

## On $w\alpha$ -Closed Sets in Topological Spaces

A. Jayalakshmi<sup>\*1</sup> & C. Janaki<sup>2</sup>

<sup>1</sup>Asst.Professor, Dept of Mathematics, Sree Narayana Guru College, Coimbatore (TN), India

<sup>2</sup>Asst.Professor, Dept of Mathematics, L. R. G Govt. Arts College for Women, Tirupur (TN), India

(Received on: 30-05-12; Revised & Accepted on: 19-06-12)

### ABSTRACT

In this paper, a new class of sets called  $w\alpha$ -closed set is introduced and their properties are studied. Moreover the notions of  $w\alpha$ -continuity and  $w\alpha$ -irresoluteness are introduced.

**Mathematics Subject Classification:** 54C10, 54A05.

**Keywords:**  $w\alpha$ -closed sets,  $w\alpha$ -open sets,  $w\alpha$ -continuous and  $w\alpha$ -irresolute.

### 1. Introduction

N. Levine [11] introduced generalized closed sets in general topology as a generalization of closed sets. Stone [21] introduced regular open sets. N. Levine [10,11], Cameron [5], Sundaram and Sheik john [20], Bhattacharyya and lahiri [4], Nagaveni [16], Palaniappan and Rao [18], Maki, Devi and Balachandran [12], J.K Park and J.H Park [19], S.S. Benchalli and R.S. Wali [3] introduced and investigated semi-open sets, generalized closed sets, weakly closed sets, semi generalized closed sets, weakly generalized closed sets, generalized pre-regular closed sets, generalized  $\alpha$ -closed sets,  $\alpha$ -generalized closed sets, mildly generalized closed sets and regular  $\omega$ -closed sets respectively. Regular  $\alpha$ -open sets and regular generalized  $\alpha$ -closed sets have been introduced and investigated by A. Vadivel and K. Vairamanickam [22]. In this paper, we define and study the properties of  $w\alpha$ -closed sets.

Throughout this paper, space  $(X, \tau)$  (or simply  $X$ ) always means a topological space on which no separation axioms are assumed unless explicitly stated. For a subset  $A$  of a space  $X$ ,  $cl(A)$ ,  $int(A)$  and  $A^c$  denote the closure of  $A$ , the interior of  $A$  and the complement of  $A$  in  $X$  respectively.

### 2. Preliminaries

**Definition 2.1:** A subset  $A$  of a space  $(X, \tau)$  is called

- (i) regular open [21] if  $A = int(cl(A))$  and regular closed if  $A = cl(int(A))$ .
- (ii) pre-open [14] if  $A \subset int(cl(A))$  and pre-closed if  $cl(int(A)) \subset A$ .
- (iii) semi-open [10] if  $A \subset cl(int(A))$  and semiclosed if  $int(cl(A)) \subset A$ .
- (iv)  $\alpha$ -open [17] if  $A \subset int(cl(int(A)))$  and  $\alpha$ -closed if  $cl(int(cl(A))) \subset A$ .
- (v) semi-pre open [2, 7] if  $A \subset cl(int(cl(A)))$  and a semi-preclosed if  $int(cl(int(A))) \subset A$ .
- (vi) regular  $\alpha$ -open [22] if there is a regular open set  $U$  such that  $U \subset A \subset \alpha cl(U)$ .
- (vii) regular semi open [5] if there is a regular open set  $U$  such that  $U \subset A \subset cl(U)$ .

**Definition 2.2:** A subset  $A$  of a space  $(X, \tau)$  is called

- (i) generalized closed set (briefly,  $g$ -closed) [11] if  $cl(A) \subset U$  whenever  $A \subset U$  and  $U$  is open in  $X$ .
- (ii) generalized  $\alpha$ -closed set (briefly,  $g\alpha$ -closed) [12] if  $\alpha cl(A) \subset U$  whenever  $A \subset U$  and  $U$  is  $\alpha$ -open in  $X$ .
- (iii)  $\alpha$ -generalized closed set (briefly,  $\alpha g$ -closed) [13] if  $\alpha cl(A) \subset U$  whenever  $A \subset U$  and  $U$  is open in  $X$ .
- (iv) regular generalized closed set (briefly,  $rg$ -closed) [18] if  $cl(A) \subset U$  whenever  $A \subset U$  and  $U$  is regular open in  $X$ .
- (v) generalized pre -regular closed set (briefly,  $gpr$ -closed) [19] if  $pcl(A) \subset U$  whenever  $A \subset U$  and  $U$  is regular open in  $X$ .

**Corresponding author:** A. Jayalakshmi<sup>\*1</sup>

<sup>1</sup>Asst.Professor, Dept of Mathematics, Sree Narayana Guru College, Coimbatore (TN), India

- (vi) weakly generalized closed set (briefly,  $\omega g$ -closed) [15,16] if  $cl(int(A)) \subset U$  whenever  $A \subset U$  and  $U$  is open in  $X$ .
- (vii) weakly closed sets (briefly,  $\omega$ -closed) [1,3] if  $cl(A) \subset U$  whenever  $A \subset U$  and  $U$  is semi open in  $X$ .
- (viii) semi weakly generalized closed set (briefly,  $s\omega g$ -closed) [16] if  $cl(int(A)) \subset U$  whenever  $A \subset U$  and  $U$  is semi-open in  $X$ .
- (ix) regular weakly generalized closed set (briefly,  $r\omega g$ -closed) [16] if  $cl(int(A)) \subset U$  whenever  $A \subset U$  and  $U$  is regular open in  $X$ .
- (x) regular generalized  $\alpha$ -closed set (briefly,  $rg\alpha$ -closed) [22] if  $\alpha cl(A) \subset U$  whenever  $A \subset U$  and  $U$  is regular  $\alpha$ -open in  $X$ .
- (xi) Semi-generalized closed (briefly,  $sg$ -closed) [4,6,8] if  $scl(A) \subset U$  whenever  $A \subset U$  and  $U$  is semi open in  $X$ .
- (xii) regular  $\omega$ -closed (briefly,  $r\omega$ -closed) [3] if  $cl(A) \subset U$  whenever  $A \subset U$  and  $U$  is regular semi open in  $X$ .
- (xiii) mildly generalized closed sets (briefly, mildly  $g$ -closed) [19] if  $cl(int(A)) \subset U$ , whenever  $A \subset U$  and  $U$  is  $g$ -open in  $X$ .

**Definitions 2.3:** A map  $f : (X, \tau) \rightarrow (Y, \sigma)$  is said to be

- (i) continuous [9] if  $f^{-1}(V)$  is closed in  $X$ , for every closed set  $V$  in  $Y$ .
- (ii)  $\omega$ -continuous [16] if  $f^{-1}(V)$  is  $\omega$ -closed in  $X$ , for every closed set  $V$  in  $Y$ .
- (iii)  $\alpha$ -continuous [3] if  $f^{-1}(V)$  is  $\alpha$ -closed in  $X$ , for every closed set  $V$  in  $Y$ .
- (iv)  $g\alpha$ -continuous [12] if  $f^{-1}(V)$  is  $g\alpha$ -closed in  $X$ , for every closed set  $V$  in  $Y$ .
- (v)  $rg\alpha$ -continuous [22] if  $f^{-1}(V)$  is  $rg\alpha$ -closed in  $X$ , for every closed set  $V$  in  $Y$ .
- (vi)  $s\omega g$ -continuous [3] if  $f^{-1}(V)$  is  $s\omega g$ -closed in  $X$ , for every closed set  $V$  in  $Y$ .
- (vii)  $r\omega g$ -continuous [3] if  $f^{-1}(V)$  is  $r\omega g$ -closed in  $X$ , for every closed set  $V$  in  $Y$ .

### 3. $wg\alpha$ -closed sets in topological spaces

**Definition 3.1:** A subset  $A$  of a space  $(X, \tau)$  is called  $wg\alpha$ -closed if  $cl(int(A)) \subset U$  whenever  $A \subset U$  and  $U$  is regular  $\alpha$ -open in  $(X, \tau)$ .

The complement of the  $wg\alpha$ -closed set is  $wg\alpha$ -open set.

We denote the set of all  $wg\alpha$ -closed sets in  $(X, \tau)$  by  $WGR\alpha C(X)$  and  $wg\alpha$ -open sets in  $(X, \tau)$  by  $WGR\alpha O(X)$

**Theorem 3.2:**

1. Every  $\omega$ -closed set is  $wg\alpha$ -closed
2. Every  $\alpha$ -closed set is  $wg\alpha$ -closed
3. Every  $g\alpha$ -closed set is  $wg\alpha$ -closed
4. Every  $rg\alpha$ -closed set is  $wg\alpha$ -closed
5. Every  $s\omega g$ -closed set is  $wg\alpha$ -closed
6. Every  $wg\alpha$ -closed set is  $r\omega g$ -closed

**Proof:** straight forward.

**Remark 3.3:** Converse of the above need not be true as shown in the following examples.

**Example 3.4:** Consider  $X = \{a, b, c, d, e\}$  and  $\tau = \{X, \phi, \{a\}, \{d\}, \{e\}, \{a, d\}, \{d, e\}, \{a, e\}, \{a, d, e\}\}$   
Let  $A = \{b\}$ .  $A$  is  $wg\alpha$ -closed, but it is not  $\omega$ -closed.

**Example 3.5:** Consider  $X = \{a, b, c, d, e\}$  and  $\tau = \{X, \phi, \{a\}, \{d\}, \{e\}, \{a, d\}, \{d, e\}, \{a, e\}, \{a, d, e\}\}$   
Let  $A = \{a, d, e\}$ .  $A$  is  $wg\alpha$ -closed, but it is not  $\alpha$ -closed.

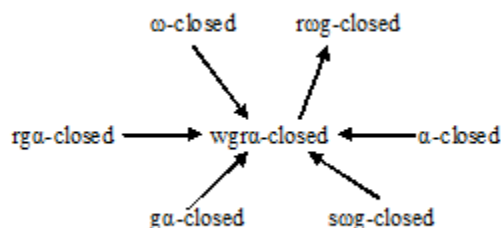
**Example 3.6:** Consider  $X = \{a, b, c, d\}$  and  $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ . Let  $A = \{a, b, d\}$ .  $A$  is  $wg\alpha$ -closed, but it is not  $g\alpha$ -closed.

**Example 3.7:** Consider  $X = \{a, b, c, d\}$  and  $\tau = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$ . Let  $A = \{c\}$ .  $A$  is  $wg\alpha$ -closed, but it is not  $rg\alpha$ -closed.

**Example 3.8:** Consider  $X = \{a, b, c, d\}$  and  $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ . Let  $A = \{a, b, c\}$ .  $A$  is  $wg\alpha$ -closed, but it is not  $sg\alpha$ -closed.

**Example 3.9:** Consider  $X = \{a, b, c, d\}$  and  $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ . Let  $A = \{b, c\}$ .  $A$  is  $r\omega g$ -closed, but it is not  $wg\alpha$ -closed.

**Remark 3.10:** The above discussions are summarized in the following diagram.



**Remark 3.11:** Union of two  $wg\alpha$ -closed sets need not be  $wg\alpha$ -closed.

**Example 3.12:** Let  $X = \{a, b, c, d\}$ ,  $\tau = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$ .  $A = \{c\}$  and  $B = \{d\}$  are  $wg\alpha$ -closed sets. But  $A \cup B$  is not  $wg\alpha$ -closed.

**Remark 3.13:** Intersection of two  $wg\alpha$ -closed sets need not be  $wg\alpha$ -closed.

**Example 3.14:** Let  $X = \{a, b, c, d\}$ ,  $\tau = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$ .  $A = \{a, b\}$  and  $B = \{a, d\}$  are  $wg\alpha$ -closed. But  $A \cap B$  is not  $wg\alpha$ -closed.

**Remark 3.15:** The following example shows that  $wg\alpha$ -closed sets are independent of mildly  $g$ -closed sets, generalized closed sets,  $\omega g$ -closed sets, semi closed sets,  $\alpha g$ -closed sets,  $sg$ -closed sets and  $r\omega$ -closed sets.

**Example 3.16:** Let  $X = \{a, b, c, d\}$  and  $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ . Then

- (i)  $wg\alpha$ -closed sets in  $(X, \tau)$  are  
 $\{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$
- (ii) mildly  $g$ -closed sets in  $(X, \tau)$  are  
 $\{X, \phi, \{d\}, \{c, d\}, \{a, d\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}\}$
- (iii) generalized closed sets in  $(X, \tau)$  are  
 $\{X, \phi, \{d\}, \{c, d\}, \{a, d\}, \{b, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$
- (iv)  $\omega g$ -closed sets in  $(X, \tau)$  are  
 $\{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, d\}, \{b, d\}, \{a, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$
- (v) semi-closed sets in  $(X, \tau)$  are  
 $\{X, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, c\}, \{b, c\}, \{a, c, d\}, \{b, c, d\}\}$
- (vi)  $\alpha g$ -closed sets in  $(X, \tau)$  are  
 $\{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, d\}, \{b, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$
- (vii)  $sg$ -closed sets in  $(X, \tau)$  are  
 $\{X, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{c, d\}, \{b, c\}, \{a, d\}, \{b, d\}, \{a, c\}, \{a, c, d\}, \{b, c, d\}\}$
- (viii)  $r\omega$ -closed sets in  $(X, \tau)$  are  
 $\{X, \phi, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$

**Theorem 3.17:** Let  $A$  be  $wg\alpha$ -closed in  $(X, \tau)$ , then  $cl(int(A)) - A$  does not contain any non-empty regular  $\alpha$ -open set.

**Proof:** Let  $F$  be a non-empty regular- $\alpha$  open set such that  $F \subset cl(int(A)) - A$ . Then  $F \subset X - A \Rightarrow A \subset X - F$ ,  $X - F$  is regular  $\alpha$ -open. Since  $A$  is  $wg\alpha$ -closed,  $cl(int(A)) \subset X - F$ . Therefore  $F \subset cl(int(A)) \cap X - cl(int(A))$ , which implies  $F = \phi$ , which is a contradiction. Hence  $cl(int(A)) - A$  does not contain any non-empty regular  $\alpha$ -open set.

**Remark 3.18:** Converse of the above theorem need not be true as shown in the following example.

**Example 3.19:** Let  $X = \{a, b, c, d, e\}$ ,  $\tau = \{X, \phi, \{a\}, \{d\}, \{e\}, \{a, d\}, \{a, e\}, \{d, e\}, \{a, d, e\}\}$ . Let  $A = \{a, b\}$ .  $cl(int(A)) - A = \{c\}$ , which is not regular  $\alpha$ -open. But  $A = \{a, b\}$  is not  $wg\alpha$ -closed.

**Corollary 3.20:** A subset  $A$  of  $X$  is  $wg\alpha$ -closed set in  $X$ , then  $cl(int(A)) - A$  does not contain any non-empty regular open set in  $X$ , but not conversely.

**Proof:** Follows from theorem 3.17 and the fact that every regular open set is regular  $\alpha$ -open.

**Example 3.21:** Let  $X = \{a, b, c, d\}$ ,  $\tau = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$ . Let  $A = \{a\}$ .  $cl(int(A)) - A = \{b\}$ , which is not regular open set. But  $A$  is not  $wg\alpha$ -closed.

**Corollary 3.22:** A subset  $A$  of  $X$  is  $wg\alpha$ -closed set in  $X$ , then  $cl(int(A)) - A$  does not contain any non-empty regular closed set in  $X$ , but not conversely.

**Proof:** Follows from theorem 3.17 and the fact that every regular open set is regular  $\alpha$ -open.

**Example 3.23:** Let  $X = \{a, b, c, d\}$ ,  $\tau = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$ . Let  $A = \{c, d\}$ ,  $cl(int(A)) - A = \{b\}$ , which is not regular closed. But  $A$  is not  $wg\alpha$ -closed.

**Theorem 3.24:** For an element  $x \in X$ , then the set  $X - \{x\}$  is  $wg\alpha$ -closed or regular  $\alpha$ -open.

**Proof:** Suppose  $X - \{x\}$  is not regular  $\alpha$ -open set. Then  $X$  is the only regular  $\alpha$ -open set containing  $X - \{x\} \Rightarrow cl(int(\{X - \{x\}\})) \subset X$ . Therefore  $X - \{x\}$  is  $wg\alpha$ -closed.

**Theorem 3.25:**  $A$  is  $wg\alpha$ -closed subset of  $X$  such that  $A \subset B \subset cl(int(A))$ , then  $B$  is  $wg\alpha$ -closed set in  $X$ .

**Proof:** If  $A$  is  $wg\alpha$ -closed subset of  $X$  such that  $A \subset B \subset cl(int(A))$ . Let  $U$  be regular  $\alpha$ -open set of  $X$  such that  $B \subset U$ , then  $A \subset U$ . Since  $A$  is  $wg\alpha$ -closed,  $cl(int(A)) \subset U$ . Now  $cl(int(B)) \subset cl(int(cl(int(A)))) = cl(int(A)) \subset U$ . Thus  $B$  is  $wg\alpha$ -closed set in  $X$ .

**Remark 3.26:** Converse of the above theorem need not be true.

**Example 3.27:** Let  $X = \{a, b, c\}$ ,  $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ . Here  $A = \{a\}$ ,  $B = \{a, c\}$  with  $A \subset B \subset cl(int(A))$ ,  $B$  is  $wg\alpha$ -closed in  $X$ , but  $A$  is not  $wg\alpha$ -closed in  $X$ .

**Theorem 3.28:** If  $A$  is  $wg\alpha$ -closed and regular open, then  $A$  is rg-closed.

**Proof:** Let  $A \subset U$  and  $U$  be regular open. By hypothesis,  $cl(A) \subset U$ . Therefore  $A$  is rg-closed.

**Theorem 3.29:** Let  $A$  be  $wg\alpha$ -closed in  $(X, \tau)$ , then  $A$  is regular closed set iff  $cl(int(A)) - A$  is regular  $\alpha$ -open.

**Proof:** Suppose  $A$  is regular closed in  $X$ . Then  $cl(int(A)) = A$  and so  $cl(int(A)) - A = \phi$ , which is regular  $\alpha$ -open in  $X$ . Conversely, Suppose  $cl(int(A)) - A$  is a regular  $\alpha$ -open in  $X$ . Then  $cl(int(A)) - A = \phi$ . Hence  $A$  is regular closed set in  $X$ .

#### 4. $wg\alpha$ -open sets

**Theorem 4.1:** A subset  $A$  of a topological space  $X$  is  $wg\alpha$ -open iff  $F \subset int(cl(A))$ , whenever  $F$  is regular  $\alpha$ -open and  $F \subset A$ .

**Proof:** Assume  $A$  is  $wg\alpha$ -open,  $A^c$  is  $wg\alpha$ -closed. Let  $F$  be a regular  $\alpha$ -open set in  $X$  contained in  $A$ .  $F^c$  is a regular  $\alpha$ -open set in  $X$  containing  $A^c$ . Since  $A^c$  is  $wg\alpha$ -closed,  $cl(int(A^c)) \subset F^c$ . Therefore  $F \subset int(cl(A))$ .

Conversely, let  $F \subset int(cl(A))$ , whenever  $F \subset A$  and  $F$  is regular  $\alpha$ -open in  $X$ . Let  $G$  be a regular  $\alpha$ -open set containing  $A^c$ , then  $G^c \subset int(cl(A))$ . Thus  $cl(int(A^c)) \subset G \Rightarrow A^c$  is  $wg\alpha$ -closed  $\Rightarrow A$  is  $wg\alpha$ -open.

**Theorem 4.2:** If  $A \subset X$  is  $wg\alpha$ -closed, then  $cl(int(A)) - A$  is  $wg\alpha$ -open.

**Proof:** Let  $A$  be  $wg\alpha$ -closed and  $F$  be regular  $\alpha$ -open.  $F \subset cl(int(A)) - A$ , then  $int(cl((cl(int(A)) - A))) = \phi$ . Thus  $F \subset int(cl((cl(int(A)) - A)))$ . Therefore  $cl(int(A)) - A$  is  $wg\alpha$ -open.

**Remark 4.3:** Converse of the above theorem need not be true as shown in the following example.

**Example 4.4:** Let  $X = \{a, b, c, d\}$ ,  $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ . Let  $A = \{a, c\}$ .  $cl(int(A)) - A = \{d\}$  is  $wg\alpha$ -open. But  $A$  is not  $wg\alpha$ -closed.

**Theorem 4.5:** If  $int(cl(A)) \subset B \subset A$  and  $A$  is  $wg\alpha$ -open, then  $B$  is  $wg\alpha$ -open.

**Proof:** Let  $int(cl(A)) \subset B \subset A$ . Thus  $X-A \subset X-B \subset cl(int(X-A))$ . Since  $X-A$  is  $wg\alpha$ -closed, by theorem 3.25,  $X-B$  is  $wg\alpha$ -closed.

**Theorem 4.6:** If  $A$  is both regular open and  $wg\alpha$ -closed then it is clopen.

**Proof:** straight forward.

### 5. $wg\alpha$ -continuous and irresolute mappings.

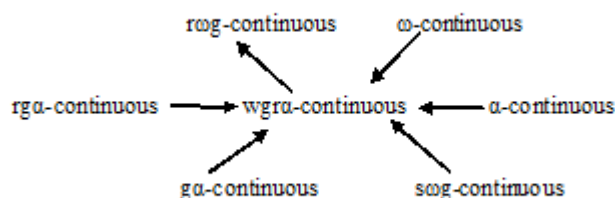
**Definition 5.1:** A map  $f:(X, \tau) \rightarrow (Y, \sigma)$  is said to be  $wg\alpha$ -continuous if  $f^{-1}(V)$  is  $wg\alpha$ -closed in  $X$ , for every closed set  $V$  in  $Y$ .

**Example 5.2:** Let  $X = \{a, b\}$ ,  $\tau = \{X, \emptyset, \{a\}, \{b\}\}$  and  $\sigma = \{X, \emptyset, \{a\}\}$ . Define  $f:(X, \tau) \rightarrow (X, \sigma)$  by  $f(a)=b$  and  $f(b)=a$ . Since every subset of  $(X, \tau)$  is  $wg\alpha$ -closed,  $f$  is  $wg\alpha$ -continuous.

**Definition 5.3:** A map  $f:(X, \tau) \rightarrow (Y, \sigma)$  is said to be  $wg\alpha$ -irresolute if  $f^{-1}(V)$  is  $wg\alpha$ -closed in  $X$ , for every  $wg\alpha$ -closed set  $V$  in  $Y$ .

**Example 5.4:** Let  $X = \{a, b, c\}$ ,  $\tau = \{X, \emptyset, \{a, b\}\}$  and  $\sigma = \{X, \emptyset, \{a, b\}, \{c\}\}$ . Define  $f:(X, \tau) \rightarrow (X, \sigma)$  by  $f(a)=c$ ,  $f(c)=a$  and  $f(b)=b$ .  $f$  is  $wg\alpha$ -irresolute.

The above results are summarized in the following diagram.



**Remark 5.5:** Every  $wg\alpha$ -irresolute function is  $wg\alpha$ -continuous, but not conversely.

**Example 5.6:** Let  $X = \{a, b, c\}$ ,  $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$  and  $\sigma = \{X, \emptyset, \{a\}\}$ . Define  $f:(X, \tau) \rightarrow (X, \sigma)$  by  $f(a)=a$ ,  $f(b)=b$  and  $f(c)=c$ . Here  $f$  is  $wg\alpha$ -continuous, but not  $wg\alpha$ -irresolute.

**Theorem 5.7:**

1. Every  $\omega$ -continuous is  $wg\alpha$ -continuous
2. Every  $g\alpha$ -continuous is  $wg\alpha$ -continuous
3. Every  $\alpha$ -continuous is  $wg\alpha$ -continuous
4. Every  $s\omega g$ -continuous is  $wg\alpha$ -continuous
5. Every  $rg\alpha$ -continuous is  $wg\alpha$ -continuous
6. Every  $wg\alpha$ -continuous is  $r\omega g$ -continuous

**Proof:** straight forward.

**Remark 5.8:** Converse of the above need not be true.

**Example 5.9:** Let  $X = \{a, b, c, d\}$ ,  $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$  and  $\sigma = \{X, \emptyset, \{b\}, \{c\}, \{b, c\}\}$ . Define  $f:(X, \tau) \rightarrow (X, \sigma)$  by  $f(a) = b$ ,  $f(b) = c$ ,  $f(c) = d$ ,  $f(d) = a$ .  $f$  is  $wg\alpha$ -continuous, but it is not  $\omega$ -continuous.

**Example 5.10:** Let  $X = \{a, b, c, d, e\}$ ,  $\tau = \{X, \emptyset, \{a\}, \{d\}, \{e\}, \{a, d\}, \{a, e\}, \{d, e\}\}$  and  $\sigma = \{X, \emptyset, \{c\}, \{c, d\}\}$ . Define  $f:(X, \tau) \rightarrow (X, \sigma)$  by  $f(a)=b$ ,  $f(b)=c$ ,  $f(c)=d$ ,  $f(d)=e$ ,  $f(e)=a$ .  $f$  is  $wg\alpha$ -continuous, but it is not  $g\alpha$ -continuous.

**Example 5.11:** Let  $X=\{a, b, c\}, \tau =\{X, \phi, \{a\}, \{c\}, \{a, c\}\}$  and  $\sigma =\{X, \phi, \{c\}, \{b, c\}\}$ . Define  $f:(X, \tau) \rightarrow (X, \sigma)$  by  $f(a)=a, f(b)=b, f(c)=c$ .  $f$  is  $w\alpha$ -continuous, but it is not  $\alpha$ -continuous.

**Example 5.12:** Let  $X=\{a, b, c, d\}, \tau =\{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$  and  $\sigma =\{X, \phi, \{d\}, \{a, c, d\}\}$ . Define  $f:(X, \tau) \rightarrow (X, \sigma)$  by  $f(a) = a, f(b) = c, f(c) = b, f(d)=d$ .  $f$  is  $w\alpha$ -continuous, but it is not  $\omega\alpha$ -continuous.

**Example 5.13:** Let  $X=\{a, b, c, d\}, \tau =\{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$  and  $\sigma =\{X, \phi, \{a, d\}, \{a, b, d\}\}$ . Define  $f:(X, \tau) \rightarrow (X, \sigma)$  by  $f(a) = b, f(b) = a, f(c) = c, f(d) = d$ .  $f$  is  $w\alpha$ -continuous, but it is not  $rg\alpha$ -continuous.

**Example 5.14:** Let  $x = \{a, b, c, d\}, \tau = \{\phi, x, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$  and  $\sigma = \{\phi, x, \{a, c\}, \{a, c, d\}\}$ . Define  $f:(X, \tau) \rightarrow (X, \sigma)$  by  $f(a) , f(b)=c, f(c)=b, f(d)=d$ .  $f$  is  $\omega\alpha$ -continuous, but it is not  $\omega\alpha$ -continuous.

**Remark 5.15:** The composition of two  $\omega\alpha$ -continuous functions need not be  $\omega\alpha$ -continuous.

**Example 5.16:** Let  $X=\{a, b, c\}, \tau =\{X, \phi, \{a\}, \{b\}, \{a, b\}\}, \sigma =\{X, \phi, \{a\}, \{a, b\}\}$  and  $\eta = \{X, \phi, \{a\}, \{a, c\}, \{a, b\}\}$ . Define  $f:(X, \tau) \rightarrow (X, \sigma)$  by  $f(a)=b, f(b)=a, f(c)=c$ . Define  $g:(X, \sigma) \rightarrow (X, \eta)$  by  $g(a)=a, g(b)=c, g(c)=b$ ,  $f$  and  $g$  are  $\omega\alpha$ -continuous, but  $g \circ f$  is not  $\omega\alpha$ -continuous.

**Theorem 5.17:** Let  $f:(X, \tau) \rightarrow (Y, \sigma)$  and  $g:(Y, \sigma) \rightarrow (Z, \eta)$  be any two maps . Then

- (i)  $g \circ f$  is  $w\alpha$ -continuous, if  $g$  is continuous and  $f$  is  $w\alpha$ -continuous
- (ii)  $g \circ f$  is  $w\alpha$ -irresolute, if  $g$  is  $w\alpha$ -irresolute and  $f$  is  $w\alpha$ -irresolute
- (iii)  $g \circ f$  is  $w\alpha$ -continuous, if  $g$  is  $w\alpha$ -continuous and  $f$  is  $w\alpha$ -irresolute

**Proof:**

(i) Let  $V$  be any closed set in  $(Z, \eta)$ . Then  $g^{-1}(V)$  is closed in  $(Y, \sigma)$ , since  $g$  is continuous. By hypothesis,  $f^{-1}(g^{-1}(V))$  is  $w\alpha$ -closed in  $(X, \tau)$ . Hence  $g \circ f$  is  $w\alpha$ -continuous.

(ii) Let  $V$  be  $w\alpha$ -closed set in  $(Z, \eta)$ . Since  $g$  is  $w\alpha$ -irresolute,  $g^{-1}(V)$  is  $w\alpha$ -closed in  $(Y, \sigma)$ . As  $f$  is  $w\alpha$ -irresolute,  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is  $w\alpha$ -closed in  $(X, \tau)$ . Hence  $g \circ f$  is  $w\alpha$ -irresolute.

(iii) Let  $V$  be closed in  $(Z, \eta)$ . Since  $g$  is  $w\alpha$ -continuous,  $g^{-1}(V)$  is  $w\alpha$ -closed in  $(Y, \sigma)$ . As  $f$  is  $w\alpha$ -irresolute,  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is  $w\alpha$ -closed in  $(X, \tau)$ . Hence  $g \circ f$  is  $w\alpha$ -continuous.

## References

- [1] Ahmad Al-Omari and Salmi Md Noorani, Regular generalized  $\omega$ -closed sets, International Journal of Mathematics and Mathematical Sciences, Volume 2007, article(ID),16292.
- [2] D. Andrijevic, Semi-preopen sets, Mat. Vesnik 38(1986), 24-32.
- [3] S.S.Benchalli and R.S.Wali, On  $rw$ -closed sets in topological spaces, Bull. Malays. Math. Sci. Soc.(2) 30(2) (2007), 99-110
- [4] P. Bhattacharya and B.K. Lahiri, Semi-generalized closed sets in topology, Indian J. Math. 29 (1987), 376-382.
- [5] D. E. Cameron, Properties of  $S$ -closed spaces, Proc. Amer Math. Soc. 72(1978), 581-586
- [6] G. Di Maio and Noiri, On  $S$ -closed spaces, Indian J. Pure Appl. Math. 18(3)(1987), 226-233.
- [7] J. Dontchev, On generalizing semi-preopen sets, Mem. Fac Sci. Kochi. Univ. Ser. A. Math. 16 (1995), 35-48.
- [8] J. Dontchev and H. Maki, On  $sg$ -closed sets semi- $\lambda$  closed sets, Questions and Answers Gen. Topology, 15(1997), 253-266.
- [9] Y. Gnanambal, On generalized preregular closed sets in topological spaces, Indian j. pure App. Math. 28 (1997), 351-360.
- [10] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70(1963), 36-41.
- [11] N. Levine, Generalized Closed sets in topology, Rend. Circ. Mat. Palermo 19(1970), 89-96
- [12] H. Maki, R. Devi and K. Balachandran, Associated topologies of generalized  $\alpha$ -closed sets and  $\alpha$ -generalized closed sets, Mem. Sci. Kochi Univ. Ser. A. Math. 15(1994), 51-63.

- [13] H. Maki, R. Devi and K. Balachandran, Generalized  $\alpha$ -closed sets in topology, Bull. Fukuoka Univ. Ed. Part-III 42(1993), 13-21.
- [14] A.S.Mashhour, M.E.Abd. El-Monsef and S. NEI-Deeb, On pre continuous mappings and weak pre-continuous mappings, proc. Math. Phys. Soc. Egypt 53(1982), 47-53.
- [15] C. Mukundhan and N. Nagaveni, A Weaker Form of a Generalized closed set, Int. J. Contemp. Math. Sciences, Vol. 6, 2011, no.20, 949-961.
- [16] N. Nagaveni, studies on generalizations of Homeomorphisms in topological spaces, Ph.D. Thesis, Bharathiar University, Coimbatore, 1999.
- [17] O.Njastad, On some classes of nearly open sets, Pacific J. Math. 15(1965), 961-970.
- [18] N.Palaniappan and K.C.Rao, Regular generalized closed sets, Kyungpook Math. J. 33 (1993), 211-219.
- [19] J. K. Park and J.H. Park, Mildly generalized closed sets, almost normal and mildly normal spaces, Chaos, Solitons and Fractals 20(2004), 1103-1111.
- [20] P. Sundaram and M. Sheik John, On w-closed sets in topology, Acta Ciencia Indica 4(2000), 389-392
- [21] M.Stone, Applications of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc , 41(1937),374-481.
- [22] A. Vadivel and K. Vairamanickam, rg $\alpha$ -closed sets and rg $\alpha$ -open sets in topological spaces, Int. Journal of Math. Analysis, Vol.3, 2009, no.37, 1803-1819.

**Source of support: Nil, Conflict of interest: None Declared**