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APPLICATION OF THE MELLIN TYPE INTEGRAL TRANSFORM IN THE RANGE [0, 1/a]

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ABSTRACT

In this paper Laplace operators are used to solve the Mellin Type Integral Transform which can be a technique for solving boundary and initial value problems. This Transforms is applicable in finite interval. This work intends to understand how Laplace operators leads to properties and relations with the Mellin Type Integral Transform. The main aim of this work is to find the relation between Laplace transform and the Mellin Type Integral Transform in [0,1/a], the result have been modified by applying suitable functions which leads to the results of Mellin Type Integral Transform in the interval 0≤t≤1/a. We illustrate the advantages and use of this transformation Cauchy's Differential Equation have been solved. We have also studied graphical representation of Mellin Type Integral Transform using MATLAB.

Keywords: *Laplace Transform, Mellin Transform, Integral Transform, Finite Transform.*

AMS Mathematical Classification: *65D32,33C90,26D07,26D99(2000),41A60,44A35.*

1. INTRODUCTION

The Laplace Transform is used to study the properties of Mellin Type Integral Transform in the range 0 to 1/a and also to show the validity of properties like Linear Property, Scaling Property, Power Property, theorems like Inversion Theorem, Convolution Theorem, Parseval's Theorem, First and Second Shifting Theorems. We obtain the Mellin Type Integral Transform of the n^{th} order derivative of f(t) with respect to t and Cauchy's Linear Differential equation $L_2(F(t)) = t^2 f''(t) + t f'(t) + f(t) = 0$ is solved by using this Integral Transform. The solution of differential equation is graphically presented by MATLAB.

2. PRELIMINARY RESULTS

Let f(x) be a given function of x which is defined for all x ≥0 and 's' is parameter, then Laplace

Transform is $L[f(x)] = \int_0^\infty e^{-sx} f(x)dx$ (1)

Considring x=log (1/at) then $L[f(t)] = \int_0^{1/a} a^s t^{s-1} f(t)dt$, denoted by M [f (t),s,0,1/a]

$$M[f(t), s, 0, 1/a] = \int_0^{1/a} a^s t^{s-1} f(t)dt \quad (2)$$

Thus obtained is a The Mellin Type Integral Transform in [0, 1/a], a>0

3. PROPERTIES

3.1 LINEAR PROPERTY

The Mellin Type Integral Transform is a Linear operator, that is for any functions f(t) and g(t), We have

$$M\left[\alpha f(t) + \beta g(t), s, 0, \frac{1}{a}\right] = \alpha M\left[f(t), s, 0, \frac{1}{a}\right] + \beta M\left[f(t), s, 0, \frac{1}{a}\right] \quad (3)$$

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3.2 SCALING PROPERTY

The Mellin Type Integral Transform in [0, 1/a] is

$M\left[f(t),s,0,\frac{1}{a}\right]=\int_0^{1/a} a^s\ t^{s-1}f(t)dt$

Then $M\left[f(bt),s,0,\frac{1}{a}\right]=(b^{-s})\int_0^{1/a} a^s\ t^{s-1}f(t)dt=(b^{-s})M[f(t),s,0,\frac{b}{a}]$ (4)

3.3 POWER PROPERTY

The Mellin Type Integral Transform in [0, 1/a] is

$M\left[f(t),s,0,\frac{1}{a}\right]=\int_0^{1/a} a^s\ t^{s-1}f(t)dt$

Then $M\left[f(t^b),s,0,\frac{1}{a}\right]=\frac{1}{b}\ M[f(t),\frac{s}{b}\ ,0,\frac{1}{a^b}]$ (5)

4. MAIN RESULTS

4.1 INVERSION THEOREM

Theorem: The Mellin Type Integral Transform in [0, 1/a] is

$M\left[f(t),s,0,\frac{1}{a}\right]=\int_0^{1/a} a^s\ t^{s-1}f(t)dt$

Then $f(t)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{t^{-s}}{s}M\left[f(t),s,0\ ,\frac{1}{a}\right]ds$

Proof: Assume that $M\left[f(t),s,0,\frac{1}{a}\right]$ is a regular equation in the strip $|Re(s)|<r$ ('r' to real number) of the s-plane and that 0 < c < v, c-I ∞ ≤ s≤ c + i∞, where c is constant then

$$M\left[f(t),s,0,\frac{1}{a}\right]=\int_0^{1/a} a^s\ t^{s-1}f(t)dt$$

$$=\int_0^{1/a} a^s\, t^{s-1}\left[\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} x^{-s}\, M\left[f(t),s,0,\frac{1}{a}\right]ds\right]dt$$

$$=\left[\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} x^{-s}\, M\left[f(t),s,0,\frac{1}{a}\right]ds\right]\int_0^{1/a} a^s\, t^{s-1}dt$$

$$=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{x^{-s}}{s}M\left[f(t),s,0,\frac{1}{a}\right]ds$$

$$f(x)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{x^{-s}}{s}M\left[f(t),s,0,\frac{1}{a}\right]ds \quad (6)$$

4.2 CONVOLUTION THEOREM

Theorem: The Mellin Type Integral Transform in [0, 1/a] is

$M\left[f(t),s,0,\frac{1}{a}\right]=\int_0^{1/a} a^s\ t^{s-1}f(t)dt$ and $M\left[g(t),s,0,\frac{1}{a}\right]=\int_0^{1/a} a^s\ t^{s-1}g(t)dt$

Then $M^{-1}\left[f(t),s,0,\frac{1}{a}\right]=f(t)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{x^{-s}}{s}M\left[f(t),s,0,\frac{1}{a}\right]ds$

$M^{-1}\left[g(t),s,0,\frac{1}{a}\right]=g(t)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{x^{-s}}{s}M\left[g(t),s,0,\frac{1}{a}\right]ds$

$$M\left[f(t)g(x-t),s,0,\frac{1}{a}\right]=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{x^{-s}}{s}M\left[f(t),s,0,\frac{1}{a}\right]M\left[g(x-t),s,\frac{1}{a}-t\ ,t\right]ds \quad (7)$$

4.3 PARSEVALS THOREM

Theorem: The Mellin Type Integral Transform in [0, 1/a] is

$M\left[f(t), s, 0, \frac{1}{a}\right] = \int_0^{1/a} a^s\ t^{s-1} f(t)dt$ and $M\left[g(t), s, 0, \frac{1}{a}\right] = \int_0^{1/a} a^s\ t^{s-1} g(t)dt$

Then $M^{-1}\left[f(t), s, 0, \frac{1}{a}\right] = f(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \frac{x^{-s}}{s} M\left[f(t), s, 0, \frac{1}{a}\right] ds$

$M^{-1}\left[g(t), s, 0, \frac{1}{a}\right] = g(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \frac{x^{-s}}{s} M\left[g(t), s, 0, \frac{1}{a}\right] ds$

$$M\left[f(t)g(t), s, 0, \frac{1}{a}\right] = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \frac{x^{-s}}{s} M\left[f(t), s, 0, \frac{1}{a}\right] M\left[g(t), s, \frac{1}{a} - t\ , t\right] ds \quad (8)$$

4.4 DEFINITIONS

Heviside Unit Step Function

For U(t-a)=H(t-a) = 1, when t > a

= 0, when t < a

U(t-a) or H(t-a) is known as the Heviside Unit Step Function.

4.5 FIRST SHIFTING THEOREM

The Mellin Type Integral Transform in [0, 1/a] is

$M\left[f(t), s, 0, \frac{1}{a}\right] = \int_0^{1/a} a^s\ t^{s-1} f(t)dt$

$$\text{Then } M\left[t^n f(t), s, 0, \frac{1}{a}\right] = M[f(t), s+n, 0\ , \frac{1}{a}] \quad (9)$$

4.6 SECOND SHIFTING THEOREM

The Mellin Type Integral Transform in [0, 1/a] is

$M\left[f(t), s, 0, \frac{1}{a}\right] = \int_0^{1/a} a^s\ t^{s-1} f(t)dt$

Then $M\left[f(t-b)U(t-b), s, 0, \frac{1}{a}\right] = M[f(u), s, -b, \frac{1}{a} - b]$

Proof: $M\left[f(t-b)U(t-b), s, 0, \frac{1}{a}\right] = \int_0^{1/a} a^s\ t^{s-1} f(t-b)U(t-b)dt$

Let u=t-b then $= \int_{-b}^{\frac{1}{a}-b} a^s (u+b)^{s-1} f(u)U(u)du$

If U(t-b)=1, when t>b $= \int_{-b}^{\frac{1}{a}-b} a^s (u+b)^{s-1} f(u)du$

$$= M[f(u), s, -b, \frac{1}{a} - b] \quad \text{where kernel is } (u+b)^{s-1} \quad (10)$$

5. DERIVATIVES

5.1 The Mellin Type Integral Transforms of first order derivative of f(t) w.r.t. t

Suppose that f(t) is continuous for all t ≥ 0 satisfying (2) for some value γ and m and it has derivative $f'(t)$ which is piecewise continuous on every finite interval in the range of t ≥ 0 . Then the Mellin Type (finite) Integral Transforms of the derivative of f(t) exists when $s > \gamma$ and $|f(t)| \leq m\ e^{\gamma t}$ for all t ≥ 0 for some constants.

Proof: Considering the case when $f'(t)$ is continuous for all t≥0, then integrating by parts, we get

$$M\left[f'(t), s, 0, \frac{1}{a}\right] = \int_0^{1/a} a^s\ t^{s-1} f'(t)dt$$

$$= af\left(\frac{1}{a}\right) - (s-1)M[f(t), s-1, 0, \frac{1}{a}] \quad (11)$$

5.2 The Mellin Type Integral Transforms of n^{th} order derivative of f(t) w.r.t. t

$$M\left[f''(t), s, 0, \frac{1}{a}\right] = \int_0^{1/a} a^s\ t^{s-1} f''(t)dt$$

$$= af'\left(\frac{1}{a}\right) - (s-1)a^2 f\left(\frac{1}{a}\right) + (s-1)(s-2)M[f(t), s-2, 0, \frac{1}{a}] \quad (12)$$

Similarly, $M\left[f'''(t), s, 0, \frac{1}{a}\right] = af''\left(\frac{1}{a}\right) - (s-1)a^2 f'\left(\frac{1}{a}\right) + (s-1)(s-2)a^3 f\left(\frac{1}{a}\right)$
$-(s-1)(s-2)(s-3)M[f(t), s-3, 0, \frac{1}{a}]$ (13)

$$M\left[f^n(t), s, 0, \frac{1}{a}\right] = af^{n-1}\left(\frac{1}{a}\right) - (s-1)a^2 f^{n-2}\left(\frac{1}{a}\right) \ldots\ldots\ldots\ldots - (s-1)(s-2)\ldots\ldots(s-n)M[f(t), s, 0, \frac{1}{a}] \quad (14)$$

6. APPLICATION

Consider Cauchy differential equation $L_2\big(F(t)\big) = t^2 f''(t) + tf'(t) + f(t)$,

$$M\left[tf'(t), s, 0, \frac{1}{a}\right] = \int_0^{1/a} a^s\ t^s f'(t)dt$$

$$= f\left(\frac{1}{a}\right) - sM[f(t), s, 0, \frac{1}{a}]$$

$$M\left[t^2 f''(t), s, 0, \frac{1}{a}\right] = \int_0^{1/a} a^s\ t^{s+1} f''(t)dt$$

$$= \frac{1}{a} f'\left(\frac{1}{a}\right) - (s+1)f\left(\frac{1}{a}\right) + s(s+1)M[f(t), s, 0, \frac{1}{a}]$$

$$M[L_2\big(F(t)\big), s, 0, \frac{1}{a}] = M[t^2 f''(t) + tf'(t) + f(t), s, 0, \frac{1}{a}]$$

$$= \frac{1}{a} f'\left(\frac{1}{a}\right) - sf\left(\frac{1}{a}\right) + (s^2+1)M[f(t), s, 0, \frac{1}{a}]$$

If $L_2\big(F(t)\big) = 0$ then $M[L_2\big(F(t)\big), s, 0, \frac{1}{a}] = 0$

$$\frac{1}{a} f'\left(\frac{1}{a}\right) - sf\left(\frac{1}{a}\right) + (s^2+1)M\left[f(t), s, 0, \frac{1}{a}\right] = 0$$

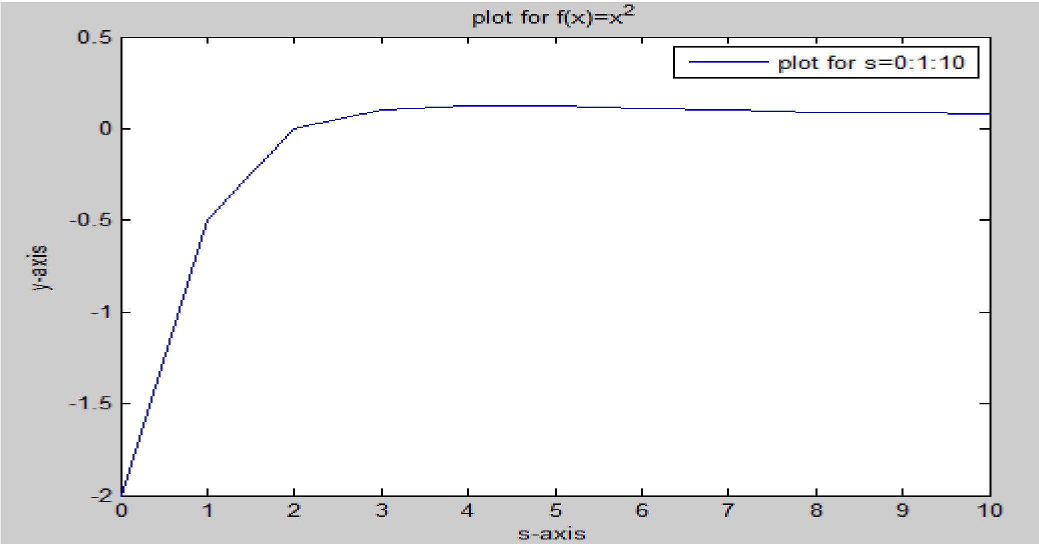
$$M\left[f(t), s, 0, \frac{1}{a}\right] = \frac{1}{s^2+1}[sf\left(\frac{1}{a}\right) - \frac{1}{a} f'\left(\frac{1}{a}\right)] \quad (15)$$

7. GRAPHICAL REPRESENTATION:

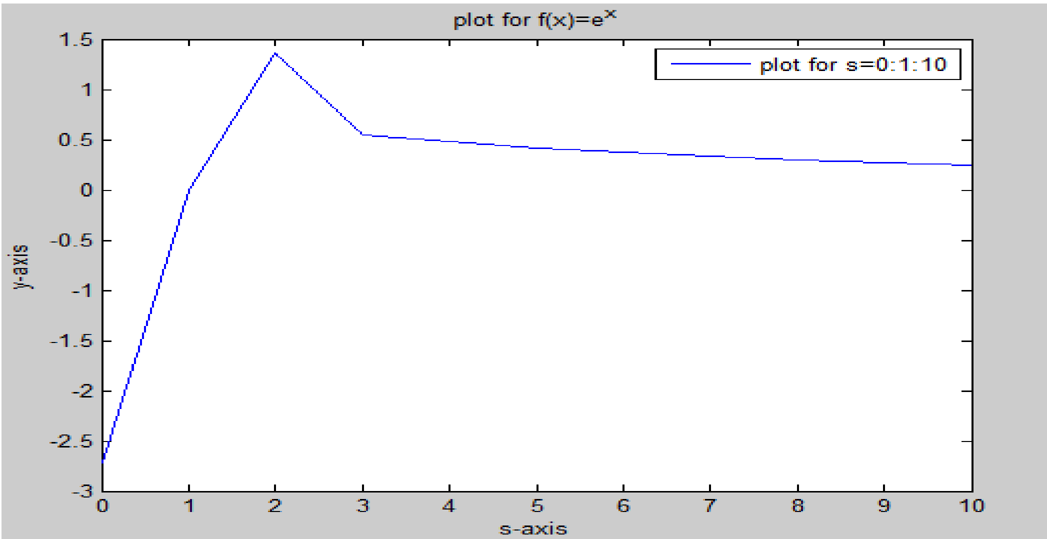
Consider the equation

$$M\left[f(t), s, 0, \frac{1}{a}\right] = \frac{1}{s^2+1}[sf\left(\frac{1}{a}\right) - \frac{1}{a} f'\left(\frac{1}{a}\right)]$$

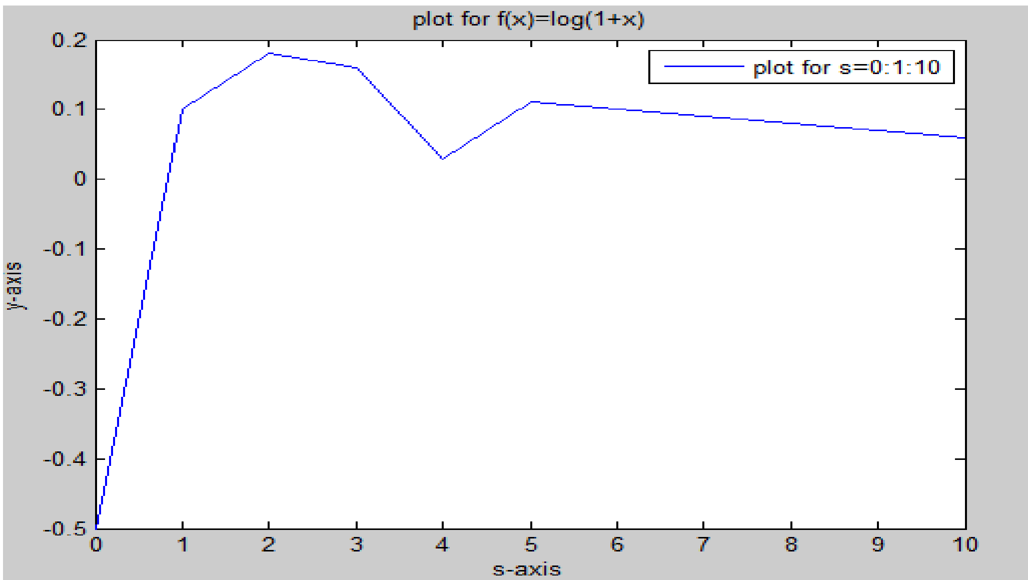
(a) $f(x)=x^2$ then $f(a) = a^2$ and let $a=1$



(b) $f(x)=e^x$ then $f(a)=e^a$ and let $a=1$



(c) $f(x)=\log(1+x)$ then $f(a)=\log(1+a)$ and let $a=1$



8. CONCLUSION:

We have been seen how the properties of the Laplace operator holds good for the Mellin Type Integral Transform in [0,1/a]. The Cauchy Differential Equation is solved and we have given graphical representation of the solution of differential equation by MATLAB. It is observed that Mellin Type Integral Transform in [0,1/a] gives better results than other Integral Transforms.

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