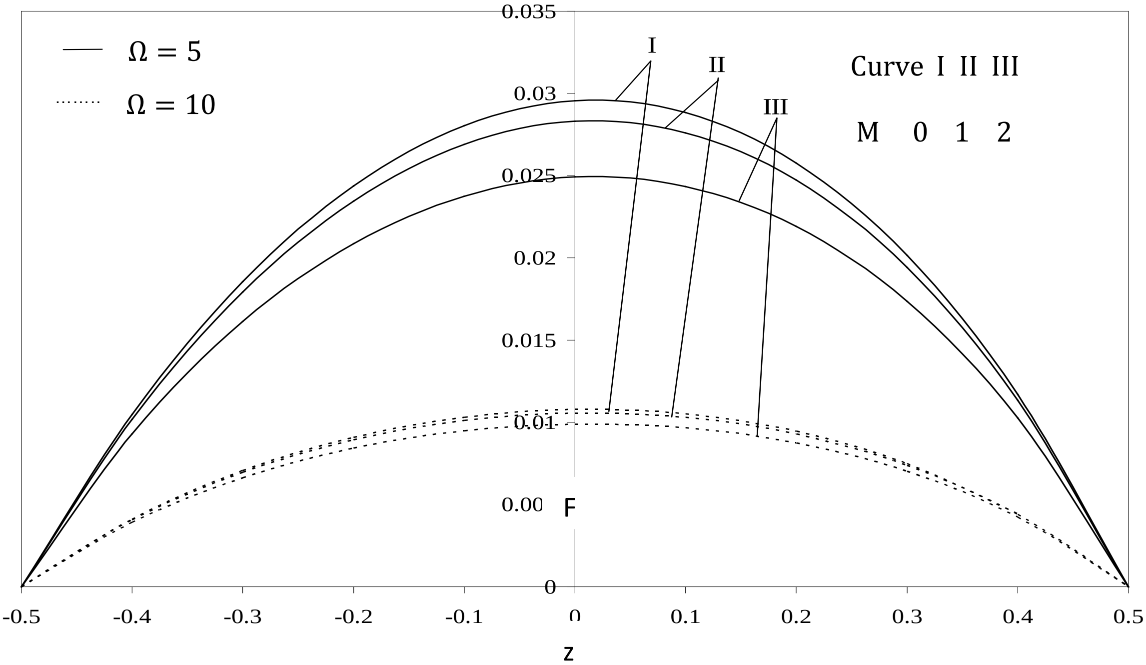

Fig. (9): Dust particles velocity variations for R = 1, λ = 0.5, ω = 5, A = 5, G = 1 and t = 0.

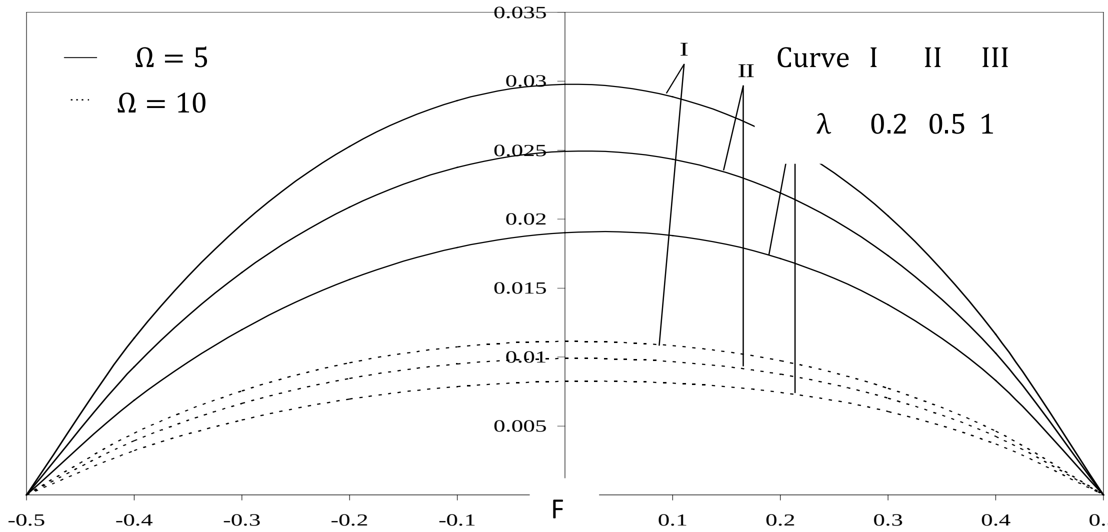

Fig.(10): Dust particles velocity variations for $R = 1$, $M = 2$, $\omega = 5$, $A = 5$, $G = 1$ and $t = 0$.

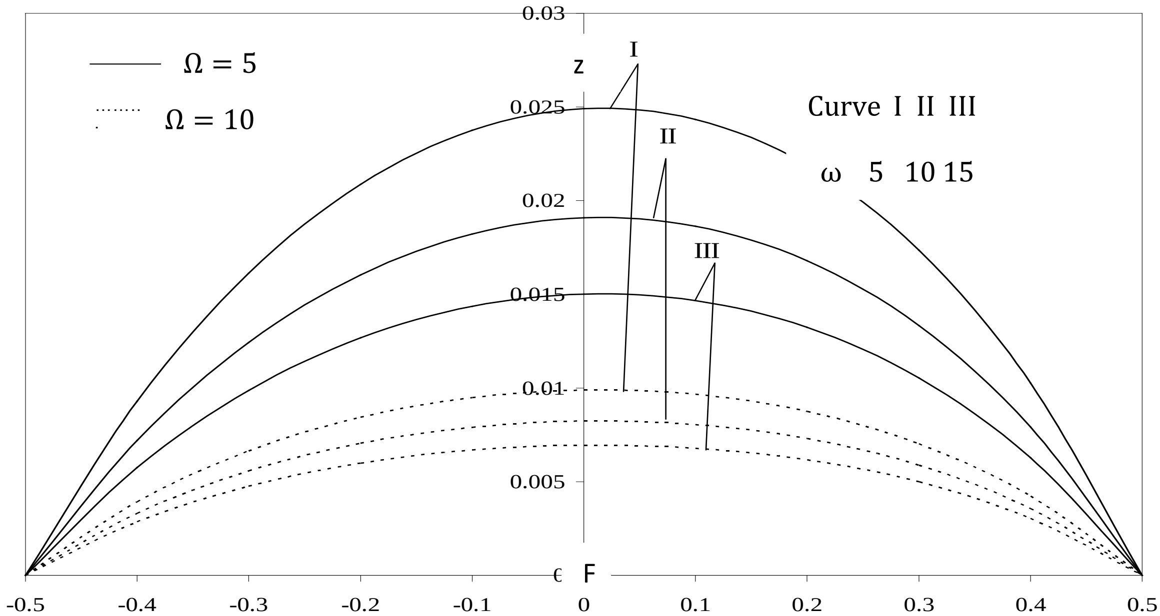

Fig. (11): Dust particles velocity variations for $R = 1$, $M = 2$, $\lambda = 0.5$, $A = 5$, $G = 1$ and $t = 0$

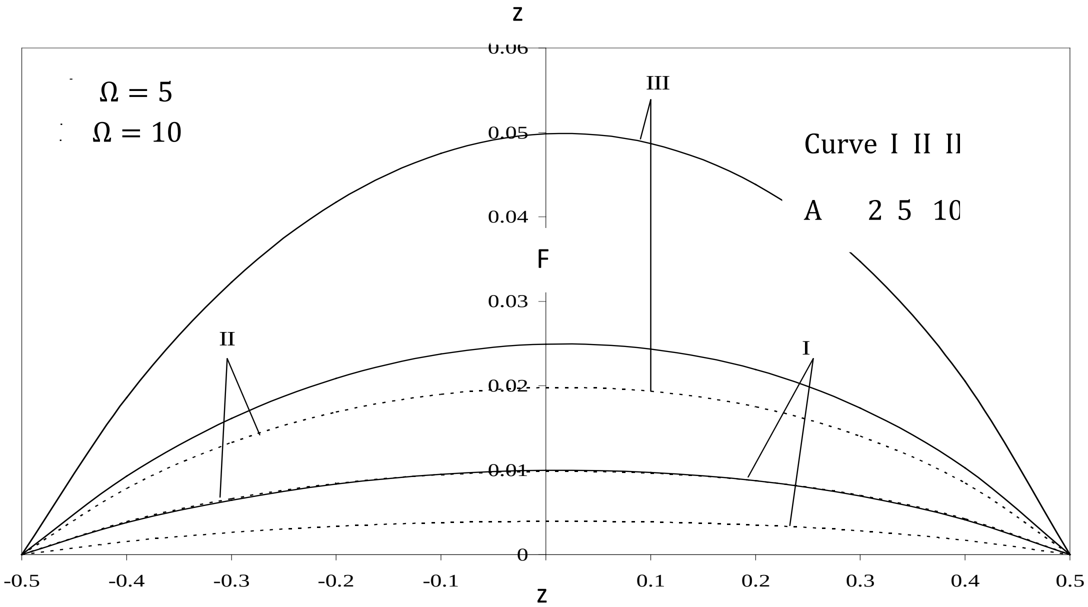

Fig. (12): Dust particles velocity variations for $R = 1$, $M = 2$, $\lambda = 0.5$, $\omega = 5$, $G = 1$ and $t = 0$.

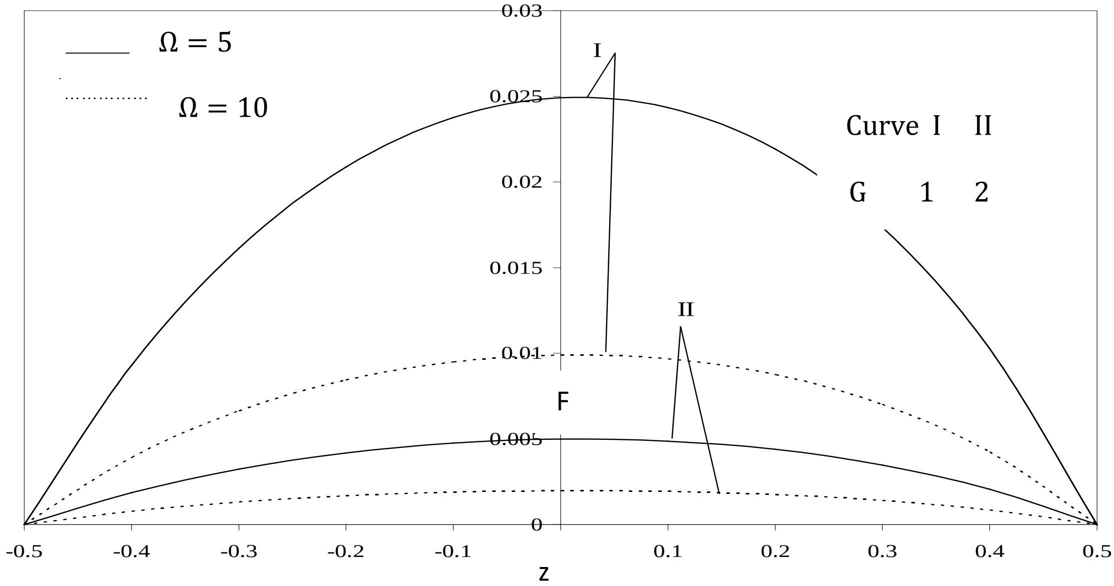

Fig. (13): Dust particles velocity variations for $R = 1$, $M = 2$, $\lambda = 0.5$, $\omega = 5$, $A = 5$ and $t = 0$.

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8. APPENDIX

$$m^2 = (R + M^2 - i(2\Omega + \lambda\omega)) - \frac{R}{(1+iG\,(2\Omega+\lambda\omega))},$$

$$m_1 = \frac{\lambda+\sqrt{\lambda^2+4m^2}}{2}, \quad m_2 = \frac{\lambda-\sqrt{\lambda^2+4m^2}}{2}.$$

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